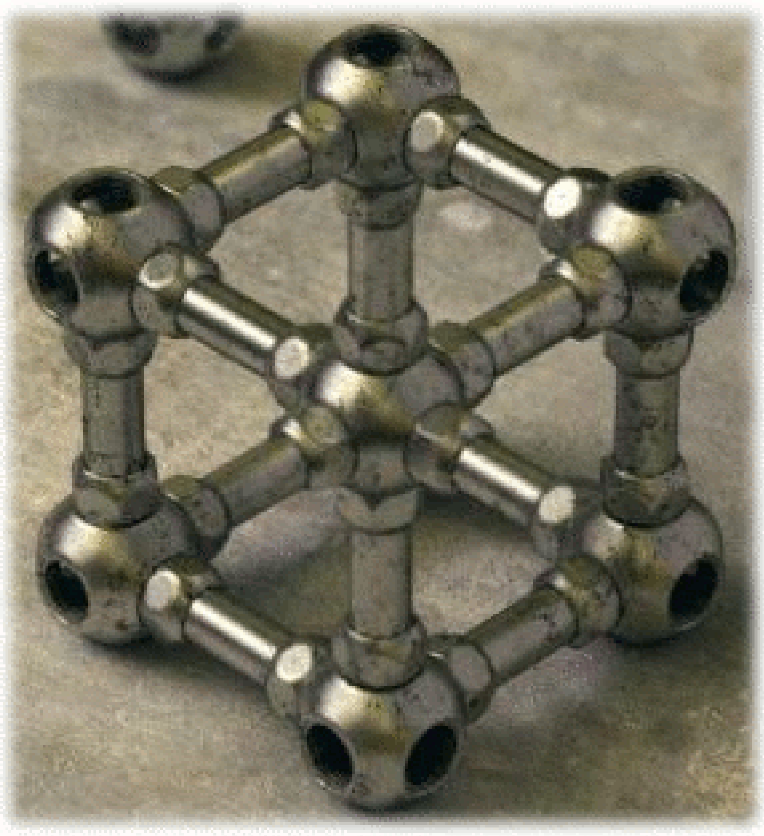

AGM revision in description logics [1]

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AGM Revision in Description Logics*

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Abstract

The AGM theory for belief revision cannot be directly applied to most description logics. For contraction, the problem lies on the fact that many logics are not compliant with the recovery postulate. For revision, the problem is that several interesting logics are not closed under negation.

In this work, we present solutions for both problems: we recall a previous solution proposing to substitute the recovery postulate of contraction by relevance and we present a construction for revision that does not depend on negation, together with a set of postulates and a representation theorem.

1 Introduction

The development of Semantic Web technologies has atracted the attention of the artificial intelligence community to the importance to represent conceptual knowledge in the web. This development has reached a peak with the adoption of OWL as the standard language to represent ontologies on the web. Since knowledge on the web is not static, another area has gained popularity in the past few years: ontology evolution. The main challenge of ontology evolution is to study how ontologies should behave in a dynamic environment. Belief revision theory has been facing this challenge for propositional logic for more than twenty years and, hence, it would be interesting to try to apply these techniques to ontologies.

In this work we apply the most influential work in belief revision, the AGM paradigm, to description logics. First, in section 2, an introduction to AGM theory is presented. In section 3, we show how to adapt the AGM postulates for contraction so that they can be used with description logics. In section 4, we present a construction for AGM-style revision that does not depend on the negation of axioms. Finally we conclude and point towards future work.

2 AGM Paradigm

In the AGM paradigm [Alchourrón *et al.*, 1985], the beliefs of an agent are represented by a *belief set*, a set of formulas closed under logical consequence. The consequence operator Cn is assumed to be tarskian, compact, satisfy the deduction theorem and supraclassicality. We will sometimes refer to these properties as the *AGM-assumptions*. Three operations are defined: expansion, contraction and revision. Given a belief set K and a formula α, the expansion $K + \alpha$ is defined as $K + \alpha = Cn(K \cup \{\alpha\})$. Contraction consists in removing some belief from the belief set and revision consist in adding a new belief in such a way that the resulting set is consistent. Contraction and revision are not uniquely defined, but are constrained by a set of *rationality postulates*. The AGM basic postulates for contraction are:

(closure) $K - \alpha = Cn(K - \alpha)$

(success) If $\alpha \notin Cn(\emptyset)$ then $\alpha \notin K - \alpha$

(inclusion) $K - \alpha \subseteq K$

(vacuity) If $\alpha \notin K$ then $K - \alpha = K$

(recovery) $K \subseteq K - \alpha + \alpha$

(extensionality) If $Cn(\alpha) = Cn(\beta)$ then $K - \alpha = K - \beta$

Of the six postulates, five are very intuitive and widely accepted. The recovery postulate has been debated in the literature since the very beginning of AGM theory [Makinson, 1987]. Nevertheless, the intuition behind the postulate, that unnecessary loss of information should be avoided, is commonly accepted.

