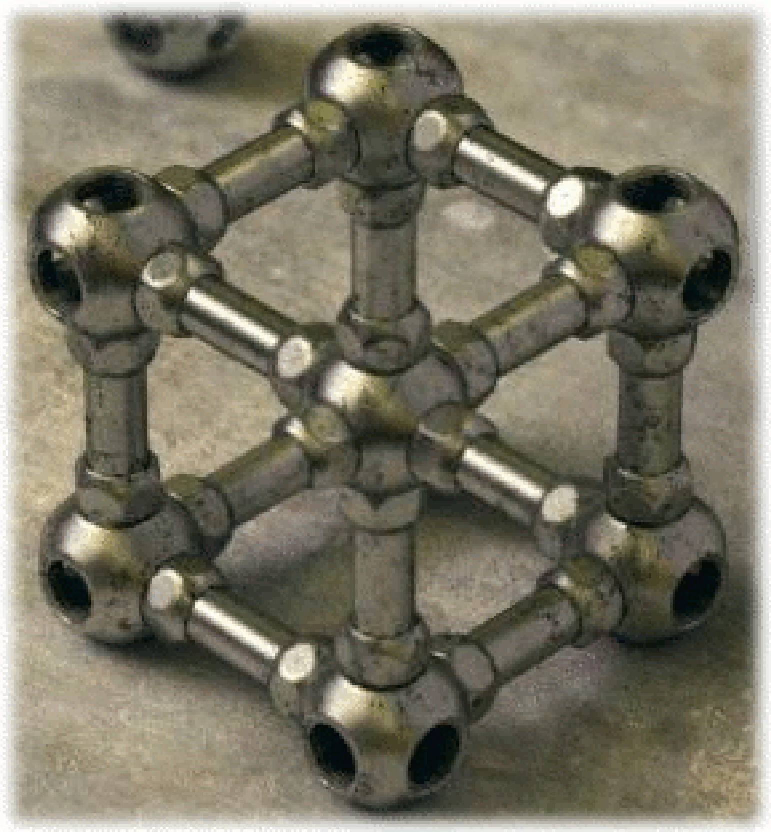

More About AGM Revision in Description Logics [1]

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More About AGM Revision in Description Logics

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1 Introduction

Belief revision studies the dynamics of beliefs defining some operations in logically closed sets (belief sets): expansion, revision and contraction. Revision, in particular deals with the problem of accommodating consistently a newly received piece of information.

Most of the works on belief revision following the seminal paper [1] assume that the underlying logic of the agent satisfies some assumptions. In [5] we showed how to apply revision of belief sets to logics that are not closed under negation. We have, however, assumed that the logic satisfies a property called distributivity. In the present work we show a list of description logics that are not closed under negation and study which of them are distributive.

1.1 AGM paradigm

The most influential work in belief revision is [1]. In this work the authors defined a number of *rationality postulates* for contraction and revision, now known as the AGM postulates. The authors then showed *constructions* for these operations and proved that the constructions are equivalent to the postulates (*representation theorem*)

Most works in belief revision assume some properties on the underlying logic: *compactness*, *Tarskianicity*, *deduction* and *supraclassicality*, which we will refer to as the AGM assumptions. The last two together are equivalent to the following two properties together for Tarskian logics:

Definition 1 (distributivity) *A logic* $\langle \mathscr{L}, Cn\rangle$ *is* distributive *iff for all sets of formulas* $X, Y, W \in 2^{\mathscr{L}}$*, we have that* $Cn(X \cup (Cn(Y) \cap Cn(W))) = Cn(X \cup Y) \cap Cn(X \cup W)$.

Definition 2 (closure under negation) *A logic* $\langle \mathscr{L}, Cn\rangle$ *is* closed under negation *iff for all* $A \in 2^{\mathscr{L}}$ *there is a* $B \in 2^{\mathscr{L}}$ *such that* $Cn(A \cup B) = \mathscr{L}$ *and* $Cn(A) \cap Cn(B) = Cn(\emptyset)$*. The set* B *is then called a* negation *of* A.

AGM revision in non-classical logics: In [5] we argued that some description logics are not closed under negation and, hence, do not satisfy the AGM assumptions. Furthermore, the most common way to define revision is via Levi identity ($K * \alpha = K - \neg\alpha + \alpha$), which assumes the existence of the negation of α. We proposed then a new construction and a set of postulates for revision for logics that are not closed under negation.

We used two postulates, borrowed from the belief base literature:

(relevance) If $\beta \in K \setminus K * \alpha$ then there is K' such that $K \cap (K * \alpha) \subseteq K' \subseteq K$ and $K' \cup \{\alpha\}$ is consistent, but $K' \cup \{\alpha, \beta\}$ is inconsistent.

