Resolvability properties of certain topological spaces

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QUESTION. What happens if these properties are relaxed?

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(ii) X is an SD space if every non-isolated point $x \in X$ is an SD limit.

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(i) $D \subset X$ is strongly discrete if there are pairwise disjoint open sets $\{U_x : x \in D\}$ with $x \in U_x$ for $x \in D$.

EXAMPLE: Countable discrete sets in T_3 spaces are SD.

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Every T_1 crowded SD space is ω-resolvable.

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PROBLEM. (Ceder and Pearson, 1967)

Are ω-resolvable spaces maximally resolvable?

[J-S-Sz]

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This left a number of questions open.

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Moreover, filtration spaces determine the resolvability behavior of all MN (or DSD) spaces.

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THEOREM. [J-S-Sz]

If F is an ultrafiltration and $\mu \geq \omega$ is a regular cardinal s.t. $F(t)$ is μ-descendingly complete for all $t \in T = \mathrm{dom}(F)$,

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COROLLARY. [J-S-Sz]

If $\mathcal{F} \in \mathrm{un}(\kappa)$ is a measure and $F(t) = \mathcal{F}$ for all $t \in \mathrm{dom}(F) = \kappa^{<\omega}$ then $X(F)$ is hereditarily ω_1-irresolvable.

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Full λ-filtrations were considered in [J-S-Sz].

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NOTE. For maximal resolvability, the cases $\kappa = \lambda$ are of interest.

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Lemma 1. [J-S-Sz]

If λ is regular, X is DSD with $|X| = \Delta(X) = \lambda$, and there are "dense many" points in X that are not CAPs of any SD set of size λ, then X is λ-resolvable.

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For any $\lambda \geq \omega$, if X is any space s.t. every point in X is the CAP of some SD set of size λ, then there is a full λ-filtration F and a one-one continuous map

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The singular case (proved in [J-M]) is similar but more complicated.

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COROLLARY [J-M]

If every $\mathcal{F} \in \mathrm{un}(\mu)$ is maximally decomposable whenever $\omega \le \mu \le \lambda$, then $X(F)$ is λ-resolvable for any λ-filtration F.