In [Alchourrón *et al.*, 1985], besides the postulates, the authors also present a construction for contraction (*partial-meet contraction*) which, for logics satisfying the AGM assumptions, is equivalent to the set of postulates in the following sense: every partial-meet contraction satisfies the six postulates and every operation that satisfies the postulates can be constructed as a partial-meet contraction.

The operation of revision is also constrained by a set of six basic postulates:

(closure) $K * \alpha = Cn(K * \alpha)$

(success) $\alpha \in K * a$

(inclusion) $K * \alpha \subseteq K + \alpha$

(vacuity) If $K + \alpha$ is consistent then $K * \alpha = K + \alpha$

(consistency) If α is consistent then $K * \alpha$ is consistent.

(extensionality) If $Cn(\alpha) = Cn(\beta)$ then $K * \alpha = K * \beta$

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In AGM theory, usually revision is constructed based on contraction and expansion, using the *Levi Identity*: $K * \alpha = (K - \neg\alpha) + \alpha$. The revision obtained using a partial-meet contraction is equivalent to the six basic postulates for revision.

3 AGM Contraction and Relevance

Although very elegant, the AGM paradigm cannot be applied to every logic. [Flouris *et al.*, 2004] define a logic to be *AGM-compliant* if it admits a contraction operation satisfying the six AGM postulates.

The authors also showed that the logics behind OWL ($\mathcal{SHIF}(\mathbf{D})$ and $\mathcal{SHOIN}(\mathbf{D})$) are not AGM-compliant. For this reason it was proposed in [Flouris *et al.*, 2006] that a new set of postulates for contraction should be defined. A contraction satisfying this new set of postulates should exist in any logic (*existence criteria*) and this new set of postulates should be equivalent to the AGM postulates for every AGM-compliant logic (*AGM-rationality criteria*).

In [Ribeiro and Wassermann, 2006] we have shown a set of postulates that partially fulfills these criteria, the AGM postulates with the *recovery* postulate exchanged by:

(relevance) If $\beta \in K \setminus K - \alpha$, then there is K' s. t. $K - \alpha \subseteq K' \subseteq K$ and $\alpha \notin Cn(K')$, but $\alpha \in Cn(K' \cup \{\beta\})$.

The relevance postulate was proposed in [Hansson, 1989] in order to capture the minimal change intuition, i.e. to avoid unnecessary loss of information. We have proven the following result:

Representation Theorem 3.1 *[Ribeiro and Wassermann, 2006] For every belief set K closed under a tarskian and compact logical consequence, $-$ is a partial meet contraction operation over K iff $-$ satisfies* closure*,* success*,* inclusion*,* vacuity*,* extensionality *and* relevance*.*

From this theorem, it follows that the *existence* criteria for this set of postulates is valid for tarskian and compact logics.

Corollary 3.2 *[Ribeiro and Wassermann, 2006] Every tarskian and compact logic is compliant with the AGM postulates if* recovery *is substituted by* relevance*.*

Furthermore, since partial meet contraction is equivalent to the AGM postulates for every logic that satisfies the AGM assumptions [Hansson, 1999], we have the following weaker version of the *AGM-rationality* criteria:

Corollary 3.3 *For every logic that satisfies the AGM assumptions* relevance *is equivalent to* recovery *in the presence of the other AGM postulates.*

4 AGM Revision without Negation

In [Ribeiro and Wassermann, 2008] we pointed out that the main problem to apply AGM revision to description logics is the absence of negation of axioms in some logics, since constructions for revision are usually based on the Levi identity. For example: there is not a consensus on which should be the negation of $R \sqsubseteq S$ in $\mathcal{SHOIN}(\mathbf{D})$. For this reason we should try to define constructions that do not depend on the negation of axioms. In [Ribeiro and Wassermann, 2008] we have proposed such constructions for belief bases (sets not necessarily closed under logical consequence). In this section we define a construction for revision without negation that can be used for revising belief sets.

Satisfying the AGM postulates for revision is quite easy. For example if we get $K * \alpha = K + \alpha$ if $K + \alpha$ is consistent and $K * \alpha = Cn(\alpha)$ otherwise, we end up with a construction that satisfies all the AGM postulates for revision.