(uniformity) If for all $K' \subseteq K$, $K' \cup \{\alpha\}$ is inconsistent iff $K' \cup \{\beta\}$ is inconsistent then $K \cap K * \alpha = K \cap K * \beta$

The set of rationality postulates we considered is: closure, success, inclusion, consistency, relevance and uniformity. The following proposition is an evidence that this is a good choice of rationality postulates:

Proposition 3 *For logics that satisfy the AGM assumptions, closure, success, inclusion, consistency, relevance and uniformity are equivalent to the original AGM postulates for revision: closure, success, consistency, vacuity and extentionality.*

We proposed also a construction inspired in some ideas from [4]:

Definition 4 (Maximally consistent set w.r.t α) *[4]* $X \in K \downarrow \alpha$ *iff* $X \subseteq K$, $X \cup \{\alpha\}$ *is consistent and if* $X \subset X' \subseteq K$ *then* $X' \cup \{\alpha\}$ *is inconsistent.*

Definition 5 (Selection function) *[1] A* selection function *for* K *is a function* γ *such that if* $K \downarrow \alpha \neq \emptyset$*, then* $\emptyset \neq \gamma(K \downarrow \alpha) \subseteq K \downarrow \alpha$*. Otherwise,* $\gamma(K \downarrow \alpha) = \{K\}$.

The construction of a *revision without negation* is defined as $K *_\gamma \alpha = \bigcap \gamma(K \downarrow \alpha) + \alpha$.

We proved that, *for distributive logics*, this construction is completely characterized by the set of rationality postulates we are considering i.e. we proved the representation theorem relating the construction to the set of postulates [5].

1.2 Description Logics

Description logics (DLs) forms a family of formalisms to represent terminological knowledge. The *signature* of a description logic is a tuple $\langle N_C, N_R, N_I\rangle$ of concept names, roles names and individual names of the language [2]. From a signature it is possible to define *complex concepts* via a *description language*. Each DL has its own description language that admits a certain set of constructors.

The *semantic* of a DL is defined using an *interpretation* $\mathcal{I} = \langle .^{\mathcal{I}}, \Delta^{\mathcal{I}}\rangle$ such that $\Delta^{\mathcal{I}}$ is a non-empty set called *domain* and $.^{\mathcal{I}}$ is an *interpretation function*. For each concept name the interpretation associates a subset of the domain, for each role name a binary relation in the domain and for each individual an element of the domain. The interpretation is then extended to complex concepts.

A *sentence* in a DL is a restriction to the interpretation. A *TBox* is a set of sentences of the form $C_1 \sqsubseteq C_2$ that restricts the interpretation of concepts[2], an *ABox* is a set of sentences of the form $C(a)$, $R(a, b)$,

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[2] Assuming that the logic admits GCI axioms

$a = b$ and $a \neq b$ that restricts the interpretation of individuals. Some DLs, like $\mathcal{ALCH}$, admits also an *RBox* which is a set of sentences of the form $R \sqsubseteq S$ that restricts the interpretations of roles.

Let $\Sigma = \langle \mathcal{T}, \mathcal{A}, \mathscr{R} \rangle$ be a tuple where $\mathcal{T}$, $\mathcal{A}$ and $\mathscr{R}$ are a Tbox, an ABox and an RBox respectively. A sentence α is a *consequence* of Σ ($\Sigma \models \alpha$ or $\alpha \in Cn(\Sigma)$) iff for all interpretations $\mathcal{I}$ if $\mathcal{I}$ satisfies Σ then $\mathcal{I}$ satisfies α.

Two characteristics the DLs we are considering that will be important in this work are: in $\mathcal{ALC}$ every sentence in the TBox is equivalent to a sentence of the form $\top \sqsubseteq C$ for a concept C [3] and in $\mathcal{ALCO}$ every sentence in the ABox is equivalent to a sentence of the form $\top \sqsubseteq C$.

2 Properties of Description Logics

The main contribution of this work is to show a set of description logics that are not closed under negation and which of them are distributive i.e. we show a set of logics such that representation theorem for revision without negation is applicable. It turns out that most DLs that admits GCI axioms (GCI axioms allow complex concepts in both sides of the sentence) are not closed under negation, but many of them are also not distributive.

Classic negation in DLs: We will say that two roles R and S are *unrelated* iff neither $R \sqsubseteq S \in Cn(\emptyset)$ nor $S \sqsubseteq R \in Cn(\emptyset)$. The main result of this section proves that if the signature of a DL $\langle \mathscr{L}, Cn \rangle$ has infinitely many unrelated roles and admits $\forall$, $\sqcup$, $\neg$ and GCI axioms then $\langle \mathscr{L}, Cn \rangle$ is not closed under negation.

