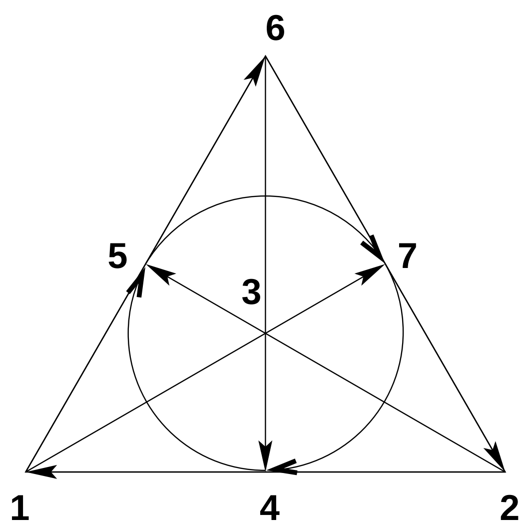
C O N F E R E N C E

Lie and Jordan algebras, their Representations and Applications, II

Brasil, Guarujá, 3–8 May 2004

Abstracts



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Monday, May 3

9:00 - 10:00	**Victor Kac** Non-linear algebras

coffee break

10:20 - 11:20	**Ivan Shestakov** Subalgebras and automorphisms of polynomial rings
11:40 - 12:40	**Yuly Billig** Representations for toroidal Lie algebras and Lie algebras of vector field

lunch

14:30 - 15:30	**Fyodor Malikov** Vertex algebras and the Calabi-Yau/Landau-Ginzburg correspondence

coffee break

Session: Structure theory and Representations

15:50 - 16:20	**Murray Bremner** Dimension formulas for free nonassociative algebras
16:30 - 17:00	**Bruce Allison** Coordinate algebras for extended affine Lie algebras of rank 1
17:10 - 17:40	**Brian D. Boe** Representation type of the blocks of category $\mathcal{O}_S$
17:50 - 18:20	**Vyacheslav Futorny** Harish-Chandra modules for Yangians

Tuesday, May 4

9:00 - 10:00	**Gus Lehrer** Generalised Euler Characteristics of Varieties of Tori in Lie Groups

coffee break

10:20 - 11:20	**Olivier Mathieu** On modular unipotent representations of $GL(n)$
11:40 - 12:40	**José M. Pérez-Izquierdo** Sabinin algebras: The basis of a Nonassociative Lie theory

lunch

Sessions: (Structure theory / Representations and Applications)

14:30 - 15:00	**Francisco César Polcino Milies** Higman's theorem and f-unitary Moufang loops of units **Jose Liberati** Conformal subalgebras of $Cend_n$ and gc_n
15:05 - 15:35	**Dessislava Kochloukova** Homological properties of Lie algebras **Fernando Levstein** The Terwilliger algebra of some association schemes
15:40 - 16:10	**Luis Antonio Peresi** Elements of minimal degree in the center of the free alternative algebra **Duncan Melville** Verma-type modules for affine Lie superalgebras

coffee break

16:30 - 17:00	**Natalia Zhukavets** Universal multiplicative envelope of free Malcev superalgebra on one odd generator **Abdel Latif Mortajine** Covariants of prehomogeneous vector spaces
17:05 - 17:35	**Henrique Guzzo Jr** The b-radical in Bernstein-Jordan algebras **Maria Trushina** Irreducible representation of a certain Jordan superalgebra
17:40 - 18:10	**Andrei Zavarnitsine** Maximal subloops of finite simple Moufang loops **Iryna Kashuba** Representation type of Jordan algebras

Thursday, May 6

9:00 - 10:00	**Anatoly M. Vershik** Canonical states on the Lie group, cohomology and isometric imbedding

coffee break

10:20 - 11:20	**Daniel Nakano** Varieties of nilpotent matrices for simple Lie algebras
11:40 - 12:40	**Issai Kantor** Generalized representations of Jordan algebras

lunch

Sessions: (Structure theory / Applications)

