Varieties of associative algebras satisfying semigroup identities

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Given a ring $\langle R, +, \cdot\rangle$, the *circle composition* $\circ$ on R is defined by letting $a \circ b = a + b - ab$ for all $a, b \in R$. If R is associative, then the circle composition is an associative operation too. In this case one can assign to $\langle R, +, \cdot\rangle$ two semigroups: the multiplicative semigroup $\langle R, \cdot\rangle$ and the *adjoint* semigroup $\langle R, \circ\rangle$. We study varieties consisting of algebras and rings whose multiplicative (adjoint) semigroups satisfy nontrivial identities. In the case of algebras over an infinite field different characterizations of such varieties can be deduced from certain results of A. V. Mikhalev, I. Z. Golubchik, D. M. Riley, M. C. Wilson, L. M. Samoilov, Yu. N. Maltsev. We consider the case of algebras over a finite field and the case of rings and find descriptions of such varieties in the language of forbidden algebras. In particular, these descriptions imply that if the multiplicative (adjoint) semigroup of a ring satisfies a nontrivial semigroup identity then it also satisfies the identity $x^{n+m}yx^n = x^nyx^{n+m}$ for some positive integers n, m.