Strong pseudocompact properties

S. Garcia-Ferreira
Coauthors: Y. F. Ortiz-Castillo and A. H. Tomita

Centro de Ciencias Matemáticas
Universidad Nacional Autónoma de México
sgarcia@matmor.unam.mx

Sao Sebastiao, Brazil, 12-16 August 2013

Content

Pseudocompactness

Our spaces will be Tychonoff.

Definition[E. Hewitt, 1948]

A space X is called *pseudocompact* if for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there is $x \in X$ such that $\{n \in \mathbb{N} : V \cap U_n \neq \emptyset\}$ is infinite for all neighborhood V of x.

Pseudocompactness

Our spaces will be Tychonoff.

Definition[E. Hewitt, 1948]

A space X is called *pseudocompact* if for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there is $x \in X$ such that $\{n \in \mathbb{N} : V \cap U_n \neq \emptyset\}$ is infinite for all neighborhood V of x.

Pseudocompactness

Our spaces will be Tychonoff.

Definition[E. Hewitt, 1948]

A space X is called *pseudocompact* if for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there is $x \in X$ such that $\{n \in \mathbb{N} : V \cap U_n \neq \emptyset\}$ is infinite for all neighborhood V of x.

Content

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

The Stone-Čech compactification $\beta(\mathbb{N})$ of the discrete space of natural numbers $\mathbb{N}$ will be identified with the set of all ultrafilters on $\mathbb{N}$ and its the remainder $\mathbb{N}^*$ will be identified with the set of all free ultrafilters on $\mathbb{N}$.

For $A \subseteq \mathbb{N}$, we let $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ and $A^* = \{p \in \mathbb{N}^* : A \in p\}$.

The family $\{\hat{A} : A \subseteq \mathbb{N}\}$ is a base of clopen subsets of $\beta(\mathbb{N})$, and $\{A^* : A \in [\mathbb{N}]^\omega\}$ is a base of clopen subsets of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\bar{f}(q) = p$, where $\bar{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\bar{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

$\beta(\mathbb{N})$

Rudin-Keisler Pre-order

Given two ultrafilters $p, q \in \mathbb{N}^*$, we say that $p \leq_{RK} q$ if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(q) = p$, where $\overline{f} : \beta\mathbb{N} \to \beta\mathbb{N}$ is the Stone extension of f.

Let $p, q \in \mathbb{N}^*$. We say that p and q are *RK-comparable* if either $p \leq_{RK} q$ or $q \leq_{RK} p$. They are *RK-equivalent* if $p \leq_{RK} q$ and $q \leq_{RK} p$ (in symbols, $p \sim q$). It is known that p and q are RK-equivalent iff there is a bijection $f : \mathbb{N} \to \mathbb{N}$ such that $\overline{f}(p) = q$.

For $p \in \mathbb{N}^*$, the set $T(p) = \{q \in \mathbb{N}^* : p \sim q\}$ is called the *type* of p. For each $p \in \mathbb{N}^*$, $T(p)$ will be considered as a subspace of $\mathbb{N}^*$.

Content

p-limit points

Definition[J. Ginsburg and V. Saks, 1975]

Given a space X, $p \in \mathbb{N}^*$ and a sequence $(S_n)_{n\in\mathbb{N}}$ of nonempty subsets of X, we say that $z \in X$ is a p-limit point of $(S_n)_{n\in\mathbb{N}}$, in symbols $z = p - lim_{n\to\infty} S_n$, if $\{n \in \mathbb{N} : S_n \cap U \neq \emptyset\} \in p$ for each neighborhood U of z.

Folklore

If $(x_n)_{n\in\mathbb{N}}$ is a sequence of points of X, the we say that x is a p-limit point of this sequence if $x = p - lim_{n\to\infty}\{x_n\}$. In this case, we simply write $x = p - lim_{n\to\infty} x_n$.

$L(p, (S_n)_{n\in\mathbb{N}})$ will denote the set of p-limit points of the sequence $(S_n)_{n\in\mathbb{N}}$.

p-limit points

Definition[J. Ginsburg and V. Saks, 1975]

Given a space X, $p \in \mathbb{N}^*$ and a sequence $(S_n)_{n\in\mathbb{N}}$ of nonempty subsets of X, we say that $z \in X$ is a p-limit point of $(S_n)_{n\in\mathbb{N}}$, in symbols $z = p - lim_{n\to\infty} S_n$, if $\{n \in \mathbb{N} : S_n \cap U \neq \emptyset\} \in p$ for each neighborhood U of z.

Folklore

If $(x_n)_{n\in\mathbb{N}}$ is a sequence of points of X, the we say that x is a p-limit point of this sequence if $x = p - lim_{n\to\infty}\{x_n\}$. In this case, we simply write $x = p - lim_{n\to\infty} x_n$.

$L(p, (S_n)_{n\in\mathbb{N}})$ will denote the set of p-limit points of the sequence $(S_n)_{n\in\mathbb{N}}$.

p-limit points

Definition[J. Ginsburg and V. Saks, 1975]

Given a space X, $p \in \mathbb{N}^*$ and a sequence $(S_n)_{n\in\mathbb{N}}$ of nonempty subsets of X, we say that $z \in X$ is a p-limit point of $(S_n)_{n\in\mathbb{N}}$, in symbols $z = p - lim_{n\to\infty} S_n$, if $\{n \in \mathbb{N} : S_n \cap U \neq \emptyset\} \in p$ for each neighborhood U of z.

Folklore

If $(x_n)_{n\in\mathbb{N}}$ is a sequence of points of X, the we say that x is a p-limit point of this sequence if $x = p - lim_{n\to\infty}\{x_n\}$. In this case, we simply write $x = p - lim_{n\to\infty} x_n$.

$L(p, (S_n)_{n\in\mathbb{N}})$ will denote the set of p-limit points of the sequence $(S_n)_{n\in\mathbb{N}}$.

p-limit points

Definition[J. Ginsburg and V. Saks, 1975]

Given a space X, $p \in \mathbb{N}^*$ and a sequence $(S_n)_{n\in\mathbb{N}}$ of nonempty subsets of X, we say that $z \in X$ is a p-limit point of $(S_n)_{n\in\mathbb{N}}$, in symbols $z = p - lim_{n\to\infty} S_n$, if $\{n \in \mathbb{N} : S_n \cap U \neq \emptyset\} \in p$ for each neighborhood U of z.

Folklore

If $(x_n)_{n\in\mathbb{N}}$ is a sequence of points of X, the we say that x is a p-limit point of this sequence if $x = p - lim_{n\to\infty}\{x_n\}$. In this case, we simply write $x = p - lim_{n\to\infty} x_n$.

$L(p, (S_n)_{n\in\mathbb{N}})$ will denote the set of p-limit points of the sequence $(S_n)_{n\in\mathbb{N}}$.

p-limit points

Definition[J. Ginsburg and V. Saks, 1975]

Given a space X, $p \in \mathbb{N}^*$ and a sequence $(S_n)_{n\in\mathbb{N}}$ of nonempty subsets of X, we say that $z \in X$ is a p-limit point of $(S_n)_{n\in\mathbb{N}}$, in symbols $z = p - lim_{n\to\infty} S_n$, if $\{n \in \mathbb{N} : S_n \cap U \neq \emptyset\} \in p$ for each neighborhood U of z.