14:30 - 15:00	**Plamen Koshlukov** Polynomial identities with involution in matrix algebras **Jiří Patera** Applications of orbit functions of compact semisimple Lie groups
15:05 - 15:35	**Alexander Grishkov** A new series of simple finite dimensional Lie algebras over a field of characteristic 2 **Dmitry Gitman** Quantization on Riemannian Manifolds
15:40 - 16:10	**Esther Garcia** Towards a Socle for Lie algebras I **Frank Michael Forger** Finite groups and the generacy of the genetic code

coffee break

16:30 - 17:00	**Miguel Gomez Lozano** Towards a Socle for Lie algebras II **Semyon Konstein** Example of simple Lie superalgebra with several independent invariant bilinear forms
17:05 - 17:35	**Juan Carlos Gutiérrez Fernández** On commutative power-associative nilalgebras **Andrei Smirnov** Self-adjoint extensions of the Dirac operator with the Aharonov-Bohm potential
17:40 - 18:10	**Maribel Tocon Barroso** The ideal of the Lesieur-Croisot elements of a Jordan algebra **Pavel Kolesnikov** Irreducible conformal subalgebras of $Cend_n$ and gc_n
18:15 - 18:45	**Lucia Satie Ikemoto Murakami** On right alternative bimodules **Roldao da Rocha** Octonions and the standard model

Friday, May 7

9:00 - 10:00	**Wilhelm Kaup** Jordan algebras and Cauchy-Riemann geometry

coffee break

10:20 - 11:20	**Kevin McCrimmon** The role of identities in Jordan algebras
11:40 - 12:40	**Alberto Elduque** Symmetric composition algebras and Freudental's Magic Square

lunch

Sessions: (Structure theory / Representations)

14:30 - 15:00	**Kurt Meyberg** Survey on R-algebras **María Concepción López-Díaz** On representations of exceptional simple Jordan superalgebras of characteristic 3
15:05 - 15:35	**Alicia Labra** A note on a class of commutative algebra **Gil Salgado** Lie superalgebras based on $gl(n)$
15:40 - 16:10	**Cristina Draper Fontanals** Models of F_4 **Natig Atakishiyev** Representation of the quantum algebra $U_q(su_{1,1})$ and the duality property of q-orthogonal polynomials

coffee break

16:30 - 17:00	**Fabian Martin-Herce** Irreducible Lie-Yamaguti algebras and L-projection **Mari Sano** A combinatorial description of the syzygies of certain Weyl modules
17:05 - 17:35	**Marines Guerreiro** A Lie algebra over a field of characteristic 2 **Michael Dokuchaev** Associativity of crossed products by partial actions
17:40 - 18:10	**Ibrahim Mashhour** From Cayley Dickson algebra to $G_2(q)$ **Viktor Bekkert** Harish-Chandra modules for generalized crossed products
18:15 - 18:45	**Victor Bovdi** Modular group algebras with maximal Lie nilpotency indices

Saturday, May 8

9:00 - 10:00	**Arturo Pianzola** Twisted loop algebras

coffee break

10:20 - 11:20	**Marc Rosso** Tensor products of irreducible representation of quantum groups at root of 1
11:40 - 12:40	**Ivan Dimitrov** Borel subalgebras of $gl(\infty)$

Contents

Coordinate Algebras for Extended Affine Lie Algebras of Rank 1

Bruce Allison

University of Virginia/Alberta

Extended affine Lie algebras (or EALAs for short) are natural generalizations of affine Kac-Moody Lie algebras. The nullity of an EALA $\mathcal{E}$ is defined to be the rank of the group Λ generated by the null roots of $\mathcal{E}$. In the case of an affine Kac-Moody Lie algebra this nullity is 1.

In general an EALA can be constructed by a process of double extension from its centreless core $\mathcal{K}$ (a twisted loop algebra when the nullity is 1). Moreover $\mathcal{K}$ is graded by a possibly nonreduced finite root system Δ, and consequently $\mathcal{K}$ can be coordinatized by a infinite dimensional Λ-graded nonassociative algebra $\mathcal{A}$ called a torus. The simplest example occurs when Δ has type A_ℓ ($\ell \geq 3$), in which case $\mathcal{A}$ is an associative quantum torus. In this talk we will discuss the tori that occur when the root system Δ has rank 1, or equivalently has type A_1 or BC_1.

In type A_1, the coordinate tori are Jordan tori which have been classified by Yoji Yoshii. Examples include the Albert torus, whose central closure is a finite dimensional Albert algebra over its centre. In recent work, Yoshii, John Faulkner and I have been investigating the coordinate tori, called structurable tori, that occur in type BC_1. This talk will report on some progress in that work.

