MAT-2453 — Cálculo Diferencial e Integral para Engenharia I — 3ª Prova

POLI – USP — 10/11/2014 — Gabarito

Questão 1 (Valor: 3.5 pontos)**.** Calcule as seguintes integrais:

a. $\displaystyle\int \frac{\ln(x^2-4)}{x^2}\,dx$

b. $\displaystyle\int_0^{\frac{\pi^2}{32}} \frac{\operatorname{arctg}\sqrt{2x}}{(1+2x)\sqrt{2x}}\,dx$

Solução. **a.** Integrando por partes, com $f'(x) = \frac{1}{x^2}$ e $g(x) = \ln(x^2-4)$, temos $f(x) = -\frac{1}{x}$, $g'(x) = \frac{2x}{x^2-4}$ e então

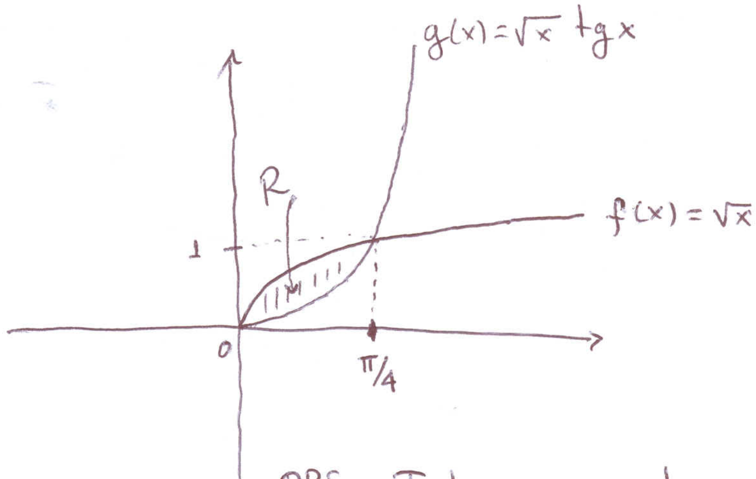
$$\begin{aligned}\int \frac{1}{x^2}\ln(x^2-4)\,dx &= -\frac{1}{x}\ln(x^2-4) + \int \frac{2}{x^2-4}\,dx \\ &= -\frac{1}{x}\ln(x^2-4) - \frac{1}{2}\int \frac{1}{x+2}\,dx + \frac{1}{2}\int \frac{1}{x-2}\,dx \\ &= -\frac{1}{x}\ln(x^2-4) - \frac{1}{2}\ln(x+2) + \frac{1}{2}\ln(x-2) + k.\end{aligned}$$

b. Fazendo $u = \operatorname{arctg}\sqrt{2x}$ temos $du = \frac{dx}{\sqrt{2x}(1+2x)}$, donde

$$\int_0^{\frac{\pi^2}{32}} \frac{\operatorname{arctg}\sqrt{2x}}{(1+2x)\sqrt{2x}}\,dx = \int_0^{\operatorname{arctg}(\frac{\pi}{4})} u\,du = \frac{u^2}{2}\Big|_0^{\operatorname{arctg}(\frac{\pi}{4})} = \frac{\operatorname{arctg}^2(\frac{\pi}{4})}{2}.$$

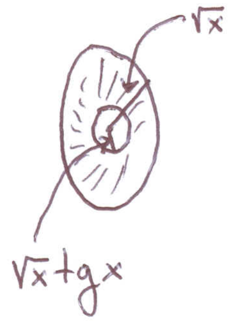
Questão 2, Prova A&B

Volume do solido obtido rotacionando a região limitada por $f(x)=\sqrt{x}$ e $g(x)=\sqrt{x}\,tg\,x$, $x \geq 0$ ao redor do eixo x



OBS: Todas as outras regiões não são limitadas.

