COMPUTATIONAL APPROACH TO ALGEBRAS WITH POLYNOMIAL IDENTITIES

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A polynomial $f(x_1, \ldots, x_m)$ in the free associative algebra $K\langle x_1, x_2, \ldots\rangle$ is a polynomial identity for the algebra R over the field K if $f(r_1, \ldots, r_m) = 0$ for all $r_1, \ldots, r_m \in R$. If $f(x_1, \ldots, x_m) \neq 0$ in $K\langle x_1, x_2, \ldots\rangle$, then R is called a PI-algebra. The class of PI-algebras enjoys nice structural and combinatorial properties similar to those of commutative algebras and finite dimensional algebras.

We present a survey on PI-algebras over a field of characteristic 0 from computational point of view. We describe several computer methods for calculation with polynomial identities of matrices and related objects. Among the other applications, these methods have been successfully used to discover new central polynomials of low degree for matrices of any order, to show that all polynomial identities of degree $2n + 2$ for the $n \times n$ matrix algebra for $n = 3, 4, 5$ are consequences of the standard identity s_{2n}, etc. Further, we discuss generators and defining relations for the free algebra modulo the polynomial identities of the Grassmann algebra and the 2×2 matrix algebra, as well as generic trace matrix algebras of small order.

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