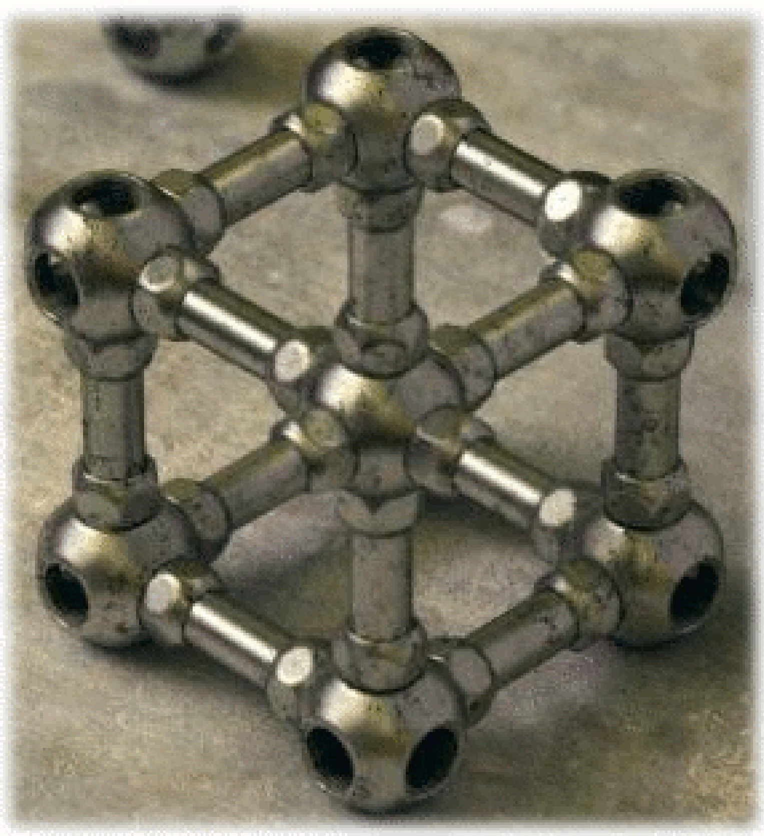


Qualitative Robot Localisation Using Information from Cast Shadows [1]

Author(s):

Paulo Santos
Hannah M. Dee
Valquiria Fenelon

[1]This work was supported by Fapesp Project LogProb, grant 2008/03995-5, SÃčo Paulo, Brazil.

Qualitative robot localisation using information from cast shadows

Paulo Santos Hannah M. Dee Valquiria Fenelon

***Abstract*— Recently, cognitive psychologists and others have turned their attention to the formerly neglected study of shadows, and the information they purvey. These studies show that the human perceptual system values information from shadows very highly, particularly in the perception of depth, even to the detriment of other cues. However with a few notable exceptions, computer vision systems have treated shadows not as signal but as noise. This paper makes a step towards redressing this imbalance by considering the formal representation of shadows. We take one particular aspect of reasoning about shadows, developing the idea that shadows carry information about a fragment of the viewpoint of the light source. We start from the observation that the region on which the shadow is cast is occluded by the caster with respect to the light source and build a qualitative theory about shadows using a region-based spatial formalism about occlusion. Using this spatial formalism and a machine vision system we are able to draw simple conclusions about domain objects and egolocation for a mobile robot.**

I. Introduction

The purpose of this work is to develop a qualitative spatial reasoning framework about cast shadows (or shadows for short) to be used for mobile robot egolocation. Qualitative spatial reasoning (QSR) aims at the logical formalisation of space from elementary entities such as regions, line segments, directions amongst others [1].

This work falls within the logic-based knowledge representation (KR) subfield of Artificial Intelligence [2], whose main goals are: the logic formalisation of reasoning processes, capable of inferring knowledge from representations of the world; the construction of a medium for pragmatically efficient computation, in which the formal representation provides the means to organise domain knowledge allowing for efficient (and consistent) queries, updates and revisions of the knowledge base; or the rigorous treatment of ontological commitments, which provide the base rules that guide reasoning about the world (for instance, what should or should not be considered as the effects of actions, what are the base spatial entities in a domain and so on) [3]. To illustrate, from the following traditional discourse: "all men are mortals and Socrates is a man", we can *infer* the implicit fact that "Socrates is mortal". This can be represented in a formal language as: $\{(\forall\ x\ men(x) \rightarrow mortal(x)) \ \wedge\ men(Socrates)\}$, from which the inference rule *modus ponens* allows the deduction of $mortal(Socrates)$. Research in Knowledge Representation works in an analogous way: given a domain, the task is to find a suitable formal representation and inference methods from which only sound facts can be inferred. Besides this, qualitative spatial reasoning reduces the domain to their elementary spatial constructs.

