3-Calabi-Yau down-up algebras

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Down-up algebras $A(\alpha, \beta, \gamma)$ have been introduced by G. Benkart and T. Roby in [1] motivated by the algebra generated by the down and up operators on a differential partially ordered set.

Given $\alpha, \beta, \gamma \in \mathbb{C}$, the algebra $A(\alpha, \beta, \gamma)$ is generated over $\mathbb{C}$ by two elements d, u, subject to the relations

$$d^2u - \alpha dud - \beta ud^2 - \gamma d,$$
$$du^2 - \alpha udu - \beta du^2 - \gamma u.$$

Starting from Bardzell's resolution for monomial algebras and using Bergman's Diamond Lemma we obtain a free bimodule resolution of $A(\alpha, \beta, 0)$ of length 3 which we use to prove that $A(\alpha, \beta, 0)$ is 3-Calabi-Yau if and only if $\beta = -1$.

References

[1] Benkart, Georgia; Roby, Tom Down-up algebras. J. Algebra 209 (1998), no. 1, pp.305–344.