Seção Transversal em x



$$A(x) = \pi\left(x - x\,tg^2 x\right)$$

$$\Rightarrow V = \int_0^{\pi/4} \pi\left(x - x\,tg^2 x\right)dx = \frac{\pi x^2}{2}\bigg|_0^{\pi/4} - \pi\int_0^{\pi/4} x\,tg^2 x\,dx$$

$$= \frac{\pi^3}{32} - \pi\int_0^{\pi/4} x\left(\sec^2 x - 1\right)dx = \frac{\pi^3}{32} - \pi\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx + \pi\int_0^{\pi/4} x\,dx =$$

$$= \frac{\pi^3}{32} + \frac{\pi x^2}{2}\bigg|_0^{\pi/4} - \pi\underbrace{\int_0^{\pi/4} x\sec^2 x\,dx}_{\substack{\text{Por partes:}\\ u=x,\ du=dx \\ dv=\sec^2 x\,dx \Rightarrow v=tg\,x}} = \frac{\pi^3}{16} - \pi\,x\,tg\,x\bigg|_0^{\pi/4} + \pi\int_0^{\pi/4} tg\,x\,dx =$$

continua →

$$= \frac{\pi^3}{16} - \frac{\pi^2}{4} + \pi \underbrace{\int_0^{\pi/4} \frac{\operatorname{sen} x}{\cos x}\, dx}_{\text{por substituição}} = \frac{\pi^3}{16} - \frac{\pi^2}{4} - \pi \int_1^{\sqrt{2}/2} \frac{du}{u}$$

$u = \cos x$

$du = -\operatorname{sen} x\, dx$

$$= \frac{\pi^3}{16} - \frac{\pi^2}{4} - \pi \ln|u| \Big|_1^{\sqrt{2}/2}$$

$$= \boxed{\frac{\pi^3}{16} - \frac{\pi^2}{4} - \pi \ln\left(\frac{\sqrt{2}}{2}\right)}$$

Questão 3.

a. Determine o comprimento da curva $y = \ln\left(\sec^5\left(\frac{x}{5}\right)\right)$, para $0 \leq x \leq \frac{5\pi}{4}$.

$$f(x) = \ln\left(\sec^5\left(\tfrac{x}{5}\right)\right) = 5\ln\left(\sec\left(\tfrac{x}{5}\right)\right)$$

$$f'(x) = 5\frac{1}{\sec\left(\frac{x}{5}\right)} \cdot \sec\left(\frac{x}{5}\right)\ \mathrm{tg}\left(\frac{x}{5}\right) \cdot \frac{1}{5} = \ \mathrm{tg}\left(\frac{x}{5}\right)$$

$$C = \int_0^{\frac{5\pi}{4}} \sqrt{1 + \left(\mathrm{tg}\left(\frac{x}{5}\right)\right)^2}\, dx = \int_0^{\frac{5\pi}{4}} \sec\left(\frac{x}{5}\right)\, dx = \left[5\ln\left|\sec\left(\frac{x}{5}\right) + \mathrm{tg}\left(\frac{x}{5}\right)\right|\right]_0^{\frac{5\pi}{4}} =$$

$$= 5\ln\left(\sqrt{2}+1\right) - 5\ln(1) = 5\ln\left(\sqrt{2}+1\right).$$

b. Calcule $F'(x))$, onde $F(x) = \displaystyle\int_0^{\arccos x} \ln\left(\sec^x t\right)\, dt$.

$$F(x) = \int_0^{\arccos x} \ln\left(\sec^x t\right)\, dt = x\int_0^{\arccos x} \ln\left(\sec t\right) dt$$

$$F'(x) = \int_0^{\arccos x} \ln\left(\sec t\right)\, dt + x\ln\left(\sec(\arccos x)\right)\left(\frac{-1}{\sqrt{1-x^2}}\right) =$$

$$= \int_0^{\arccos x} \ln\left(\sec t\right)\, dt + \frac{x\ln x}{\sqrt{1-x^2}}.$$