From the formalisation of shadows we aim to provide a computer vision system with inference methods capable of concluding facts about the position, orientation and motion of objects in the world from the visual observation of objects and their shadows. As we show in Section II, the cognitive (informational) content of shadows is great, and we humans use this in our day to day perception of depth and motion. A shadow is caused when an object (a "caster") comes between a light source and a surface (a "screen"). Self shading is what occurs when the caster and screen are the same object, and the informational content of such shadows has been investigated at length within the computer vision community (as *shape from shading*). However, cast shadows (in which the caster and screen are different objects) have usually received less attention in scene understanding. We shall concern ourselves in this paper with cast shadows, restricting the investigation to the more common case in which the caster and screen are largely opaque. The present work takes one particular aspect on reasoning about cast shadows, developing the idea that shadows provide the viewpoint of the light source. In other words, from the viewpoint of the light source the shadow is completely occluded by the caster. From this observation we construct a qualitative theory about shadows upon the spatial theory about occlusion known as the region occlusion calculus (ROC)[4] and use it for qualitative egolocation for a mobile robot.

Fig. 1. Sometimes shadows carry information about objects outside of view, via the "viewpoint" of the light source; artwork *Shadow Stone*, by Andy Goldsworthy.

Section II discusses prior work in shadow reasoning, from within Philosophy, Psychology and Computer Vision to motivate the following sections. The region occlusion calculus (ROC) is discussed in Section III. Section IV presents an

Paulo Santos is with Elect. Eng. Dep., FEI, Sao Paulo, Brazil, email: psantos@fei.edu.br

Hannah Dee is with School of Computing, University of Leeds, UK, email: hannah@comp.leeds.ac.uk.

Valquiria Fenelon is with Elect. Eng. Dep., FEI, Sao Paulo, Brazil

extension of ROC to deal with shadows (which we call Perceptual Qualitative Relations about Shadows – PQRS), and Section V provides examples of inferences within the extended theory. Conclusions are drawn on Section VI.

II. Shadows in the cognitive sciences

The importance of cast shadows in our depth perception was intensely exploited in Renaissance paintings [5]. However the cognitive processes behind the perception of the external world using shadows as cues have only recently begun to be investigated [6], [7]. Casati in [8] points out that cast shadows carry information about the presence and location of the light source and the caster when they are inside or outside the observer's field of view. Shadows also carry information on the intensity of the source, the shape of the caster and the texture of the screen, and it is possible to hypothesise the distance between the caster and the screen given whether or not the caster and the shadow are in contact with each other. Another important fact about the information content of shadows is that they can be seen as providing the observer with *a second viewpoint*: that of the light source, as the shadow depicts the projection of the caster's terminator line. The psychological work reported in [9] suggest that the human perceptual system is biased to use shadow information on the interpretation of 3D motion and that shadow information can even over-ride notions of conservation of object size. This justifies the development of a shadow processing stage in any cognitively plausible vision system.

In the field of computer vision much shadow detection work is centred around the idea of shadow as noise. When subtracting background from video to find objects of interest, shadows are a major source of false positives, hence shadow detection becomes important for noise reduction (see, e.g., [10]). When we consider systems which *use* shadows there are only a handful: [11] use known 3D locations and their cast shadows to perform camera calibration and light location (using known casters and screen to tell about light source); [12] uses the moving shadows cast by known vertical objects (flagpoles, the side of buildings) to determine the 3D shape of objects on the ground (using the shadow to tell about the shape of the screen). Perhaps most relevant to the current paper is the work of Balan *et al.* [13], who use shadows as a source of information for detailed human pose recognition: they show that using a single shadow from a fixed light source can provide a similar disambiguation effect as in using additional cameras.