Folklore

If $(x_n)_{n\in\mathbb{N}}$ is a sequence of points of X, the we say that x is a p-limit point of this sequence if $x = p - lim_{n\to\infty}\{x_n\}$. In this case, we simply write $x = p - lim_{n\to\infty} x_n$.

$L(p, (S_n)_{n\in\mathbb{N}})$ will denote the set of p-limit points of the sequence $(S_n)_{n\in\mathbb{N}}$.

Pseudocompactness

Remark

Let $\{x_n : n \in \mathbb{N}\} \subseteq X$. Then $x \in X$ is an accumulation point of $\{x_n : n \in \mathbb{N}\}$ iff there is $p \in \mathbb{N}^*$ such that $x = p - lim_{n \to \infty} x_n$.

Remark

A space X is called pseudocompact iff for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there $p \in \mathbb{N}^*$ such that $L(p, (U_n)_{n \in \mathbb{N}}) \neq \emptyset$.

Pseudocompactness

Remark

Let $\{x_n : n \in \mathbb{N}\} \subseteq X$. Then $x \in X$ is an accumulation point of $\{x_n : n \in \mathbb{N}\}$ iff there is $p \in \mathbb{N}^*$ such that $x = p - lim_{n \to \infty} x_n$.

Remark

A space X is called pseudocompact iff for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there $p \in \mathbb{N}^*$ such that $L(p, (U_n)_{n \in \mathbb{N}}) \neq \emptyset$.

Pseudocompactness

Remark

Let $\{x_n : n \in \mathbb{N}\} \subseteq X$. Then $x \in X$ is an accumulation point of $\{x_n : n \in \mathbb{N}\}$ iff there is $p \in \mathbb{N}^*$ such that $x = p - lim_{n \to \infty} x_n$.

Remark

A space X is called pseudocompact iff for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there $p \in \mathbb{N}^*$ such that $L(p, (U_n)_{n \in \mathbb{N}}) \neq \emptyset$.

Pseudocompactness

Remark

Let $\{x_n : n \in \mathbb{N}\} \subseteq X$. Then $x \in X$ is an accumulation point of $\{x_n : n \in \mathbb{N}\}$ iff there is $p \in \mathbb{N}^*$ such that $x = p - lim_{n\to\infty} x_n$.

Remark

A space X is called pseudocompact iff for every countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty open subset of X there $p \in \mathbb{N}^*$ such that $L(p, (U_n)_{n\in\mathbb{N}}) \neq \emptyset$.

p-pseudocompactness

Definition[J. Ginsburg and V. Saks, 1975]

For $p \in \mathbb{N}^*$, a topological space X is called *p-pseudocompact* if every sequence of nonempty open subsets of X has a p-limit point. A space X is called *ultrapseudocompact* if X is p-pseudocompact for all $p \in \mathbb{N}^*$.

Clearly, for each $p \in \mathbb{N}^*$, p-pseudocompactness $\Rightarrow$ pseudompactness. p-compactness is preserved under arbitrary products, for all $p \in \mathbb{N}^*$.

p-pseudocompactness

Definition[J. Ginsburg and V. Saks, 1975]

For $p \in \mathbb{N}^*$, a topological space X is called *p-pseudocompact* if every sequence of nonempty open subsets of X has a p-limit point. A space X is called *ultrapseudocompact* if X is p-pseudocompact for all $p \in \mathbb{N}^*$.

Clearly, for each $p \in \mathbb{N}^*$, p-pseudocompactness $\Rightarrow$ pseudompactness. p-compactness is preserved under arbitrary products, for all $p \in \mathbb{N}^*$.

p-pseudocompactness

Definition[J. Ginsburg and V. Saks, 1975]

For $p \in \mathbb{N}^*$, a topological space X is called *p-pseudocompact* if every sequence of nonempty open subsets of X has a p-limit point. A space X is called *ultrapseudocompact* if X is p-pseudocompact for all $p \in \mathbb{N}^*$.

Clearly, for each $p \in \mathbb{N}^*$, p-pseudocompactness $\Rightarrow$ pseudompactness. p-compactness is preserved under arbitrary products, for all $p \in \mathbb{N}^*$.

p-pseudocompactness

Definition[J. Ginsburg and V. Saks, 1975]

For $p \in \mathbb{N}^*$, a topological space X is called *p-pseudocompact* if every sequence of nonempty open subsets of X has a p-limit point. A space X is called *ultrapseudocompact* if X is p-pseudocompact for all $p \in \mathbb{N}^*$.

Clearly, for each $p \in \mathbb{N}^*$, p-pseudocompactness $\Rightarrow$ pseudompactness.

p-compactness is preserved under arbitrary products, for all $p \in \mathbb{N}^*$.

p-pseudocompactness

Definition[J. Ginsburg and V. Saks, 1975]

For $p \in \mathbb{N}^*$, a topological space X is called *p-pseudocompact* if every sequence of nonempty open subsets of X has a p-limit point. A space X is called *ultrapseudocompact* if X is p-pseudocompact for all $p \in \mathbb{N}^*$.

Clearly, for each $p \in \mathbb{N}^*$, p-pseudocompactness $\Rightarrow$ pseudompactness. p-compactness is preserved under arbitrary products, for all $p \in \mathbb{N}^*$.

strong p-pseudocompactness

Definition[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarúa, 2013]

for $p \in \mathbb{N}^*$, a space X is called *strongly p-pseudocompact*, if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

Clearly, strongly p-pseudocompactness $\Rightarrow$ p-pseudocompactness, for each $p \in \mathbb{N}^*$. Strongly p-pseudocompactness is preserved under arbitrary products.

strong p-pseudocompactness

Definition[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarúa, 2013]

for $p \in \mathbb{N}^*$, a space X is called *strongly p-pseudocompact*, if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

Clearly, strongly p-pseudocompactness $\Rightarrow$ p-pseudocompactness, for each $p \in \mathbb{N}^*$. Strongly p-pseudocompactness is preserved under arbitrary products.

strong p-pseudocompactness

Definition[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarúa, 2013]

for $p \in \mathbb{N}^*$, a space X is called *strongly p-pseudocompact*, if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

Clearly, strongly p-pseudocompactness $\Rightarrow$ p-pseudocompactness, for each $p \in \mathbb{N}^*$. Strongly p-pseudocompactness is preserved under arbitrary products.

strong p-pseudocompactness

Definition[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarúa, 2013]

for $p \in \mathbb{N}^*$, a space X is called *strongly p-pseudocompact*, if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

Clearly, strongly p-pseudocompactness $\Rightarrow$ p-pseudocompactness, for each $p \in \mathbb{N}^*$. Strongly p-pseudocompactness is preserved under arbitrary products.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

p-pseudocompactness

Given a space X, $K(X)$ denotes the hyperspace of nonempty compact subsets of X with the Vietories topoloy.