Representations of the quantum algebra $U_q(\mathrm{su}_{1,1})$ and the duality property of q-orthogonal polynomials

N.M. Atakishiyev, A.U. Klimyk

UNAM, Mexico and *Bogolyubov Institute for Theoretical Physics, Ukraine*

We discuss an application of the discrete series representations of the quantum algebra $U_q(\mathrm{su}_{1,1})$ (which is a real form of the Drinfeld–Jimbo quantum algebra $U_q(\mathrm{sl}_2)$) to the study of properties of q-orthogonal polynomials. To achieve this, we employ two operators of the representation space, constituting a generalized Leonard pair. One of these operators is related to a three-term recurrence relation for a given set of orthogonal polynomials and the second one to a q-difference equation for them. This approach allows us to consider a given set of q-orthogonal polynomials and a dual family with respect to this set. By means of these two operators and a notion of duality, one can prove orthogonality relations for given sets of polynomials and their duals.

Our results can be viewed as an extension of the notion of duality, which is well known for orthogonal polynomials on a finite set of points, to the case of countable sets of points of orthogonality. In this way one can pair known families of q-orthogonal polynomials into dual families. For some families of polynomials this notion of duality leads to families of q-orthogonal polynomials, which have not been discussed in the literature. The q-orthogonal polynomials, dual to the big q-Jacobi polynomials, may be an instance of such novel sets.

Harish-Chandra modules for generalized crossed products

Viktor Bekkert

UFMG, Brazil

(joint work with G. Benkart and Vyacheslav Futorny)

Let $\mathcal{M}$ be a monoid with the unit element e, D be a commutative ring with a unit element 1, $\vartheta : \mathcal{M} \to \mathsf{Aut}\, D$ be a monoid morphism and $\xi : \mathcal{M} \times \mathcal{M} \to D$ be a 2-cocycle, i.e., it satisfies the conditions

$$\xi(f,g)\xi(fg,h) = \vartheta_g(\xi(g,h))\xi(f,gh),$$

$$\xi(g,e) = \xi(e,g) = 1,$$

for $f,g,h \in \mathcal{M}$. Here ϑ_g denotes the image of an element $g \in \mathcal{M}$ under the morphism ϑ. We consider a $\mathcal{M}$-graded ring $R = D\{\vartheta, \xi\} = \oplus_{g\in\mathcal{M}} R_g = \oplus_{g\in\mathcal{M}} Dx_g$ with Dx_g being a free left D-module and $x_e = 1$. The multiplicative structure of R is given by: for $g,h \in \mathcal{M}$ and $r_g, r_h \in D$

$$(r_g x_g)(r_h x_h) = r_g \vartheta_g(r_h)\xi(g,h)x_{gh}.$$

The class of graded rings considered contains the nth Weyl algebra A_n, quantized Weyl algebras $A_n^{q,\Lambda}$, q-deformed Heisenberg algebra and others.

A module $M \in R$-mod is called *Harish-Chandra module* (with respect to D) if $M = \bigoplus_{\mathfrak{m}\in\mathsf{Specm} D} M(\mathfrak{m})$, where

$$M(\mathfrak{m}) = \{x \in M \mid \text{there exists}\, k \geq 0, \text{ such that}\, \mathfrak{m}^k x = 0\}.$$

A module $M \in R$-mod is called *weight module* (with respect to D) if $M = \bigoplus_{\mathfrak{m}\in\mathsf{Specm} D} M(\mathfrak{m})$, where

$$M(\mathfrak{m}) = \{x \in M \mid \mathfrak{m}x = 0\}.$$

We investigate Harish-Chandra modules for such generalized crossed products, reducing the classification of such simple modules to the determination of maximal ideals in certain polynomial algebras and maximal left ideals in certain iterated skew-polynomial algebras. We also study the representation type of the blocks of locally-finite Harish-Chandra module categories and of locally-finite weight module categories.

Representations for toroidal Lie algebras and Lie algebras of vector fields

Yuly Billig

Carleton University, Canada

In this talk we will present new results and conjectures in representation theory of toroidal Lie algebras and the Lie algebras of vector fields on a torus.

