

EIGENVALUE COINCIDENCES AND K -ORBITS ON THE FLAG VARIETY

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This talk is based on joint work with Sam Evens which relates the Gelfand-Zeitlin integrable system on $gl(n, C)$ to orbits of $K = GL(n - 1, C) \times GL(1, C)$ on the flag variety of $gl(n, C)$. For an n by n matrix $x \in gl(n, C)$, let x_{n-1} be the $n-1$ by $n-1$ matrix in the upper left corner of x . We study the variety $X(j)$ consisting of matrices x such that x and x_{n-1} share at least j eigenvalues in common counting repetitions. We show that $X(j)$ is an equidimensional variety of codimension j . The irreducible components of $X(j)$ are given by the K -saturation of Borel subalgebras which generate K -orbits of codimension j in the flag variety of $gl(n, C)$. Further, we use this description of the varieties $X(j)$ for $j = 0, \dots, n - 1$ to study the K -action on $gl(n, C)$. We compute the closed K -orbits on $gl(n, C)$ and show that ring of invariants $C[gl(n, C)]^K$ is polynomial and find explicit generators.