Theorem[J. Angoa, Y. F. Ortiz-Castillo and A. Tamariz-Mascarua, 2013]

For a topological space X, the following statements are equivalent:

- X is pseudo-ω-bounded,
- $K(X)$ is pseudocompact,
- $K(X)$ is p-pseudocompact for some $p \in \mathbb{N}^*$, and
- $K(X)$ is strongly p-pseudocompact for some $p \in \mathbb{N}^*$.

Strong pseudocompactness

Definition[Salvador-Yasser]

A space X is called *strongly pseudocompact* if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are $p \in \mathbb{N}^*$, a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

It is easy to check that the strong pseudocompactness of $K(X)$ is equivalent to any statement of the previous theorem.

There is an ultrapseudocompact, non-strongly pseudocompact space.

Strong pseudocompactness

Definition[Salvador-Yasser]

A space X is called *strongly pseudocompact* if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are $p \in \mathbb{N}^*$, a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

It is easy to check that the strong pseudocompactness of $K(X)$ is equivalent to any statement of the previous theorem.

There is an ultrapseudocompact, non-strongly pseudocompact space.

Strong pseudocompactness

Definition[Salvador-Yasser]

A space X is called *strongly pseudocompact* if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are $p \in \mathbb{N}^*$, a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

It is easy to check that the strong pseudocompactness of $K(X)$ is equivalent to any statement of the previous theorem.

There is an ultrapseudocompact, non-strongly pseudocompact space.

Strong pseudocompactness

Definition[Salvador-Yasser]

A space X is called *strongly pseudocompact* if for each sequence $(U_n)_{n\in\mathbb{N}}$ of nonempty open subsets of X there are $p \in \mathbb{N}^*$, a sequence $(x_n)_{n\in\mathbb{N}}$ of points in X and $x \in X$ such that $x = p - lim_{n\to\infty} x_n$ and $x_n \in U_n$ for all $n \in \mathbb{N}$.

It is easy to check that the strong pseudocompactness of $K(X)$ is equivalent to any statement of the previous theorem.

There is an ultrapseudocompact, non-strongly pseudocompact space.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup\{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup\{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup\{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup\{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup\{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup\{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup \{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup \{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup\{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup\{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Ultrapseudocompactness

Theorem[P. Simon, 1985]

There are $2^{\mathfrak{c}}$ weak P-points of $\mathbb{N}^*$ pairwise RK-incomparable.

Construction

Pick three RK-incomparable weak P-points q, r and t of $\mathbb{N}^*$ and fix an ω-partition $\{A_n : n \in \mathbb{N}\}$ of $\mathbb{N}$. Put $Z = \cup_{n\in\mathbb{N}} A_n^*$ and $Y = Fr(Z)$. Let us consider the following sets:

$$Q = \bigcup\{\overline{\{q_n : n \in \mathbb{N}\}} \cap Y : q_n \in T(q) \cap Z)\},$$

and, for each $i \in \mathbb{N}$,

$$R_i = \bigcup\{\overline{\{r_n : n \in \mathbb{N}\}} \setminus \{r_n : n \in \mathbb{N}\} : \{r_n : n \in \mathbb{N}\} \in [A_i^* \cap T(r)]^\omega\}$$

and $C_i = [A_i^* \cap T(t)] \cup R_i$. Our space will be $X = \left[\bigcup_{n\in\mathbb{N}} C_n\right] \cup Q$.

Strong pseudocompactness

Countable compactness $\Rightarrow$ strong pseudocompactness $\Rightarrow$ pseudocompactness.

If $p \in \mathbb{N}^*$ is a weak P-point, then $T(p)$ is a pseudocompact space that is not strongly pseudocompact since every countable subset is discrete.

Strong pseudocompactness

Countable compactness $\Rightarrow$ strong pseudocompactness $\Rightarrow$ pseudocompactness.

If $p \in \mathbb{N}^*$ is a weak P-point, then $T(p)$ is a pseudocompact space that is not strongly pseudocompact since every countable subset is discrete.

Strong pseudocompactness

Countable compactness $\Rightarrow$ strong pseudocompactness $\Rightarrow$ pseudocompactness.

If $p \in \mathbb{N}^*$ is a weak P-point, then $T(p)$ is a pseudocompact space that is not strongly pseudocompact since every countable subset is discrete.

Strong pseudocompactness

Countable compactness $\Rightarrow$ strong pseudocompactness $\Rightarrow$ pseudocompactness.

If $p \in \mathbb{N}^*$ is a weak P-point, then $T(p)$ is a pseudocompact space that is not strongly pseudocompact since every countable subset is discrete.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \vec{1}$ and $\vec{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \overrightarrow{1}$ and $\overrightarrow{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\bigcup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\bigcup_{n \leq k} A_k} \longrightarrow \vec{1}$ and $\vec{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \overrightarrow{1}$ and $\overrightarrow{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \overrightarrow{1}$ and $\overrightarrow{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \vec{1}$ and $\vec{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

A strongly pseudocompact group that is not countably compact.

For each $x \in \{0,1\}^{\mathfrak{c}}$, we define $sup(x) = \{\xi < \mathfrak{c} : x(\xi) \neq 0\}$ and let $\chi_A : \mathfrak{c} \to \{0,1\}$ be the characteristic function of $A \subseteq \mathfrak{c}$. Fix a partition $\{P_n : n \in \mathbb{N}\}$ of $\mathfrak{c}$ in subsets of size $\mathfrak{c}$. Consider the subgroup G of $\{0,1\}^{\mathfrak{c}}$ generated by the set
$\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\} \cup \{\chi_{\cup_{n \leq k} A_k} : n \in \mathbb{N}\}$.

Since $\{x \in \{0,1\}^{\mathfrak{c}} : |sup(x)| \leq \omega\}$ is ω-bounded and dense in G, G is strongly pseudocompact. As $\chi_{\cup_{n \leq k} A_k} \longrightarrow \overrightarrow{1}$ and $\overrightarrow{1} \notin G$, G cannot be countably compact.

Strong pseudocompactness

For each $p \in \mathbb{N}$, strong p-pseudocompactness $\Rightarrow$ strong pseudocompactness.

A countable compact space whose square is not pseudocompact is an example of a strongly pseudocompact space that is not p-pseudocompact for any $p \in \mathbb{N}^*$. In 1967, for each $1 < n \in \mathbb{N}$, Z. Frolík constructed a space X such that X^n is countably compact and X^{n+1} is not pseudocompact.

There is another strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$ constructed by using RF-order on $\mathbb{N}^*$.

Strong pseudocompactness

For each $p \in \mathbb{N}$, strong p-pseudocompactness $\Rightarrow$ strong pseudocompactness.

A countable compact space whose square is not pseudocompact is an example of a strongly pseudocompact space that is not p-pseudocompact for any $p \in \mathbb{N}^*$. In 1967, for each $1 < n \in \mathbb{N}$, Z. Frolík constructed a space X such that X^n is countably compact and X^{n+1} is not pseudocompact.

There is another strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$ constructed by using RF-order on $\mathbb{N}^*$.