Representation type of the blocks of category $\mathcal{O}_S$

Brian D. Boe

University of Georgia, USA

We investigate the representation type of the blocks of the relative (parabolic) category $\mathcal{O}_S$ for complex semisimple Lie algebras. The main result provides a classification of the blocks in the "mixed" case when the simple roots corresponding to the singular set and S do not meet (including, as a special case, a complete classification of the blocks corresponding to regular weights). This is joint work with Dan Nakano.

Modular group algebras with maximal Lie nilpotency indices

Victor Bovdi

University of Debrecen, Hungary

Let R be an associative algebra with identity. The ring R can be treated as a Lie algebra under the Lie product $[x, y] = xy - yx$, where $x, y \in R$. Set $[x_1, \dots, x_n] = [[x_1, \dots, x_{n-1}], x_n]$, where $x_1, \dots, x_n \in R$. The *n-th lower Lie power* $R^{[n]}$ of R is the associative ideal generated by all Lie products $[x_1, ..., x_n]$, where $R^{[1]} = R$ and $x_1, ..., x_n \in R$. The *n-th upper Lie nilpotent power* $R^{(n)}$ of R is the associative ideal generated by all Lie products $[x, y]$, where $R^{(1)} = R$ and $x \in R^{(n-1)}$, $y \in R$.

The ring R is called *Lie nilpotent* if there exists m such that $R^{[m]} = 0$. The minimal integers m, n such that $R^{[m]} = 0$ and $R^{(n)} = 0$ are called *the lower Lie nilpotency indices* and *the upper Lie nilpotency indices* of R and they are denoted by $t_L(R)$ and $t^L(R)$, respectively.

Let $R = KG$ be the Lie nilpotent group algebra with $char(K) = p > 0$. If KG is Lie nilpotent [3] then $t_L(KG) \le t^L(KG) \le |G'| + 1$. Moreover, if $char(K) > 3$, then $t_L(KG) = t^L(KG)$ (see [1]). But the question, when is $t_L(KG) = t^L(KG)$ for $char(K) = 2, 3$ is still open.

As Shalev [2] proved, if G is a finite p-group and $char(K) \ge 5$, then $t_L(KG) = |G'| + 1$ if and only if G' is cyclic. We proved the following:

Theorem (V. Bovdi, E.Spinelli). *Let KG be a Lie nilpotent group algebra with $char(K) = p > 0$. Then $t_L(KG) = |G'| + 1$ if and only if one of the following conditions holds:*

- *G' is cyclic;*
- *$p = 2$ and G' is noncyclic of order 4 and $\gamma_3(G) \neq 1$.*

Corollary. *Let KG be a Lie nilpotent group algebra with $char(K) > 0$ such that $t^L(KG) = |G'| + 1$. Then $t_L(KG) = t^L(KG)$.*

References

[1] Bhandari, A. K.; Passi, I.B.S. *Lie Nilpotency indices of group algebras*, Bull.London Math. Soc. **24** (1992), 68–70.

[2] Shalev, A. *The nilpotency class of the unit group of a modular group algebra. III.*, Arch. Math. (Basel) **60** (2) (1993), 136–145.

[3] Sharma, R.K., Vikas, Bist *A note on Lie nilpotent group rings*, Bull. Austral. Math. Soc. **45** (1992), 503–506.

Dimension formulas for free nonassociative algebras

Murray Bremner

University of Saskatchewan, Canada

There is a well-developed theory relating free associative algebras and free Lie algebras. Two of the most important results are (1) Friedrich's criterion, which characterizes the Lie elements of a free associative algebra as the elements which are "primitive" in the sense of Hopf algebras, and (2) Witt's dimension formula, which gives the dimension of the subspace of Lie polynomials of any degree in the free associative algebra on any number of generators. Within the last few years Ivan Shestakov has proved some important theorems about free nonassociative algebras which have made it possible to generalize these two results. I will begin by reviewing the associative theory, and then introduce the nonassociative theory in which Akivis algebras play the role of Lie algebras. In the nonassociative case, the Akivis elements and the primitive elements do not coincide. Every Akivis element is primitive, but there are primitive elements which are not Akivis. I will give examples of Akivis and primitive elements up to degree 5 in one variable. Using Shestakov's theorem, we obtain a fast algorithm for computing the dimension of the Akivis elements of any degree in the free nonassociative algebra on any number of generators. Using a theorem of Shestakov and Umirbaev, we prove a closed formula for the dimension of the primitive elements in the free nonassociative algebra. This is joint work with Irvin Henztel of Iowa State University and Luiz Peresi of Universidade de Sao Paulo.

