Uniform Entropy in the Realm of Topological Groups

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[1] Joint research with D. Alcaraz, Universidad Politécnica de Cartagena and D. Dikranjan, Università d'Udine

The outcomes on h-jumps endomorphisms are joint results with D. Alcaraz (Universidad de Cartagena, Spain) and D. Dikranjan (Università di Udine, Italy). The remainder of the results is a joint work with D. Dikranjan (Università di Udine, Italy)

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$$h_T(\alpha,\mathcal{U}) = \lim_{n\to\infty} \frac{\log N\left(\bigwedge_{i=0}^{n-1} \alpha^{-i}(\mathcal{U})\right)}{n}$$

where, as usual, $\bigwedge_{i=0}^{n-1} \alpha^{-i}(\mathcal{U})$ stands for the cover

$$\mathcal{U} \cap \alpha^{-1}(\mathcal{U}) \cap \alpha^{-2}(\mathcal{U}) \cap \cdots\cdots \cap \alpha^{-(n-1)}(\mathcal{U})\,.$$

$N\left(\bigwedge_{i=0}^{n-1} \alpha^{-i}(\mathcal{U})\right) \equiv$ the smallest cardinality of a finite subcover of $\bigwedge_{i=0}^{n-1} \alpha^{-i}(\mathcal{U})$.

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$$(\alpha^j(x), \alpha^j(y)) \in U \quad 0 \leq j < n$$

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Let (G, τ) be a topological group. If $\alpha \colon G \to G$ is a continuous endomorphism, then the uniform entropy of α with respect to the right uniformity on G coincides with the uniform entropy of α with respect to the left uniformity on G.

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Given an integer k, let m_k^G denote the continuous endomorphism

$$m_k^G(x) = kx \quad x \in G\,.$$

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Let G be a compact connected group G such that the commutator of G is finite dimensional. Then G is finite dimensional if and only if every continuous endomorphism has finite entropy.

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- $G = \prod_{n=1}^{\infty} L_n$. (Notice that every continuous automorphism on G is a product of continuous automorphisms of the single components L_n).

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• $bG \equiv$ its Weil completion (*the Bohr compactification of* G)

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- Case 1. G is not bounded torsion.
- Case 2. $G = \bigoplus_{\kappa} \mathbb{Z}(n)$, where κ is an infinite cardinal and $n > 1$.
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A group of the form $G = \prod_p \mathbb{Z}_p^{n_p} \times F_p$, where $n_p \geq 0$ is an integer and F_p is a finite p-group for every prime p, is called an *Orsatti group*.

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An abelian group G is an Orsatti group if and only if the $\mathbb{Z}$-topology on G is compact.

- The $\mathbb{Z}$-topology of G is the group topology of G having as basic neighborhoods of 0 all subgroups of G of the form nG, $n > 0$.

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Every endomorphism on an Orsatti group has zero entropy.

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(2) G/pG is finite for every prime p.

(3) For every prime p, every endomorphism on G/pG has finite entropy.

(4) Every endomorphism on a quotient of G has finite entropy.

(5) There exists a closed subgroup G_1 of G containing $c(G)$, such that $G = G_1 \times \prod_p \mathbb{Z}_p^{n_p}$ and $G_1/c(G) \cong \prod_p F_p$, where n_p is a non-negative integer and F_p is a finite p-group for every prime p.

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In case these conditions hold, G is metrizable.

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(P1) Are the classes (of compact abelian groups in) $\mathfrak{E}$ and $\mathfrak{E}_0$ closed with respect to: (i) taking finite products, and (ii) taking extensions?

(P2) Is there a *metrizable* abelian group which has h-endomorphisms?

(P3) (J. Peters, 1981) Let G be a locally compact abelian group. Is the function

$$\alpha \in Aut(G) \to h_B(\alpha) \in [0, \infty]$$

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Muito obrigado pela sua atenção !!