Strong pseudocompactness

For each $p \in \mathbb{N}$, strong p-pseudocompactness $\Rightarrow$ strong pseudocompactness.

A countable compact space whose square is not pseudocompact is an example of a strongly pseudocompact space that is not p-pseudocompact for any $p \in \mathbb{N}^*$. In 1967, for each $1 < n \in \mathbb{N}$, Z. Frolík constructed a space X such that X^n is countably compact and X^{n+1} is not pseudocompact.

There is another strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$ constructed by using RF-order on $\mathbb{N}^*$.

Strong pseudocompactness

For each $p \in \mathbb{N}$, strong p-pseudocompactness $\Rightarrow$ strong pseudocompactness.

A countable compact space whose square is not pseudocompact is an example of a strongly pseudocompact space that is not p-pseudocompact for any $p \in \mathbb{N}^*$. In 1967, for each $1 < n \in \mathbb{N}$, Z. Frolík constructed a space X such that X^n is countably compact and X^{n+1} is not pseudocompact.

There is another strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$ constructed by using RF-order on $\mathbb{N}^*$.

Strong pseudocompactness

For each $p \in \mathbb{N}$, strong p-pseudocompactness $\Rightarrow$ strong pseudocompactness.

A countable compact space whose square is not pseudocompact is an example of a strongly pseudocompact space that is not p-pseudocompact for any $p \in \mathbb{N}^*$. In 1967, for each $1 < n \in \mathbb{N}$, Z. Frolík constructed a space X such that X^n is countably compact and X^{n+1} is not pseudocompact.

There is another strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$ constructed by using RF-order on $\mathbb{N}^*$.

Rudin-Frolík order

Definition(Rudin-Frolík order)

For each $p, q \in \mathbb{N}$, we say that $p <_{RF} q$ if there is an embedding $f : \mathbb{N} \to \mathbb{N}^*$ such that $\bar{f}(p) = q$.

Frolík proved that $\bar{f}(p) \not\sim p$ for every $p \in \mathbb{N}$ and every embedding $f : \mathbb{N} \to \mathbb{N}^*$.

For each $p, q \in \mathbb{N}$, we say that $p \leq_{RF} q$ if either $p <_{RF} q$ or $p \sim q$. We know that $\leq_{RF} \Rightarrow \leq_{RK}$ and they are different.

Rudin-Frolík order

Definition(Rudin-Frolík order)

For each $p, q \in \mathbb{N}$, we say that $p <_{RF} q$ if there is an embedding $f : \mathbb{N} \to \mathbb{N}^*$ such that $\overline{f}(p) = q$.

Frolík proved that $\overline{f}(p) \not\sim p$ for every $p \in \mathbb{N}$ and every embedding $f : \mathbb{N} \to \mathbb{N}^*$.

For each $p, q \in \mathbb{N}$, we say that $p \leq_{RF} q$ if either $p <_{RF} q$ or $p \sim q$. We know that $\leq_{RF} \Rightarrow \leq_{RK}$ and they are different.

Rudin-Frolík order

Definition(Rudin-Frolík order)

For each $p, q \in \mathbb{N}$, we say that $p <_{RF} q$ if there is an embedding $f : \mathbb{N} \to \mathbb{N}^*$ such that $\bar{f}(p) = q$.

Frolík proved that $\bar{f}(p) \not\sim p$ for every $p \in \mathbb{N}$ and every embedding $f : \mathbb{N} \to \mathbb{N}^*$.

For each $p, q \in \mathbb{N}$, we say that $p \leq_{RF} q$ if either $p <_{RF} q$ or $p \sim q$. We know that $\leq_{RF} \Rightarrow \leq_{RK}$ and they are different.

Rudin-Frolík order

Definition(Rudin-Frolík order)

For each $p, q \in \mathbb{N}$, we say that $p <_{RF} q$ if there is an embedding $f : \mathbb{N} \to \mathbb{N}^*$ such that $\overline{f}(p) = q$.

Frolík proved that $\overline{f}(p) \not\sim p$ for every $p \in \mathbb{N}$ and every embedding $f : \mathbb{N} \to \mathbb{N}^*$.

For each $p, q \in \mathbb{N}$, we say that $p \leq_{RF} q$ if either $p <_{RF} q$ or $p \sim q$. We know that $\leq_{RF} \Rightarrow \leq_{RK}$ and they are different.

Rudin-Frolík order

Definition(Rudin-Frolík order)

For each $p, q \in \mathbb{N}$, we say that $p <_{RF} q$ if there is an embedding $f : \mathbb{N} \to \mathbb{N}^*$ such that $\overline{f}(p) = q$.

Frolík proved that $\overline{f}(p) \not\sim p$ for every $p \in \mathbb{N}$ and every embedding $f : \mathbb{N} \to \mathbb{N}^*$.

For each $p, q \in \mathbb{N}$, we say that $p \leq_{RF} q$ if either $p <_{RF} q$ or $p \sim q$. We know that $\leq_{RF} \Rightarrow \leq_{RK}$ and they are different.

Rudin-Frolík order

Lemma

The $\leq_{RF}$ satisfies the following properties.

- The weak P-points of $\mathbb{N}^*$ are $\leq_{RF}$-minimal.
- For every $p \in \mathbb{N}^*$, the RF-predecessors of p are linearly ordered by $\leq_{RF}$.

Rudin-Frolík order

Lemma

The $\leq_{RF}$ satisfies the following properties.

- The weak P-points of $\mathbb{N}^*$ are $\leq_{RF}$-minimal.
- For every $p \in \mathbb{N}^*$, the RF-predecessors of p are linearly ordered by $\leq_{RF}$.

Rudin-Frolík order

Lemma

The $\leq_{RF}$ satisfies the following properties.

- The weak P-points of $\mathbb{N}^*$ are $\leq_{RF}$-minimal.
- For every $p \in \mathbb{N}^*$, the RF-predecessors of p are linearly ordered by $\leq_{RF}$.

Rudin-Frolík order

Lemma

The $\leq_{RF}$ satisfies the following properties.

- The weak P-points of $\mathbb{N}^*$ are $\leq_{RF}$-minimal.
- For every $p \in \mathbb{N}^*$, the RF-predecessors of p are linearly ordered by $\leq_{RF}$.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^*(s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^* (s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^*(s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^*(s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^*(s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong pseudocompactness

Definition

For each $p \in \mathbb{N}^*$, we define

$$B_p = \{q \in \mathbb{N}^* : \exists s \in \mathbb{N}^*(s \leq_{RF} q \wedge s \leq_{RF} p) \vee (p \leq_{RF} q \vee q \leq_{RF} p)\}.$$

and $X_p = \mathbb{N}^* \setminus B_p$. Observe that $p \notin X_p$ for every $p \in \mathbb{N}^*$.

Lemma

If $p, q \in \mathbb{N}^*$, then X_p is strongly q-pseudocompact iff $q \notin B_p$. In particular, X_p cannot be strongly p-pseudocompact.