Associativity of crossed products by partial actions

M. Dokuchaev
IME-USP, Brazil

Partial actions of groups have been defined by Ruy Exel (see [2]) and became a powerful tool in the theory of operator algebras. In the most general setting of partial actions on abstract sets the definition is as follows:

Definition 1. *Let G be a group with identity element e and $\mathcal{X}$ be a set. A partial action α of G on $\mathcal{X}$ is a collection of subsets $\mathcal{D}_g \subseteq \mathcal{X}$ $(g \in G)$ and bijections $\alpha_g : \mathcal{D}_{g^{-1}} \to \mathcal{D}_g$ such that*

(i) $\mathcal{D}_e = \mathcal{X}$ *and* α_e *is the identity map of* $\mathcal{X}$*;*
(ii) $\mathcal{D}_{(gh)^{-1}} \supseteq \alpha_h^{-1}(\mathcal{D}_h \cap \mathcal{D}_{g^{-1}})$*;*
(iii) $(\alpha_g \circ \alpha_h)(x) = \alpha_{gh}(x)$ *for each* $x \in {\alpha_h}^{-1}(\mathcal{D}_h \cap \mathcal{D}_{g^{-1}})$.

It also can be defined as a partial homomorphism (partial representation) from G to the symmetric inverse semigroup $\mathcal{I}(\mathcal{X})$ of $\mathcal{X}$ (see [2]).

In order to define a partial action α of a group G on a unital associative K-algebra $\mathcal{A}$ over a commutative ring K we suppose in Definition 1 that each $\mathcal{D}_g$ $(g \in G)$ is an ideal of $\mathcal{A}$ and that every map $\alpha_g : \mathcal{D}_{g^{-1}} \mapsto \mathcal{D}_g$ is an isomorphism of algebras. Using partial actions one can generalize the concept of crossed product. For simplicity we assume that the twisting is trivial, so we give the definition in the context of corresponding skew group rings.

Definition 2. *Given a partial action α of G on $\mathcal{A}$, the skew group ring $\mathcal{A}_\alpha * G$ corresponding to α is the set of all formal sums $\{\sum_{g\in G} a_g\delta_g : a_g \in \mathcal{D}_g\}$, where δ_g are symbols. The addition is defined by the obvious way and the multiplication is determined by $(a_g\delta_g)\cdot(b_h\delta_h) = \alpha_g(\alpha_{g^{-1}}(a_g)b_h)\delta_{gh}$.*

It turns out that $\mathcal{A}_\alpha * G$ is not necessarily associative. There are examples showing that in general it does not even have associative powers. Our main result on associativity of $\mathcal{A}_\alpha * G$ is obtained in colaboration with Ruy Exel [1] and asserts that $\mathcal{A}_\alpha * G$ is always associative provided that $\mathcal{A}$ is semiprime.

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Models of F_4

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Some new constructions of the exceptional Lie algebra F_4 will be given, based on semisimple subalgebras of maximal rank.

q-commutators

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Let $(A, \circ)$ be an algebra with a vector space A and a multiplication $\circ$. A q-commutator $\circ_q$ on A is defined by $a \circ_q b = a \circ b + q\, b \circ a$. Let $(a,b,c) = a \circ (b \circ c) - (a \circ b) \circ c$ be an associator. For some category of PI-algebras $\mathfrak{L}$ denote by $\mathfrak{L}^{(q)}$ a category of algebras $A^{(q)} = (A, \circ_q)$, where $A \in \mathfrak{L}$. Let $[a,b] = a \circ b - b \circ a$, and $\{a,b\} = a \circ b - b \circ a$.