This happens because there is no postulate in AGM revision that guaranties minimal loss of information We can define a minimality criterium for revision using a postulate similar to the relevance postulate for contraction:

(relevance) If $\beta \in K \setminus K * \alpha$ then there is K' such that $K \cap (K * \alpha) \subseteq K' \subseteq K$ and $K' \cup \{\alpha\}$ is consistent, but $K' \cup \{\alpha, \beta\}$ is inconsistent.

In order to define a construction that satisfies relevance, we will use the definition of maximally consistent sets with respect to a sentence, which are the maximal subsets of K that, together with α, are consistent:

Definition 4.1 (Maximally consistent set w.r.t α) *[Delgrande, 2008] $X \in K \downarrow \alpha$ iff: i.$X \subseteq K$. ii. $X \cup \{\alpha\}$ is consistent. iii. If $X \subset X' \subseteq K$ then $X' \cup \{\alpha\}$ is inconsistent.*

A selection function chooses at least one element of this set:

Definition 4.2 (Selection function) *[Alchourrón* et al.*, 1985] A* selection function *for K is a function γ such that: If $K \downarrow \alpha \neq \emptyset$, then $\emptyset \neq \gamma(K \downarrow \alpha) \subseteq K \downarrow \alpha$, otherwise, $\gamma(K \downarrow \alpha) = \{K\}$.*

We define partial-meet revision without negation by the intersection of the elements chosen by the selection function followed by an expansion by α:

Definition 4.3 (Revision without negation)

$$K *_\gamma \alpha = \bigcap \gamma(K \downarrow \alpha) + \alpha$$

The revision presented above satisfies the six AGM postulates for revision plus relevance:

Theorem 4.4 *$K *_\gamma \alpha$ satisfies the six basic AGM postulates for revision and relevance.*

Proof: *Closure*, *success* and *inclusion* follow directly from the construction. *Vacuity* follows from the fact that if $K + \alpha$ is consistent then $K \downarrow \alpha = \{K\}$. *Extensionality* follows from the fact that if $Cn(\alpha) = Cn(\beta)$ then for all sets X, $X \cup \{\alpha\}$ is inconsistent iff $X \cup \{\beta\}$ is also inconsistent. To prove *consistency*, assume that α is consistent and that $\bigcap \gamma(K \downarrow \alpha) + \alpha$ is inconsistent. Then since $\bigcap \gamma(K \downarrow \alpha) \subseteq X \in K \downarrow \alpha$, by monotonicity $X \cup \{\alpha\}$ is inconsistent, which contradicts the definition. For *relevance*, let $\beta \in K \setminus K *_\gamma \alpha$. Then there exists $X \in \gamma(K \downarrow \alpha)$ such that $\beta \notin X + \alpha$. We also know that for all $X' \in \gamma(K \downarrow \alpha)$, it holds that $\bigcap \gamma(K \downarrow \alpha) + \alpha \subseteq X' + \alpha$ and hence, $K \cap (\bigcap \gamma(K \downarrow \alpha) + \alpha) \subseteq K \cap (X' + \alpha)$. This holds in particular for X as above. Take $K' = K \cap (X + \alpha)$. □

It also satisfies a postulate that for classical logics follows from the AGM postulates, but not in the general case:

(uniformity) If for all $K' \subseteq K$, $K' \cup \{\alpha\}$ is inconsistent iff $K' \cup \{\beta\}$ is inconsistent then $K \cap K * \alpha = K \cap K * \beta$

In order to prove the representation theorem, we need to make use of two properties of consequence operations:

1. Inconsistent explosion: Whenever K is inconsistent, then for all formulas α, $\alpha \in Cn(K)$
2. Distributivity: For all sets of formulas X, Y and W, $Cn(X \cup (Cn(Y) \cap Cn(W))) = Cn(X \cup Y) \cap Cn(X \cup W)$

We also need the following lemmas:

Lemma 4.5 *For any monotonic and compact logic if $X \subseteq K$ and $X \cup \{\alpha\}$ is consistent then there is X' s.t. $X \subseteq X' \in K \downarrow \alpha$.*[1]

Lemma 4.6 *[Delgrande, 2008]$K \downarrow \alpha = K \downarrow \beta$ iff for all $X \subseteq K$, $X \cup \{\alpha\}$ is inconsistent iff $X \cup \{\beta\}$ is inconsistent.*

Lemma 4.7 *If Cn satisfies distributivity and * satisfies success, inclusion and consistency, then $(K \cap K * \alpha) + \alpha = K + \alpha \cap K * \alpha + \alpha = K * \alpha$*