Theorem 6 *Consider a DL $\langle \mathscr{L}, Cn \rangle$ that admits the constructors $\neg$, $\forall$, $\sqcup$ and general concept inclusion axioms in the TBox. If there is an infinite number of unrelated roles, then $\langle \mathscr{L}, Cn \rangle$ is not closed under negation*

The proof of this theorem comes from the fact that if $\langle \mathscr{L}, Cn \rangle$ admits $\sqcup$, $\neg$ then every sentence can be written as $\top \sqsubseteq C$ and from the following lemmas:

Lemma 7 *Let A and B be concepts such that $\top \sqsubseteq A$ and $\top \sqsubseteq B$ are not tautologies and let R be a role name that is unrelated with any role that appears in A or B. Then $Cn(\emptyset) \subset Cn(\top \sqsubseteq A \sqcap \forall R.B) \subseteq Cn(\top \sqsubseteq A) \cap Cn(\top \sqsubseteq B)$.*

Lemma 8 *If $Cn(\top \sqsubseteq A) = Cn(\emptyset)$ and $\top \sqsubseteq B$ is a negation of $\top \sqsubseteq A$ then $Cn(\top \sqsubseteq B) = \mathscr{L}$*

As a corollary of this result we have that many well known description logics are not closed under negation. Hence, for all these logics the AGM results are not applicable:

Corollary 9 *The following DLs are not closed under negation: $\mathcal{ALC}$, $\mathcal{ALCO}$, $\mathcal{ALCH}$, OWL-lite and OWL-DL.*

Distributivity in DLs: In this section we show a list of distributive and non-distributive DLs. We start with an example showing that the logic $\mathcal{ALC}$ is not distributive in general.

Example 10: Let $X = \{a = b\}$, $Y = \{C(a)\}$ and $Z = \{C(b)\}$, then $Cn(Y) \cap Cn(Z) = Cn(\emptyset)$. Hence $C(a) \notin Cn(X \cup (Cn(Y) \cap Cn(Z)))$, but $C(a) \in Cn(X \cup Y) \cap Cn(X \cup Z)$.

The example above depends on the existence of the ABox. In fact, $\mathcal{ALC}$ with empty ABox is distributive:

Proposition 11 *Consider a DL $\langle \mathscr{L}, Cn \rangle$ such that for every sentence $\alpha \in \mathscr{L}$ there is a sentence $\alpha' \in \mathscr{L}$ such that $Cn(\alpha) = Cn(\alpha')$ and α' has the form $\top \sqsubseteq C$ for some concept C. Then $\langle \mathscr{L}, Cn \rangle$ is distributive.*

Since in $\mathcal{ALCO}$ the ABox can be written in terms of the TBox, $\mathcal{ALCO}$ is distributive even in the presence of the ABox.

Finally, if we consider a logic $\langle \mathscr{L}, Cn \rangle$ that admits role hierarchy, but does not admit role constructors, then $\langle \mathscr{L}, Cn \rangle$ is not distributive. Consider the following example:

Example 12: Let $X = \{R \sqsubseteq S_1, R \sqsubseteq S_2\}$, $Y = \{S_1 \sqsubseteq S_3\}$ and $Z = \{S_2 \sqsubseteq S_3\}$. We have that $Cn(Y) \cap Cn(Z) = Cn(\emptyset)$. Hence $R \sqsubseteq S_3 \notin Cn(X \cup (Cn(Y) \cap Cn(Z)))$, but $R \sqsubseteq S_3 \in Cn(X \cup Y) \cap Cn(X \cup Z)$.

Besides $\mathcal{ALCH}$, the logics behind OWL 1 ($\mathcal{SHOIN}$ for OWL-DL and $\mathcal{SHIF}$ for OWL-lite), OWL-2 ($\mathcal{SROIQ}$) and the OWL profiles OWL-RL and OWL-QL admit role hierarchy, but do not admit role constructors. None of these logics are distributive.

The following table sums up the results of this section:

Description Logic	Negation	Distributivity
$\mathcal{ALC}$	no	no
$\mathcal{ALC}$ without ABox	no	yes
$\mathcal{ALCO}$	no	yes
$\mathcal{ALCH}$, OWL-lite, OWL-DL	no	no
OWL-QL, OWL-RL and OWL 2	?	no

3 Conclusion and future work

In this work we continued the work started in [5] by showing for which DLs the AGM revision without negation can be applied. We showed that most DLs that admits GCIs are not closed under negation, but most of them are also not distributive. We showed that $\mathcal{ALC}$ with empty TBox and $\mathcal{ALCO}$ are two exceptions. These logics are distributive and not closed under negation. Hence, the representation theorem presented in [5] holds for $\mathcal{ALC}$ with empty ABox and $\mathcal{ALCO}$.

In addition to that, we showed that the postulates used in [5] are equivalent to the AGM postulates if the underlying logic satisfies the AGM assumptions. This is a good evidence that we chose a good set of rationality postulates.

As future work we should look for a construction that can be characterized by this set of postulates (or a similar one) not only in distributive, but in any Tarskian compact logic.

REFERENCES

[1] C. Alchourrón, P. Gärdenfors, and D. Makinson, 'On the logic of theory change', *Journal of Symbolic Logic*, **50**(2), 510–530, (1985).

[2] *The Description Logic Handbook*, eds., F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider, Cambridge University Press, 2003.

[3] F. Baader and U. Sattler, 'An overview of tableau algorithms for description logics', *Studia Logica*, **69**(1), 5–40, (2001).

[4] J. P. Delgrande, 'Horn clause belief change: Contraction functions', in *Proceedings of KR*, eds., G.Brewka and J. Lang, pp. 156–165, (2008).

[5] M. M. Ribeiro and R. Wassermann, 'AGM revision in description logics', in *Proceedings of ARCOE*, (2009).