Lemma

For every $p \in \mathbb{N}^*$ there is $t \in \mathbb{N}^*$ such that X_p is strongly t-pseudocompact.

Strong peudocompactness

Example

If X is the countably compactification of the disjoint topological sum $\sqcup_{p \in \mathbb{N}^*} X_p$ by adding just one point, then X is a strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$.

Strong peudocompactness

Example

If X is the countably compactification of the disjoint topological sum $\sqcup_{p \in \mathbb{N}^*} X_p$ by adding just one point, then X is a strongly pseudocompact space that is not strongly p-pseudocompact for any $p \in \mathbb{N}^*$.

Content

Weak P-points

Theorem

The space of weak P-points of $\mathbb{N}^*$ is ultrapseudocompact and non-strongly pseudocompact

Notation

Let $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ be an ω-partition of $\mathbb{N}$ and let $p \in \mathbb{N}^*$. We define

$$\mathcal{F}(\mathcal{P}, p) = \{\bigcup_{n\in A}(P_n \setminus F_n) : A \in p \text{ and } \forall n \in A(F_n \in [\mathbb{N}]^{<\omega})\}.$$

It is easy to check that $\mathcal{F}(\mathcal{P}, p)$ is a filter base. We write $\mathcal{F}_p$ instead of $\mathcal{F}(\mathcal{P}, p)$ when a confusion is not possible.

Weak P-points

Theorem

The space of weak P-points of $\mathbb{N}^*$ is ultrapseudocompact and non-strongly pseudocompact

Notation

Let $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ be an ω-partition of $\mathbb{N}$ and let $p \in \mathbb{N}^*$. We define

$$\mathcal{F}(\mathcal{P}, p) = \{\bigcup_{n \in A}(P_n \setminus F_n) : A \in p \text{ and } \forall n \in A(F_n \in [\mathbb{N}]^{<\omega})\}.$$

It is easy to check that $\mathcal{F}(\mathcal{P}, p)$ is a filter base. We write $\mathcal{F}_p$ instead of $\mathcal{F}(\mathcal{P}, p)$ when a confusion is not possible.

Weak P-points

Theorem

The space of weak P-points of $\mathbb{N}^*$ is ultrapseudocompact and non-strongly pseudocompact

Notation

Let $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ be an ω-partition of $\mathbb{N}$ and let $p \in \mathbb{N}^*$. We define

$$\mathcal{F}(\mathcal{P}, p) = \{\bigcup_{n \in A}(P_n \setminus F_n) : A \in p \text{ and } \forall n \in A(F_n \in [\mathbb{N}]^{<\omega})\}.$$

It is easy to check that $\mathcal{F}(\mathcal{P}, p)$ is a filter base. We write $\mathcal{F}_p$ instead of $\mathcal{F}(\mathcal{P}, p)$ when a confusion is not possible.

Weak P-points

Theorem

The space of weak P-points of $\mathbb{N}^*$ is ultrapseudocompact and non-strongly pseudocompact

Notation

Let $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ be an ω-partition of $\mathbb{N}$ and let $p \in \mathbb{N}^*$. We define

$$\mathcal{F}(\mathcal{P}, p) = \{\bigcup_{n \in A}(P_n \setminus F_n) : A \in p \text{ and } \forall n \in A(F_n \in [\mathbb{N}]^{<\omega})\}.$$

It is easy to check that $\mathcal{F}(\mathcal{P}, p)$ is a filter base. We write $\mathcal{F}_p$ instead of $\mathcal{F}(\mathcal{P}, p)$ when a confusion is not possible.

Weak P-points

Theorem

The space of weak P-points of $\mathbb{N}^*$ is ultrapseudocompact and non-strongly pseudocompact

Notation

Let $\mathcal{P} = \{P_n : n \in \mathbb{N}\}$ be an ω-partition of $\mathbb{N}$ and let $p \in \mathbb{N}^*$. We define

$$\mathcal{F}(\mathcal{P}, p) = \{\bigcup_{n \in A}(P_n \setminus F_n) : A \in p \text{ and } \forall n \in A(F_n \in [\mathbb{N}]^{<\omega})\}.$$

It is easy to check that $\mathcal{F}(\mathcal{P}, p)$ is a filter base. We write $\mathcal{F}_p$ instead of $\mathcal{F}(\mathcal{P}, p)$ when a confusion is not possible.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \overline{f}[\mathbb{N}^*]$, $\overline{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \overline{f}[\mathbb{N}^*]$, $\overline{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \bar{f}[\mathbb{N}^*]$, $\bar{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \overline{f}[\mathbb{N}^*]$, $\overline{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \overline{f}[\mathbb{N}^*]$, $\overline{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Theorem-I

Let $A \subseteq \mathbb{N}^*$. Then:

- A is ultrapseudocompact iff for every $p \in \mathbb{N}^*$ and every ω-partition $\mathcal{P}$ of $\mathbb{N}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$.
- If $A = T(A)$, for every $p \in \mathbb{N}^*$ there exists $q \in A$ such that $p \leq_{RK} q$ iff for every function with infinite fibers $f : \mathbb{N} \to \mathbb{N}$ and every $p \in \overline{f}[\mathbb{N}^*]$, $\overline{f}^{-1}(p) \cap A \neq \emptyset$.
- If A is ultrapseudocompact, then for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$.

Weak P-points

Remark

The first clause of previous theorem is very important since to prove that a subset $A \subseteq \mathbb{N}^*$ is ultrapseudocompact is enough to check the condition:
"for every $p \in \mathbb{N}^*$ and every ω-partition $\{P_n : n \in \mathbb{N}\}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$".

Unfortunately, we do not know whether or not this last conditions is equivalent to the statement
"for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$".

Theorem[J. van Mill, 1984]

There is a finite-to-one function $f : \mathbb{N} \to \mathbb{N}$ such that for all $p \in \mathbb{N}^*$ there is a weak P-point q in $\mathbb{N}^*$ such that $p \leq_{RK} q$ via f.

Weak P-points

Remark

The first clause of previous theorem is very important since to prove that a subset $A \subseteq \mathbb{N}^*$ is ultrapseudocompact is enough to check the condition:
"for every $p \in \mathbb{N}^*$ and every ω-partition $\{P_n : n \in \mathbb{N}\}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$".

Unfortunately, we do not know whether or not this last conditions is equivalent to the statement
"for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$".

Theorem[J. van Mill, 1984]

There is a finite-to-one function $f : \mathbb{N} \to \mathbb{N}$ such that for all $p \in \mathbb{N}^*$ there is a weak P-point q in $\mathbb{N}^*$ such that $p \leq_{RK} q$ via f.

Weak P-points

Remark

The first clause of previous theorem is very important since to prove that a subset $A \subseteq \mathbb{N}^*$ is ultrapseudocompact is enough to check the condition:
"for every $p \in \mathbb{N}^*$ and every ω-partition $\{P_n : n \in \mathbb{N}\}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$".

Unfortunately, we do not know whether or not this last conditions is equivalent to the statement
"for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$".