The representation theorem states the equivalence between the construction and a set of postulates:

Representation Theorem 4.8 (Revision without negation) *For any monotonic and compact logic that satisfies inconsistent explosion and distributivity, a revision operator $*$ is a revision without negation $\bigcap \gamma(K \downarrow \alpha) + \alpha$ for some selection function γ if and only if it satisfies closure, success, inclusion, consistency, relevance and uniformity.*

Proof: (construction $\Rightarrow$ postulates):
The AGM postulates and relevance are satisfied, as shown in Theorem 4.4. For *uniformity*, note that if it holds that for all $K' \subseteq K$, $K' \cup \{\alpha\}$ is inconsistent iff $K' \cup \{\beta\}$ is inconsistent, then by Lemma 4.6, $K \downarrow \alpha = K \downarrow \beta$, and hence, $\bigcap \gamma(K \downarrow \alpha) = \bigcap \gamma(K \downarrow \beta)$.

(postulates $\Rightarrow$ construction):
Let $*$ be an operator satisfying the six postulates and let $\gamma(K \downarrow \alpha) = \{X \in K \downarrow \alpha | K \cap (K * \alpha) \subseteq X\}$ if α is consistent and $\gamma(K \downarrow \alpha) = \{K\}$ otherwise. We have to prove that: 1) $\gamma(K \downarrow \alpha)$ is a selection function and 2) $K *_\gamma \alpha = K * \alpha$.

1. First we need to prove that γ is well defined, i.e., that if $K \downarrow \alpha = K \downarrow \beta$ then $\gamma(K \downarrow \alpha) = \gamma(K \downarrow \beta)$. This follows directly from Lemma 4.6 and *uniformity*.

To prove that γ is a selection function we need to prove that if $K \downarrow \alpha \neq \emptyset$, then $\emptyset \neq \gamma(K \downarrow \alpha) \subseteq K \downarrow \alpha$. If α is consistent then it follows from *consistency* that $K * \alpha$ is consistent. In this case, because of *closure* and *success* $(K * \alpha) + \alpha$ is consistent. It follows that $(K \cap K * \alpha) \cup \{\alpha\}$ is consistent and by Lemma 4.5 there is X s.t. $K \cap (K * \alpha) \subseteq X \in K \downarrow \alpha$. Hence $X \in \gamma(K \downarrow \alpha)$.

2. If α is inconsistent, it follows from *closure*, *success*, and inconsistent explosion that both $K * \alpha$ and $K *_\gamma \alpha$ are the unique inconsistent belief set.

[1]This is a generalization of the *Upper Bound Property* used in [Alchourrón *et al.*, 1985]

Otherwise, $K \cap (K * \alpha) \subseteq X$ for every $X \in \gamma(K \downarrow \alpha)$. It follows that $K \cap (K * \alpha) \subseteq \bigcap \gamma(K \downarrow \alpha)$. By monotonicity, $(K \cap (K * \alpha)) + \alpha \subseteq \bigcap \gamma(K \perp_\alpha) + \alpha$ and by Lemma 4.7, $K * \alpha \subseteq K *_\gamma \alpha$.

To prove that $K *_\gamma \alpha \subseteq K * \alpha$, we will show that $\bigcap \gamma(K \downarrow \alpha) \subseteq K * \alpha$. Let $\beta \in \bigcap \gamma(K \downarrow \alpha) \setminus K * \alpha$. Since $\bigcap \gamma(K \downarrow \alpha) \subseteq K$, by *relevance*, there is K' such that $K \cap (K * \alpha) \subseteq K' \subseteq K$ and $K' \cup \{\alpha\}$ is consistent, but $K' \cup \{\alpha, \beta\}$ is inconsistent. Since $K' \subseteq K$ and $K' \cup \{\alpha\}$ is consistent, by Lemma 4.5 we know that there is X such that $\beta \notin X$, $K' \subseteq X \in K \downarrow \alpha$. Since $K \cap (K * \alpha) \subseteq X$, by the construction of γ, $X \in \gamma(K \downarrow \alpha)$ and hence, $\beta \notin \bigcap \gamma(K \downarrow \alpha)$. □

5 Conclusion and Future Work

Although it is not possible to apply the AGM paradigm directly to description logics, it is possible to adapt it. In the case of the contraction, that means a small change in the postulates. In the case of the revision, we need to adapt both construction and postulates.

As future work we plan to compare our set of postulates for contraction with the one proposed in [Flouris *et al.*, 2006] and study the relation between the contraction and revision operators proposed. We also plan to apply the solution for contraction and revision to other logics where negation is problematic, such as Horn logic.

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