In this paper, perception (and reasoning about) cast shadows is understood as the problem of inferring spatial relations from the observation of objects and their shadows. The use of cast shadows in such processes, however, presupposes the solution of the shadow correspondence problem [6], which involves the segmentation of shadows in scenes and the connection of shadows to their relative casters [9]. Shadows, like holes, are dependent objects – without a caster, they do not occur. Matching shadows to their casters is a hard problem for various reasons: there may be various competing possibilities to match shadows and objects in a complex scene (i.e. the shadow correspondence problem is underconstrained); the screen may not be planar, which may turn a point-to-point matching into a complex non-linear registration procedure; and shadows of close objects may merge. Given these complications, we incorporated a partial solution to this problem using as heuristics the idea that a shadow connected to an object is the shadow cast by this object. A complete solution to the shadow correspondence problem is outside the scope of this paper. Instead, we concentrate on formalising the information content of shadows, using a region-based ontology from a qualitative spatial reasoning perspective. The next section presents the underlying theories with which shadows are formalised in this work.

III. Background

This section presents the qualitative spatial reasoning approaches that are used in the development of this research. A comprehensive overview of this field can be found in [1].

One of the best known QSR approaches is the Region Connection Calculus (RCC) [14]. RCC is a many-sorted first-order axiomatisation of spatial relations based on a reflexive, symmetric and non-transitive dyadic primitive relation of *connectivity* ($C/2$) between two regions. Informally, assuming two regions x and y, the relation $C(x, y)$, read as *"x is connected with y"*, is true if and only if the closures of x and y have a point in common.

Assuming the $C/2$ relation, and that x, y and z are variables for spatial regions, some mereotopological relations can be defined. Some of them are: $DC(x, y)$, which is read as "x is disconnected from y"; $EQ(x, y)$: "x is equal to y"; $O(x, y)$: "x overlaps y"; $P(x, y)$: "x is part of y"; $PO(x, y)$: "x partially overlaps y"; $PP(x, y)$: "x is a proper part of y"; $EC(x, y)$: "x is externally connected with y"; $TPP(x, y)$: "x is a tangential proper part of y"; $NTPP(x, y)$: "x is a non-tangential proper part of y"; $TPPi/2$ and $NTPPi/2$ are the inverse relations of $TPP/2$ and $NTPP/2$ respectively.

RCC represents qualitative mereotopological relations between spatial regions independently of any observer's viewpoint. In contrast [15] proposes a *lines-of-sight* calculus in order to represent relative positions between pairs of non-overlapping convex bodies as seen from a particular observer. The main interest in this formalism is the representation and manipulation of information about visual occlusion between objects. Inspired by these ideas, *Region Occlusion Calculus* (ROC) [4] was proposed to represent the various possibilities of interposition relations between two arbitrary shaped objects as an extension of the Region Connection Calculus. The relations constituting ROC are represented in Figure 2. These relations are defined over RCC relations along with the primitive relation $TotallyOccludes(x, y, v)$, which stands for "x totally occludes y wrt the viewpoint v".

In order to make explicit both the distinction between a body and the region of space it occupies, and also the distinction between a physical body to its projection wrt a viewpoint, ROC assumes, respectively, the functions r (*region*) and i (*image*). The *region* function can be understood

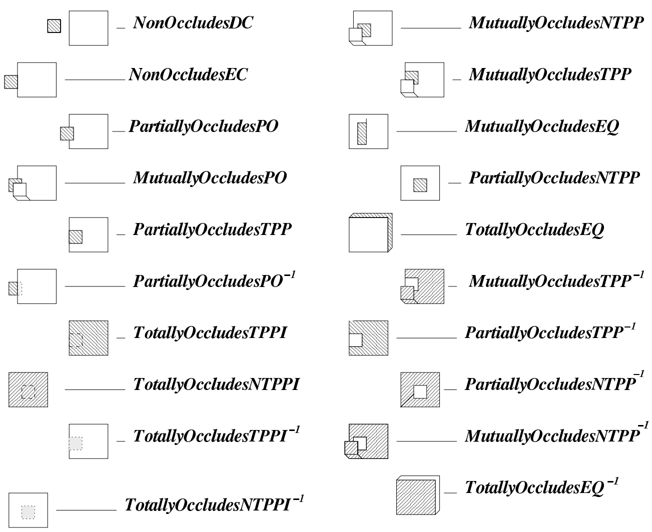


Fig. 2. The 20 ROC relations, where ϕ^{-1} is the inverse of ϕ.

as a mapping from a physical body to its occupancy region. Similarly, the *image* function is a mapping from a physical body to its relative 2D projection wrt a viewpoint. Below we present those ROC axioms (introduced in [4]) that are used in our theory of shadows.