Theorem 1. *Let $q^2 \neq 1$. Let $\mathfrak{Ass}$ be a category of associative algebras. Then the category $\mathfrak{Ass}^{(q)}$ is defined by the identity $(q-1)^2(a,c,b) + q[c,[a,b]] = 0$ if $q^2 - 4q + 1 \neq 0$. If $q^2 - 4q + 1 \neq 0$ then $\mathfrak{Ass}^{(q)}$ is equivalent to $\mathfrak{Ass}$. If $q^2 - 4q + 1 = 0$, then $\mathfrak{Ass}^{(q)}$ is equivalent to the category of alternative algebras.*

Theorem 2. *Let $q^2 \neq 1, q^3 \neq -1$. Let $\mathfrak{Alt}$ be a category of alternative algebras. Then $\mathfrak{Alt}^{(q)}$ is defined by the identities*

$$(a,b,c) + (c,b,a) = 0,$$

$$(1 - q + q^2) c \circ \{a,b\} - q\{a,b\} \circ c - (q^2+1)((c \circ a) \circ b + (c \circ b) \circ a)$$
$$+2q((a \circ c) \circ b + (b \circ c) \circ a) = 0.$$

Theorem 3. *Let $q^2 \neq -1$. Let $\mathfrak{Rsym}$ be a category of right-symmetric algebras, i.e., algebras with identity $(a,b,c) + (a,c,b) = 0$. Then $\mathfrak{Rsym}$ satisfies the identity*

$$(q-1)(-(a,c,b) + (b,c,a)) + (q^2-1)((a,b,c) - (b,a,c)) - q[[a,b],c] = 0.$$

If $q^2 + 2q - 2 \neq 0$, then this identity is basic identity for $\mathfrak{Rsym}^{(q)}$. If $q^2 + 2q - 2 = 0$, then this identity and Lie-admissible identity form basis for identities of $\mathfrak{Rsym}^{(q)}$.

Theorem 4. *Let $\mathfrak{Lei}$ be the category of Leibniz algebras, i.e., algebras with the identity $(a \circ b) \circ c = a \circ (b \circ c) - b \circ (a \circ c)$. Then $\mathfrak{Lei}^{(-1)}$ is defined by skew-symmetric identity and by three identities of degree 5 (they are too huge to be presented here: they have 9, 60 and 62 terms). T-Ideal of identities for $\mathfrak{Lei}^{(1)}$ is generated by the identity $(a \circ b) \circ (c \circ d) = 0$. If $q^2 \neq 1$, then $\mathfrak{Lei}^{(q)}$ is generated by identities*

$$(1-q) a \circ (b \circ c) + (q^3 - q + 1) a \circ (c \circ b) - q^2 b \circ (a \circ c) - (q^3 - q) c \circ (a \circ b)$$
$$-q(b \circ c) \circ a + (q^3 + q^2 - q)(c \circ a) \circ b = 0,$$

$$-a \circ \{b, c\} + q\{b, c\} \circ a = 0.$$

If $(q+2)(q^4+2q^3-q+1)=0$*, then these identities are independent.*

Similar constructions for q-commutator identities for Leibniz dual algebras, for Novikov algebras and other classes of algebras are given.

Nagata-Higman theorem for Leibniz dual algebras

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An algebra with identity $(a \circ b) \circ c = a \circ (b \circ c + c \circ b)$ is called Leibniz dual. Example: $A = K[x]$ under multiplication $a \circ b = a \int_0^x b$ is Leibniz dual. Let $a^{\cdot n}$ be a right-bracket n-power $a \circ (\cdots (a \circ a) \cdots)$, where $a \in A$. An algebra A is called nil, if for any $a \in A$ there exists $n = n(a)$ such that $a^{\cdot n} = 0$. Say that A is nil with nil-index n if $a^{\cdot n} = 0$ for any $a \in A$. A is called

nilpotent if there exists some N such that $a_1 \circ (\cdots (a_{N-1} \circ a_N) \cdots) = 0$ for any $a_1, \ldots, a_N \in A$. An algebra A is called solvable if $A^{(k)} = 0$ for some k, where $A^{(i)}$ are defined by $A^{(0)} = A, \quad A^{(i+1)} = A^{(i)} \circ A^{(i)}, i > 0$.

Theorem 1. Let K be an algebraically closed field of characteristic $p \geq 0$. Then every finite-dimensional Leibniz dual algebra is solvable.

Theorem 2. Let K be a field of characteristic $p \geq 0$ and A be a solvable Leibniz dual algebra with solvability length N. If $p = 0$ or $p > 2^N - 1$, then A is nil with nil-index 2^N. Conversely, if A is a nil Leibniz dual algebra with nil-index N and if $p = 0$ or $p > N - 1$, then A is solvable with solvability length N.