Theorem[J. van Mill, 1984]

There is a finite-to-one function $f : \mathbb{N} \to \mathbb{N}$ such that for all $p \in \mathbb{N}^*$ there is a weak P-point q in $\mathbb{N}^*$ such that $p \leq_{RK} q$ via f.

Weak P-points

Remark

The first clause of previous theorem is very important since to prove that a subset $A \subseteq \mathbb{N}^*$ is ultrapseudocompact is enough to check the condition:
"for every $p \in \mathbb{N}^*$ and every ω-partition $\{P_n : n \in \mathbb{N}\}$, there exists $q \in A$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$".

Unfortunately, we do not know whether or not this last conditions is equivalent to the statement
"for every $p \in \mathbb{N}^*$, there exists $q \in A$ such that $p \leq_{RK} q$".

Theorem[J. van Mill, 1984]

There is a finite-to-one function $f : \mathbb{N} \to \mathbb{N}$ such that for all $p \in \mathbb{N}^*$ there is a weak P-point q in $\mathbb{N}^*$ such that $p \leq_{RK} q$ via f.

Weak P-points

Theorem-II

For every ω-partition $\{P_n : n \in \mathbb{N}\}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$ there is a weak P-point point q such that $\mathcal{F}_p \subseteq q$.

Weak P-points

Theorem-II

For every ω-partition $\{P_n : n \in \mathbb{N}\}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$ there is a weak P-point point q such that $\mathcal{F}_p \subseteq q$.

Weak P-points

Corollary

The weak P-points of $\mathbb{N}$ is an ultrapseudocompact non-strongly pseudocompact space.

Proof

Obviously, X cannot be strongly pseudocompact. By Theorem II, for every ω-partition $\mathcal{P}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$, there exists $q \in X$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$. It then follows from Theorem I that X is ultrapseudocompact.

Weak P-points

Corollary

The weak P-points of $\mathbb{N}$ is an ultrapseudocompact non-strongly pseudocompact space.

Proof

Obviously, X cannot be strongly pseudocompact. By Theorem II, for every ω-partition $\mathcal{P}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$, there exists $q \in X$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$. It then follows from Theorem I that X is ultrapseudocompact.

Weak P-points

Corollary

The weak P-points of $\mathbb{N}$ is an ultrapseudocompact non-strongly pseudocompact space.

Proof

Obviously, X cannot be strongly pseudocompact. By Theorem II, for every ω-partition $\mathcal{P}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$, there exists $q \in X$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$. It then follows from Theorem I that X is ultrapseudocompact.

Weak P-points

Corollary

The weak P-points of $\mathbb{N}$ is an ultrapseudocompact non-strongly pseudocompact space.

Proof

Obviously, X cannot be strongly pseudocompact. By Theorem II, for every ω-partition $\mathcal{P}$ of $\mathbb{N}$ and every $p \in \mathbb{N}^*$, there exists $q \in X$ such that $\mathcal{F}(\mathcal{P}, p) \subseteq q$. It then follows from Theorem I that X is ultrapseudocompact.

Content

Pseudocompact groups

Theorem[Comfort-Ross, 1966]

The product of pseudocompact topological groups is pseudocompact.

Theorem[Protasov, 1994]

Every infinite totally bounded group contains a nonclosed discrete subset.

Definition

A topological group is *totally bounded* if for every neighborhood V of e there is $F \in [G]^{<\omega}$ such that $G = VF$. Every pseudocompact group is totally bounded.

Pseudocompact groups

Theorem[Comfort-Ross, 1966]

The product of pseudocompact topological groups is pseudocompact.

Theorem[Protasov, 1994]

Every infinite totally bounded group contains a nonclosed discrete subset.

Definition

A topological group is *totally bounded* if for every neighborhood V of e there is $F \in [G]^{<\omega}$ such that $G = VF$. Every pseudocompact group is totally bounded.

Pseudocompact groups

Theorem[Comfort-Ross, 1966]

The product of pseudocompact topological groups is pseudocompact.

Theorem[Protasov, 1994]

Every infinite totally bounded group contains a nonclosed discrete subset.

Definition

A topological group is *totally bounded* if for every neighborhood V of e there is $F \in [G]^{<\omega}$ such that $G = VF$. Every pseudocompact group is totally bounded.

Pseudocompact groups

Theorem[Comfort-Ross, 1966]

The product of pseudocompact topological groups is pseudocompact.

Theorem[Protasov, 1994]

Every infinite totally bounded group contains a nonclosed discrete subset.

Definition

A topological group is *totally bounded* if for every neighborhood V of e there is $F \in [G]^{<\omega}$ such that $G = VF$. Every pseudocompact group is totally bounded.

Pseudocompact groups

Theorem[Comfort-Ross, 1966]

The product of pseudocompact topological groups is pseudocompact.

Theorem[Protasov, 1994]

Every infinite totally bounded group contains a nonclosed discrete subset.

Definition

A topological group is *totally bounded* if for every neighborhood V of e there is $F \in [G]^{<\omega}$ such that $G = VF$. Every pseudocompact group is totally bounded.

Stronger pseudocompactness

Definition

We say that a space X is:

- D-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $\overline{D} \setminus (\bigcup_{n\in\mathbb{N}} \overline{U_n}) \neq \emptyset$ and $U_n \cap D \neq \emptyset$ for infinitely many $n \in \mathbb{N}$.
- F-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a non-closed discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $|U_n \cap D| < \omega$ for all $n \in \mathbb{N}$.

Stronger pseudocompactness

Definition

We say that a space X is:

- D-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $\overline{D} \setminus \left(\bigcup_{n\in\mathbb{N}} \overline{U_n}\right) \neq \emptyset$ and $U_n \cap D \neq \emptyset$ for infinitely many $n \in \mathbb{N}$.
- F-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a non-closed discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $|U_n \cap D| < \omega$ for all $n \in \mathbb{N}$.

Stronger pseudocompactness

Definition

We say that a space X is:

- D-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $\overline{D} \setminus \left(\bigcup_{n\in\mathbb{N}} \overline{U_n}\right) \neq \emptyset$ and $U_n \cap D \neq \emptyset$ for infinitely many $n \in \mathbb{N}$.
- F-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a non-closed discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $|U_n \cap D| < \omega$ for all $n \in \mathbb{N}$.

Stronger pseudocompactness

Definition

We say that a space X is:

- D-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $\overline{D} \setminus \left(\bigcup_{n\in\mathbb{N}} \overline{U_n}\right) \neq \emptyset$ and $U_n \cap D \neq \emptyset$ for infinitely many $n \in \mathbb{N}$.
- F-pseudocompact if for every infinite countable family $\{U_n : n \in \mathbb{N}\}$ of nonempty pairwise disjoint open subsets, there is a non-closed discrete set $D \subseteq \bigcup_{n\in\mathbb{N}} U_n$ such that $|U_n \cap D| < \omega$ for all $n \in \mathbb{N}$.

Strong pseudocompactness

Stronlgy pseudocompact $\Rightarrow$ F-pseudocompact $\Rightarrow$ D-pseudocompact.

Strong pseudocompactness

Stronlgy pseudocompact $\Rightarrow$ F-pseudocompact $\Rightarrow$ D-pseudocompact.