Formula $(A1)$ is the ROC axiom that states that "if x totally occludes y from a viewpoint ν, x totally occludes any part of y":

$$(A1)\ \ \forall x\ y\ z\ v\ \ [[TotallyOccludes(x,y,v) \wedge P(r(z),r(y))] \rightarrow TotallyOccludes(x,z,v)].$$

The fact "if x totally occludes y from ν, no part of y totally occludes part of x" is formalised in formula $(A2)$.

$$(A2)\ \ \forall x\ y\ z\ v\ \ [TotallyOccludes(x,y,v) \rightarrow \forall z\ u[[P(r(z),r(x)) \wedge P(r(u),r(y))] \rightarrow \neg TotallyOccludes(u,z,v)]]$$

In order to simplify notation, the following abbreviations [4] are included in the theory: $Occludes(x,y,v)$, $PartiallyOccludes(x,y,v)$, and $MututallyOccludes(x,y,v)$, whose definitions are omitted here for brevity. With these definitions the 20 ROC relations (Figure 2) can be defined [4].

IV. Perceptual qualitative relations about shadows (PQRS)

For the purposes of this work we assume a static light source, denoted by the constant symbol L, situated above the observer[1]. We also assume that the scenes are observed from an egocentric point of view that is represented by v. In order to simplify the notation we also assume that shadows are cast on a single screen Scr which does not need to be flat, since (as we shall see) shadow detection in this work does not take into account the shapes of image regions, but only the intensity of the pixels composing them. The basic part of the theory has a sort for physical bodies (including the casters, the screen and the shadows): $o_1, \ldots, o_n$; sorts for time points: $t_1, \ldots, t_n$; and spatial regions: $r_1, \ldots, r_n$. For convenience we represent shadows by the symbols $s_1, \ldots, s_n$. It is assumed throughout this paper that the variables are universally quantified, unless explicitly stated.

The set of perceptual qualitative relations about shadows (PQRS) includes the region connection calculus and a subset of the region occlusion calculus (ROC) composed of the relations $\{NonOccludesDC(o,s,\nu)$, $NonOccludesEC(o,s,\nu)$, $PartiallyOccludesPO(o,s,\nu)$, $PartiallyOccludesTPP(o,s,\nu)$, $TotallyOccludesTPPI(o,s,\nu)$, $TotallyOccludesEQ(o,s,\nu)$ and $TotallyOccludesNTPPI(o,s,\nu)\}$ for a caster o, its shadow s and a viewpoint ν. The remaining ROC relations (shown in Figure 2) have no model wrt casters and their cast shadows.

We introduce the predicate $Shadow(s,o,Scr,L)$ that denotes that s is a shadow of object o on the screen Scr from the light source L. It is also convenient to define: $Is\_a\_Shadow(s,o) \equiv \exists scr, l\ \ Shadow(s,o,scr,l)$, standing for "$s$ is a shadow of o".

We can now state as an axiom that the shadow of an object o is the region in a screen that is totally occluded by the caster from the light source viewpoint. Formally, we have:

$$(A3)\ \ Shadow(s,o,Scr,L) \leftrightarrow PO(r(s),r(Scr)) \wedge TotallyOccludes(o,s,L) \wedge \neg\exists o' TotallyOccludes(o',o,L).$$

The third conjunct of the right-hand side of Formula $(A3)$ guarantees the existence of the shadow of o.

It follows from $(A3)$ and Axiom $(A2)$ that no shadow occludes its own caster[2], as denoted by Theorem $(T1)$ below.

$$(T1)\ \ Shadow(s,o,Scr,L) \rightarrow \neg TotallyOccludes(s,o,L).$$

It is also a consequence of Axiom $(A3)$ and the ROC axioms that no shadow casts a shadow itself (cf. Theorem $(T2)$):

$$(T2)\ \ Shadow(s,o,Scr,L) \rightarrow \neg Shadow(s',s,Scr',L).$$

We can also prove that if two shadows of distinct objects partially overlap, then the objects will be in a relation of occlusion wrt the light source, as expressed in Theorem $(T3)$.

$$(T3)\ \ Shadow(s,o,Scr,L) \wedge \exists o' \neg(o=o') \wedge O(r(s),r(o)) \rightarrow Occludes(o,o',L) \vee Occludes(o',o,L).$$

Theorems $(T1)$, $(T2)$ and $(T3)$ are proved in [17].

