CENTRAL SIMPLE ALGEBRAS OF PRIME EXPONENT AND DIVIDED POWER OPERATIONS

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Abstract. Let p be a prime and F a field of characteristic different from p, containing all p-primary roots of unity. As usual denote by ${}_{\mathrm{p}}\operatorname{Br}(F)$ the p-torsion of the Brauer group $\operatorname{Br}(F)$. We give a necessary condition for $\alpha \in {}_{\mathrm{p}}\operatorname{Br}(F)$ to be cyclic in terms of Milnor's K-groups. Namely, suppose L/F is a cyclic field extension such that $\alpha_L = 0$. We show that $\gamma_i(\alpha) = 0$ for all $i \geq 2$, where

$$\gamma_i : {}_{\mathrm{p}}\operatorname{Br}(F) \simeq K_2(F)/pK_2(F) \to K_{2i}(F)/pK_{2i}(F)$$

are the divided power operations of degree p. The most part of the proof is elementary and based on the residues technique for rational function fields, but one time we have to use the very deep result of Rost and Voevodsky on bijectivity of the norm residue homomorphism $K_*(F)/p \to H^*(F, \mu_p)$.

Time permitted we will show that in general the condition above is not sufficient, and pose a few open questions.