D-pseudocompactness

Example

There is a D-pseudocompact space X that is not F-pseudocompact.

Fix $p \in \mathbb{N}^*$ and let $\{A_n : n \in \mathbb{N}\}$ be a partition of $\mathbb{N}$ in infinite sets. Let $f : \omega \to T(p)$ be an embedding and set $\bar{f}(p) = p^2$. Now let $g : \omega \to T(p^2)$ be an embedding and set $\bar{g}(p) = p^3$. Define

$$B_n = T(p) \cap A_n^* \text{ for each } n \in \mathbb{N},$$

$$C_n = T(p^2) \cap A_n^* \text{ for each } n \in \mathbb{N}, \text{ and}$$

$$R = Fr(\bigcup_{n \in \mathbb{N}} A_n^*) \setminus (T(p^2) \cup T(p^3)).$$

The space is

$$X = R \cup (\bigcup_{n \in \mathbb{N}} (B_n \cup C_n)).$$

D-pseudocompactness

Example

There is a D-pseudocompact space X that is not F-pseudocompact.

Fix $p \in \mathbb{N}^*$ and let $\{A_n : n \in \mathbb{N}\}$ be a partition of $\mathbb{N}$ in infinite sets. Let $f : \omega \to T(p)$ be an embedding and set $\bar{f}(p) = p^2$. Now let $g : \omega \to T(p^2)$ be an embedding and set $\bar{g}(p) = p^3$. Define

$$B_n = T(p) \cap A_n^* \text{ for each } n \in \mathbb{N},$$

$$C_n = T(p^2) \cap A_n^* \text{ for each } n \in \mathbb{N}, \text{ and}$$

$$R = Fr(\bigcup_{n \in \mathbb{N}} A_n^*) \setminus (T(p^2) \cup T(p^3)).$$

The space is

$$X = R \cup (\bigcup_{n \in \mathbb{N}} (B_n \cup C_n)).$$

D-pseudocompactness

Example

There is a D-pseudocompact space X that is not F-pseudocompact.

Fix $p \in \mathbb{N}^*$ and let $\{A_n : n \in \mathbb{N}\}$ be a partition of $\mathbb{N}$ in infinite sets. Let $f : \omega \to T(p)$ be an embedding and set $\bar{f}(p) = p^2$. Now let $g : \omega \to T(p^2)$ be an embedding and set $\bar{g}(p) = p^3$. Define

$$B_n = T(p) \cap A_n^* \text{ for each } n \in \mathbb{N},$$

$$C_n = T(p^2) \cap A_n^* \text{ for each } n \in \mathbb{N}, \text{ and}$$

$$R = Fr(\bigcup_{n \in \mathbb{N}} A_n^*) \setminus (T(p^2) \cup T(p^3)).$$

The space is

$$X = R \cup (\bigcup_{n \in \mathbb{N}} (B_n \cup C_n)).$$

D-pseudocompactness

Example

There is a D-pseudocompact space X that is not F-pseudocompact.

Fix $p \in \mathbb{N}^*$ and let $\{A_n : n \in \mathbb{N}\}$ be a partition of $\mathbb{N}$ in infinite sets. Let $f : \omega \to T(p)$ be an embedding and set $\bar{f}(p) = p^2$. Now let $g : \omega \to T(p^2)$ be an embedding and set $\bar{g}(p) = p^3$. Define

$$B_n = T(p) \cap A_n^* \text{ for each } n \in \mathbb{N},$$

$$C_n = T(p^2) \cap A_n^* \text{ for each } n \in \mathbb{N}, \text{ and}$$

$$R = Fr(\bigcup_{n\in\mathbb{N}} A_n^*) \setminus (T(p^2) \cup T(p^3)).$$

The space is

$$X = R \cup (\bigcup_{n\in\mathbb{N}} (B_n \cup C_n)).$$

D-pseudocompactness

Example

There is a D-pseudocompact space X that is not F-pseudocompact.

Fix $p \in \mathbb{N}^*$ and let $\{A_n : n \in \mathbb{N}\}$ be a partition of $\mathbb{N}$ in infinite sets. Let $f : \omega \to T(p)$ be an embedding and set $\bar{f}(p) = p^2$. Now let $g : \omega \to T(p^2)$ be an embedding and set $\bar{g}(p) = p^3$. Define

$$B_n = T(p) \cap A_n^* \text{ for each } n \in \mathbb{N},$$

$$C_n = T(p^2) \cap A_n^* \text{ for each } n \in \mathbb{N}, \text{ and}$$

$$R = Fr(\bigcup_{n\in\mathbb{N}} A_n^*) \setminus (T(p^2) \cup T(p^3)).$$

The space is

$$X = R \cup (\bigcup_{n\in\mathbb{N}} (B_n \cup C_n).$$

D-pseudocompactness

Question

Is there an F-pseudocompact non-strongly pseudocompact space ?

D-pseudocompactness

Question

Is there an F-pseudocompact non-strongly pseudocompact space ?

F-pseudocompact groups

Theorem

Every pseudocompact topological group is F-pseudocompact.

F-pseudocompact groups

Theorem

Every pseudocompact topological group is F-pseudocompact.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i} a_{n_j}^{-1} : i, j \in \mathbb{N} \text{ and } i < j\}$ is discrete,
- $\{a_{n_i} a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i} a_{n_j}^{-1} : i, j \in \mathbb{N} \text{ and } i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i} a_{n_j}^{-1} : i, j \in \mathbb{N} \text{ and } i < j\}$ is discrete,
- $\{a_{n_i} a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i} a_{n_j}^{-1} : i, j \in \mathbb{N} \text{ and } i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i} a_{n_j}^{-1} : i, \ j \in \mathbb{N} \ \text{ and } \ i < j\}$ is discrete,
- $\{a_{n_i} a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i} a_{n_j}^{-1} : i, \ j \in \mathbb{N} \ \text{ and } \ i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \text{ and } i < j\}$ is discrete,
- $\{a_{n_i}a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \text{ and } i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i} a_{n_j}^{-1} : i, \; j \in \mathbb{N} \text{ and } i < j\}$ is discrete,
- $\{a_{n_i} a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i} a_{n_j}^{-1} : i, \; j \in \mathbb{N} \text{ and } i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{ and } \ i < j\}$ is discrete,
- $\{a_{n_i}a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{ and } \ i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{and} \ i < j\}$ is discrete,
- $\{a_{n_i}a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{and} \ i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{ and } \ i < j\}$ is discrete,
- $\{a_{n_i}a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \ \text{ and } \ i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \text{ and } i < j\}$ is discrete,
- $\{a_{n_i}a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i}a_{n_j}^{-1} : i,\ j \in \mathbb{N} \text{ and } i < j\}}$.