We can obtain similar results to those above considering partial shadows (instead of the whole shadows represented in $Shadow/2$), i.e. shadows of parts of casters.

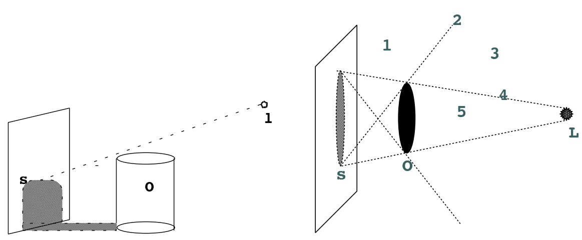
[1]An assumption likely to be made by the human perceptual system [16].

[2]Note that we are only dealing with cast, and not self attached, shadows.

Relative location

The results above relate the perspective view of shadows and their casters from the light source viewpoint. It is possible, however, to reason about shadows from arbitrary viewpoints: relating shadows with occlusion suggests the distinction of five regions defined from the lines of sight between the light source, the caster and its shadow, as represented in Figure 3(b). Therefore, any viewpoint v located on Region 1 will observe the shadow s and the object o as $NonOccludesDC(o, s, v)$; similarly, if v observes o and s from Region 3 it should see that $PartiallyOccludesPO(o, s, v)$ and from Region 5 that $TotallyOccludesNTPPI(o, s, v)$. Region 4 is the surface defined by the lines of sight from l tangential to o and s, from where v would observe $TotallyOccludesTPPI(o, s, v)$. Region 2, from where v sees $NonOccludesEC(o, s, v)$, is a bisected conic surface defined by the lines connecting opposite sides of the object and its shadow, starting at infinity and stopping at the object.

It is worth noting that, if only the top part of the shadow is considered, the five regions described above can also be defined in the case where the shadow is connected to its caster (cf. Figure 3(a)), whether or not the shadow is completely cast on the ground.



(a) (b)

Fig. 3. (a) a cast shadow connected to its caster; (b) distinct regions implied by the observation of a shadow and its caster.

By including axioms for left/right information in the theory (cf. [4] omitted here for brevity), besides of locating an observer in regions 1, 2, 3 and 4 (shown in Figure 3(b)), we would be able to say that this observer is on to the *left* (or *right*) of object o and shadow s. Let the terms $Region\ i$ (where $i \in 1, 2, 3, 4, 5$) represent the regions in Figure 3(b) and let the relation $located(r, \nu, o, s)$ represent an observer ν located at a region r wrt an object o and its shadow s. Then, we have:

$$(A4)\ \ located(Region\ \ 1, \nu, o, s) \leftarrow Is\_a\_Shadow(s, o) \wedge NonOccludesDC(o, s, v) \wedge v \neq o;$$

$$(A5)\ \ located(Region\ \ 2, \nu, o, s) \leftarrow Is\_a\_Shadow(s, o) \wedge NonOccludesEC(o, s, v) \wedge v \neq o;$$

$$(A6)\ \ located(Region\ \ 3, \nu, o, s) \leftarrow Is\_a\_Shadow(s, o) \wedge PartiallyOccludesPO(o, s, v) \wedge v \neq o;$$

$$(A7)\ \ located(Region\ \ 4, \nu, o, s) \leftarrow Is\_a\_Shadow(s, o) \wedge TotallyOccludesTPPI(o, s, v) \wedge v \neq o;$$

$$(A8)\ \ located(Region\ \ 5, \nu, o, s) \leftarrow Is\_a\_Shadow(s, o) \wedge TotallyOccludesNTPPI(o, s, v) \wedge v \neq o.$$

It is worth noting that (according to Axiom $(A3)$) PQRS cannot handle the case where the robot (located on Region 5) totally occludes the screen from the light source, since the predicate $Shadows$ is false in this case.

In conclusion, formalising the relations between shadows, casters and observers using ROC facilitates a qualitative characterisation of the space around objects, that can be used as relative location for a mobile observer.

V. EXPERIMENTS

We are currently implementing the ideas for relative location presented in Section IV on our ActivMedia PeopleBot using a monocular colour camera. Shadow detection is accomplished by mapping the images captured by the camera into a HSV colour space and thresholding on V, whereby high values (light objects) are filtered out and low values (dark objects) are casters. Shadows are located within a value range in between light and dark objects whose thresholds are found by experimentation. Noise and some spurious shadows are filtered out by morphological operations.

