To Homological Characterization of Semirings: p-Schreier Varieties of Semimodules, Serre's Problem on Projective Semimodules over Polynomial Semirings, and Related Problems

Yefim Katsov
Hanover College, USA

In 1976, D. Quillen and A. Suslin, resolving Serre's famous problem– "conjecture," independently proved that the classes of projective and free modules over the polynomial rings $R[x_1, x_2, \ldots, x_n]$, $n \in \mathbb{N} := \{0, 1, \ldots\}$, with a ground field R coincide. In this talk, we consider the Serre's problem in a more general semiring setting. In particular, calling a category $\mathcal{M}_R$ of right R-semimodules over a semiring R a *p-Schreier* (*Schreier*) *variety* iff all projective R-semimodules (all subsemimodules of free R-semimodules) are free in $\mathcal{M}_R$, we present complete descriptions of p-Schreier (Schreier) varieties $\mathcal{M}_R$ over division semirings R. Moreover, we give a complete description of proper division semirings R whose categories of semimodules $\mathcal{M}_{R(X)}$ over the polynomial semirings $R(X)$ over R, in not necessary commuting variables X, are p-Schreier varieties and show that the categories of semimodules $\mathcal{M}_{R(X)}$ over the polynomial semirings $R(X)$ over $\mathbf{N}$-valued semirings R, in particular $\mathcal{M}_{\mathbf{N}(X)}$, are p-Schreier varieties; we also show that for $\mathbf{N}$-valued semirings S, the semimodule categories $\mathcal{M}_S$ are not Schreier varieties. If time permits, we might discuss some related problems and directions for future investigations.