F-pseudocompact groups

Proof

Let G be a pseudocompact group and let $(U_n)_{n\in\mathbb{N}}$ be a sequence of nonempty pairwise disjoint open sets of G. Since G is pseudocompact we can find an accumulation point x for the sequence $(U_n)_{n\in\mathbb{N}}$. Without loss of generality, assume that $x = e \notin \overline{U_n}$ for all $n \in \mathbb{N}$. By following the proof of Protasov's Theorem, we can find a discrete set $\{a_{n_k} : k \in \mathbb{N}\}$ such that:

- $n_k < n_{k+1}$ for all $k \in \mathbb{N}$,
- The set $\{a_{n_i} a_{n_j}^{-1} : i, \ j \in \mathbb{N} \ \text{ and } \ i < j\}$ is discrete,
- $\{a_{n_i} a_{n_j}^{-1} : j \in \mathbb{N}\} \subseteq U_{n_i}$ for each $i \in \mathbb{N}$, and
- $e \in \overline{\{a_{n_i} a_{n_j}^{-1} : i, \ j \in \mathbb{N} \ \text{ and } \ i < j\}}$.

F-pseudocompact groups

Let $D = \{a_{n_i} a_{n_j}^{-1} : i, \ j \in \mathbb{N} \ \text{ and } \ i < j \leq 2n_i\}$. It is evident that $|D \cap U_n| < \omega$ for every $n \in \mathbb{N}$. Now, let U be an open neighborhood of e. Fix a symmetric open neighborhood V of e so that $V^2 \subseteq U$. Since G is totally bounded, there is a finite set F such that $\{a_{n_i} : i \in \mathbb{N}\} \subseteq VF$. If $k > |F|$, then, there are $i, j \in \mathbb{N}$ and $g \in F$ such that $k \leq i < j \leq 2n_k$ and $a_{n_i}, a_{n_j} \in Vg$. So $a_{n_i} a_{n_j}^{-1} \in D$ and $a_{n_i} a_{n_j}^{-1} \in (Vg)(Vg)^{-1}$. As $(Vg)(Vg)^{-1} = V^2 \subseteq U$ and $i < j \leq 2n_k \leq 2n_i$, we obtain that $D \cap U \neq \emptyset$. Since $e \notin D$, D is not closed. Therefore, G is F-pseudocompact.

F-pseudocompact groups

Let $D = \{a_{n_i} a_{n_j}^{-1} : i,\ j \in \mathbb{N} \text{ and } i < j \leq 2n_i\}$. It is evident that $|D \cap U_n| < \omega$ for every $n \in \mathbb{N}$. Now, let U be an open neighborhood of e. Fix a symmetric open neighborhood V of e so that $V^2 \subseteq U$. Since G is totally bounded, there is a finite set F such that $\{a_{n_i} : i \in \mathbb{N}\} \subseteq VF$. If $k > |F|$, then, there are $i, j \in \mathbb{N}$ and $g \in F$ such that $k \leq i < j \leq 2n_k$ and $a_{n_i}, a_{n_j} \in Vg$. So $a_{n_i} a_{n_j}^{-1} \in D$ and $a_{n_i} a_{n_j}^{-1} \in (Vg)(Vg)^{-1}$. As $(Vg)(Vg)^{-1} = V^2 \subseteq U$ and $i < j \leq 2n_k \leq 2n_i$, we obtain that $D \cap U \neq \emptyset$. Since $e \notin D$, D is not closed. Therefore, G is F-pseudocompact.

p-compact groups

Question

Is every pseudocompact group p-psuedocompact for some $p \in \mathbb{N}^*$?

Theorem[Salvador-Yasser, 2013]

Let $\{X_i : i \in I\}$ be a family of compact metric spaces and let $X = \prod_{i \in I} X_i$. Then, every pseudocompact dense subspace of X is ultrapseudocompact.

Thus, every dense pseudocompact subgroup of a Cantor cube $\{0, 1\}^{\alpha}$ is ultrapseudocompact, for any uncountable cardinal α, .

p-compact groups

Question

Is every pseudocompact group p-psuedocompact for some $p \in \mathbb{N}^*$?

Theorem[Salvador-Yasser, 2013]

Let $\{X_i : i \in I\}$ be a family of compact metric spaces and let $X = \prod_{i \in I} X_i$. Then, every pseudocompact dense subspace of X is ultrapseudocompact.

Thus, every dense pseudocompact subgroup of a Cantor cube $\{0,1\}^\alpha$ is ultrapseudocompact, for any uncountable cardinal α, .

p-compact groups

Question

Is every pseudocompact group p-psuedocompact for some $p \in \mathbb{N}^*$?

Theorem[Salvador-Yasser, 2013]

Let $\{X_i : i \in I\}$ be a family of compact metric spaces and let $X = \prod_{i \in I} X_i$. Then, every pseudocompact dense subspace of X is ultrapseudocompact.

Thus, every dense pseudocompact subgroup of a Cantor cube $\{0, 1\}^\alpha$ is ultrapseudocompact, for any uncountable cardinal α, .

Pseudocompact groups

Theorem[Tomita-Salvador, 2013]

Every pseudocompact torsion group is ultrapseudocompact.

Theorem[Salvador, 1992]

Suppose the existence of two selective ultrafilters RK-equivalent. Then for every weak P-point $p \in \omega^*$ there exists a pseudocompact group that is not p-pseudocompact.

Pseudocompact groups

Theorem[Tomita-Salvador, 2013]

Every pseudocompact torsion group is ultrapseudocompact.

Theorem[Salvador, 1992]

Suppose the existence of two selective ultrafilters RK-equivalent. Then for every weak P-point $p \in \omega^*$ there exists a pseudocompact group that is not p-pseudocompact.

Pseudocompact groups

Theorem[Tomita-Salvador, 2013]

Every pseudocompact torsion group is ultrapseudocompact.

Theorem[Salvador, 1992]

Suppose the existence of two selective ultrafilters RK-equivalent. Then for every weak P-point $p \in \omega^*$ there exists a pseudocompact group that is not p-pseudocompact.

Pseudocompact groups

Example[Tomita-Salvador, 2013]

There is a pseudocompact subgroup of $\{0,1\}^{\mathfrak{c}}$ that is not strongly pseudocompact.

Pseudocompact groups

Example[Tomita-Salvador, 2013]

There is a pseudocompact subgroup of $\{0,1\}^{\mathfrak{c}}$ that is not strongly pseudocompact.

Pseudocompact groups

Question

In *ZFC*, is there a pseudocompact group that is not ultraseudocompact ?

Question

Is the product of strongly pseudocompact groups strongly pseudocompact ?

Pseudocompact groups

Question

In *ZFC*, is there a pseudocompact group that is not ultraseudocompact ?

Question

Is the product of strongly pseudocompact groups strongly pseudocompact ?

Pseudocompact groups

Question

In ZFC, is there a pseudocompact group that is not ultraseudocompact ?

Question

Is the product of strongly pseudocompact groups strongly pseudocompact ?

Ofelia

FELICIDADES OFELIA POR TUS ϵ-AÑOS, WHERE $\epsilon > 0$

Ofelia

FELICIDADES OFELIA POR TUS ϵ-AÑOS, WHERE $\epsilon > 0$

Ofelia

FELICIDADES OFELIA POR TUS ϵ-AÑOS, WHERE $\epsilon > 0$