The experiments where conducted with the robot immersed in a prepared office-like environment containing target objects and where the light-source was a single sharp artificial light located above the scene at one side of the room (cf. Figure 5(a)). The robot was set to navigate through the room, stopping after a certain time interval to analyse its position wrt the object-shadow locations introduced in Section IV. In the robot setup, however, the regions represented in Figure 3(b) had to be slightly modified in order to account for the uncertainty in locating the robot on the one-dimensional regions 2 and 4. This modification is shown in Figure 4, where regions 2 and 4 are now defined as the shaded spaces surrounding the respective original regions. Note also in this figure that the regions exist "behind" the light source and behind the shadow[3], what defines them is the PQRS relation between the caster and its shadow.

Figure 5(b) shows an example of the sort of segmentation obtained from robot images.

Experiment1: Egolocation

In our first experiment, the robot collected 118 snapshots around the target object (the black bucket in Figures 5(a) and 5(b)), which was kept at the centre of the camera view. The system located the robot correctly into one of the five object-shadow regions (Figure 4) in 97 out of the 129 snapshots (as shown in the diagonal of Table I below). Out of the 32 mislocations, 60% were related to the borderlines separating two regions, whereas the remainder were due to noise from the scene background (mainly dark regions on the wall that were

[3]If the screen is the floor.

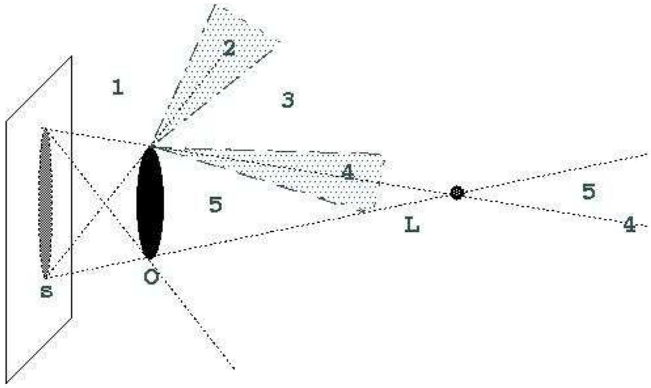


Fig. 4. Distinct regions wrt shadows used in the preliminary experiments.

(a) Robot office-like environment (b) Example image

Fig. 5. Robot office-like environment and example image

segmented as shadows and linked to the target object). These results are summarised on Table I that contains a confusion matrix for the robot localisation task. The first column shows the actual region where the robot was located and the first row, the answers given by the system; the correct answers form the main diagonal of the matrix and are marked in bold text. It is worth noting that the wrong answers located the robot in an adjacent region of the correct one, so even these carry some information about the relative robot's location.

TABLE I
SUMMARY OF THE RESULTS FOR EGOLOCATION FROM SHADOWS

	Region 1	Region 2	Region 3	Region 4	Region 5
Region 1	**40**	0	0	0	0
Region 2	11	**27**	2	0	0
Region 3	0	2	**15**	0	0
Region 4	0	5	4	**6**	0
Region 5	0	1	0	1	**9**

Experiment 2: Egolocation and object depth

Shadows (as well as occlusion) are also important cues for depth perception. The region occlusion calculus incorporates a primitive relation for $nearness$ ($N(x, y, z)$, read as "x is nearer to y than x is to z"), along with a set of axioms originally from [18] in order to relate occlusion with comparative distance. The nearness relation is related with occlusion in ROC by the following axiom:

$$(A9)\ \ \forall x\ y\ v\ [PartiallyOccludes(x, y, v) \rightarrow N(v, x, y)]$$

representing that "if a body x partially occludes a body y wrt some viewpoint v then x is nearer to v than y is to v".

TABLE II
SUMMARY OF THE EXPERIMENTS USING SHADOWS AS DEPTH CUES.

	total trials	egolocation	robot nearness	light source nearness
Region 2	32	22	23	32
Region 3	22	13	14	22
Region 4	4	4	0	0

It is easy to see that Axioms $(A3)$ and $(A9)$ imply the commonsense fact $N(L, o, s)$ (for a light source L, an object o and its shadow s) and consequently that $N(L, o, Scr)$. It is a consequence of this fact that if a shadow s (from caster o) is observed cast on an object o' ($o \neq o'$) then we know that o is nearer the light source than o', even though o' is never directly perceived by the observer (only its shadow). Formally: $Shadow(s, o, o', L) \rightarrow N(L, o, o')$.

Given the information about how some objects are nearer to the light source than others, and the qualitative egolocation (discussed in the previous sections), the robot is capable of inferring the depth of objects wrt itself. The idea is to use the observation of pairs of objects in such an arrangement wrt the light source that one of the objects is located in the middle of a long shadow (which is the combination of the shadows of both objects) and the other is not (cf. Figure 6). Thus, in this case, if the robot finds itself located in regions 2, 3 and 4 (Figure 4) it can infer that the object in the middle of the shadow (i.e., the one that has a shadow of another object cast on itself) is further from the robot than the other, because any location in regions 2, 3 and 4 will be closer to the object that is nearer to the light source. If the robot is located on region 1, however, no conclusion can be drawn wrt object's relative positions using cast shadows.

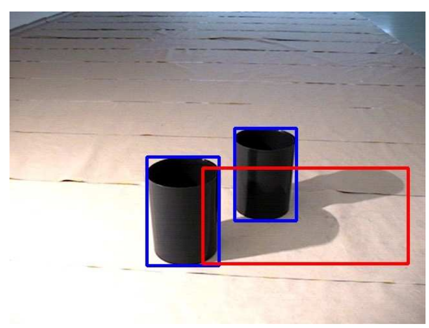


Fig. 6. Two objects and the segmented combination of their shadows.

In order to experiment these ideas in our scenario, we collected 58 poses with two objects in positions such that the shadow of one object is partially cast on the other, with the robot located in regions 2, 3 or 4 wrt the object that has no shadow cast on it (cf. Figure 6). Thus, by segmenting objects and shadows, we obtained the results shown in table II below for the robot egolocation and the relative location of the two objects in the scene.

From these results we can conclude that the system could always find the relative depth of objects wrt the light source (as shown in the last column of Table II). Moreover, whenever the robot located itself correctly in regions 2 and 3, it could decide the depth of scene objects.

In general, the classification errors were due to the threshold filters used to segment shadows from objects, the thresholds where over sensitive to changes in the intensity of the light source (note that the robot environment was also receiving natural light from the lab windows, cf. Figure 5(a)). The errors in the decision of nearness where due to the fact that the segmentation sometimes did not get a combination of the shadow of both objects in the scene, but two separate shadows instead. In these cases the split shadows perceived by the robot prevented it from determining object depth. In region 3 the caster and its shadow are partially occluded, and (in some cases) not much of the shadow is left to be pickled out by the vision system, causing the self-localisation to err. We are currently investigating two avenues to improve these results: one idea is to use the relative location diagram (Figure 3(b)) to generate expectations about where the robot is, if the expectation contradicts the observation, the robot should vary the thresholds selecting those where the best match between expectation and observation occurs; a second idea is to use Markov Random fields to segment shadows, casters and background.

VI. Discussion and open issues

This paper has identified the perception of cast shadows as an open problem within knowledge representation and computer vision, and has presented the theory Perceptual Qualitative Relations about Shadows (PQRS) which allows simple inferences about qualitative relations between caster, shadow, light source and screen. This formalism was used to prove several theorems on commonsense facts about space, and it has been experimented on the task of qualitative self-localisation by evaluating its inferences on images collected by a mobile robot vision system.

Cast shadows and their relations to our depth perceptions were extensively studied during the Renaissance [5]; however, only recently the perception of shadows has being considered as a subject for scientific enquiry [6]. This paper is a first step towards providing a rigorous account of the information content of shadows using formal knowledge representation techniques. It is worth pointing out that the choice of a qualitative theory does not preclude the use of quantitative or statistical methods, but complements them by making explicit the knowledge content from a domain. The purpose of qualitative methods is to add a more abstract layer to a robotic (or computer vision) system, whereby it is possible to make inferences about the knowledge encoded and also to prove theorems about the theory proposed.

The inclusion of probabilistic reasoning (along with logic inferences) in PQRS is on our research agenda, as the rigorous treatment of uncertainty is essential in order to extend this theory towards more natural environments. Integrating the proposed qualitative self-localisation method with other robot localisation frameworks is also a task to be done in order to enhance the robustness of our method, and to improve the efficiency of the localisation task.

PQRS can be used on environments with multiple light sources if there is a dominant source (as in our experiments), i.e., if it is possible to single out the shadows cast by one source. More research is needed to extend PQRS to work in environments with light sources of equal intensities.

We expect that the ideas presented in this work can be further integrated in intelligent vision and robotic systems in order to endow them with the basic machinery for reasoning about space using the entire spectrum of perceptual cues contained in the visual observation of the world.

VII. Acknowledgements

Paulo Santos acknowledges support from FAPESP project LogProb, 2008/03995-5, São Paulo, and also travel support from CAPES and CNPq, Brazil; Hannah Dee acknowledges support from EPSRC project LAVID, EP/D061334/1, UK; and Valquiria Fenelon is a graduate student sponsored by CAPES, Brazil.

This project was initiated on exchange visits from Hannah Dee and Paulo Santos sponsored by the British Council.

Many thanks to the two anonymous reviewers for the thoughtful comments.

References

[1] A. G. Cohn and J. Renz, "Qualitative spatial representation and reasoning," in *Handbook of Knowledge Representation*. Elsevier, 2008, pp. 551–596.

[2] F. van Harmelen, V. Lifschitz, and B. Porter, Eds., *Handbook of Knowledge Representation (Foundations of Artificial Intelligence)*. Elsevier Science, 2007.

[3] R. Davis, H. Shrobe, and P. Szolovits, "What is a knowledge representation?" *AI Magazine*, vol. 14, no. 1, pp. 17–33, 1993.

[4] D. Randell, M. Witkowski, and M. Shanahan, "From images to bodies: Modeling and exploiting spatial occlusion and motion parallax," in *Proc. of IJCAI*, Seattle, U.S., 2001, pp. 57–63.

[5] T. da Costa Kauffmann, "The perspective of shadows: The history of the theory of shadow projection," *Journal of the Warburg and Courtauld Institutes*, vol. 38, pp. 258–287, 1979.

[6] P. Mamassian, "Impossible shadows and the shadow correspondence problem," *Perception*, vol. 33, pp. 1279–1290, 2004.

[7] P. Cavanagh, "The artist as neuroscientist," *Nature*, vol. 434, pp. 301–307, 2005.

[8] R. Casati, "The shadow knows: a primer on the informational structure of cast shadows," *Perception*, vol. 33, no. 11, pp. 1385–1396, 2004.

[9] P. Mamassian, D. C. Knill, and D. Kersten, "The perception of cast shadows," *Trends in cognitive sciences*, vol. 2(8), pp. 288–295, 1998.

[10] N. Martel-Brisson and A. Zaccarin, "Learning and removing cast shadows through a multidistribution approach," *IEEE transactions on Pattern Analysis and Machine Intelligence (PAMI)*, vol. 29 (7), pp. 1134–1146, 2007.

[11] X. Cao and H. Foroosh, "Camera calibration and light source orientation from solar shadows," *Proc. Computer Vision and Pattern Recognition (CVPR)*, vol. 105, no. 1, pp. 60–72, 2007.

[12] Y. Caspi and M. Werman, "Vertical parallax from moving shadows," in *Proc. Computer Vision and Pattern Recognition (CVPR)*, New York, USA, 2006.

[13] A. O. Balan, M. J. Black, H. Haussecker, and L. Sigal, "Shining a light on human pose: On shadows, shading and the estimation of pose and shape," *Proc. International Conference on Computer Vision (ICCV)*, 2007.

[14] D. Randell, Z. Cui, and A. Cohn, "A spatial logic based on regions and connection," in *Proc. of KR*, Cambridge, U.S., 1992, pp. 165–176.

[15] A. Galton, "Lines of sight," in *Proc. of the Seventh Annual Conference of AI and Cognitive Science*, Dublin, Ireland, 1994, pp. 103–113.

[16] P. Mamassian and R. Goutcher, "Prior knowledge on the illumination position," *Cognition*, vol. 81, no. 1, pp. B1–B9, 2001.

[17] P. Santos, H. Dee, and V. Fenelon, "Notes on a qualitative theory of shadows," in *Proc. of the Cognitive Robotics Workshop*, 2008, pp. 47–54.

[18] J. van Benthem, *The Logic of Time: a model-theoretic investigation into the varieties of temporal ontology and temporal discource*. Dordrecht, The Netherlands: Kluwer Academic, 1991.