Theorem 3. Let K be a field of characteristic $p \geq 0$. Every Leibniz dual nil-algebra is nilpotent. If A is nil with nil-index n then nilpotency index of A is no more than $2^n - 1$.

Symmetric composition algebras and Freudenthal's Magic Square

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The well-known Tits construction provides models of the exceptional simple Lie algebras using, as ingredients, a unital composition algebra and a degree three central simple Jordan algebra. By varying both ingredients, Freudenthal's Magic Square is obtained.

On the other hand, since degree three simple Jordan algebras are obtained as 3×3 matrices over unital composition algebras, Tits construction can be interpreted as a construction based on two unital composition algebras. But, even though the construction is not symmetric on the two algebras, so is the outcome (the Magic Square). Several more symmetric constructions have been proposed, based on the triality phenomenon. Here a new construction of the Magic Square will be presented, based on a pair of the so called "symmetric composition algebras", which provide very simple formulas for triality.

Finally, some of these symmetric composition algebras can be constructed in terms of copies of the two-dimensional natural module for $sl(2)$. This gives a unified construction of the simple exceptional Lie algebras in terms of these tiny ingredients: copies of $sl(2)$ and of its natural module.

Finite Groups and the Degeneracy of the Genetic Code

Frank Michael Forger
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We present the recently completed full classification of the possible symmetry breaking schemes that allow to reproduce the degeneracy of the genetic code, starting from an irreducible 64-dimensional representation of a simple finite group or one of its satellites, i.e., one of its upwards and/or downwards extensions. (Based on joint work with F. Antoneli, supported by FAPESP.)

Harish-Chandra modules over the Yangians

Vyacheslav Futorny
IME-USP, Brazil

This talk is based on our joint recent results with A.Molev (University of Sydney) and S.Ovsienko (University of Kiev).

We will discuss the theory of Harish-Chandra modules for the Yangians of the general linear Lie algebra. Yangians first appeared in the works of L.Faddeev and his school in their study of integrable models. They form a family of quantum groups related to the rational solutions of Yang-Baxter equation. Yangians were introduced by V.Drinfeld in 1985 who also classified all finite-dimensional irreducible representations. In the last decade the Yangians found a number of applications in Mathematics and Physics, in particular in conformal field theory, statistical mechanics, quantum gravity, representation theory etc.

Our recent progress in the study of Harish-Chandra modules over Yangians is based on the development of the theory of Harish-Chandra categories for associative algebras and on certain generalization of a famous result of B.Kostant on the freeness of the universal enveloping algebra of a simple Lie algebra over its center.

Towards a Socle for Lie Algebras I

Esther Garcia

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(joint work with A. Fernández López and M. Gómez Lozano)

In the search of a notion of socle for Lie algebras we start with a nondegenerate 3-graded Lie algebra (L, π) with associated Jordan pair V_π. In this context, the socle of (L, π) is defined as the subalgebra of L generated by the socle of V_π. The socle $\mathrm{Soc}(L, \pi)$ turns out to be an ideal of L, can be expressed as the sum of all minimal 3-graded inner ideals of L, and is a sum of simple ideals (TKK-algebras of the simple components of the socle of V). This allows us to give a structure theory for nondegenerate 3-graded Lie algebras with essential socle in terms of TKK-algebras of simple Jordan pairs (which we have determined) and their derivation algebras.

A natural question to ask is whether $\mathrm{Soc}(L, \pi)$ depends on the grading π. In general, the answer is yes. However, $\mathrm{Soc}(L, \pi)$ turns out to be independent of *effective* gradings (no nonzero ideal of the Lie algebra is contained in the zero part of L). Actually, the socle defined by any grading is contained in the socle defined by an effective grading. This inspires the notion of Jordan socle for a – non-necessarily 3-graded – nondegenerate Lie algebra L as the sum of the socles of all 3-graded ideals of L. The Jordan socle is a 3-graded ideal of L, and if L is actually 3-graded with an effective grading, then the Jordan socle of L coincides with that defined by the effective grading. Moreover, the socle structure theory for nondegenerate 3-graded Lie algebras is still valid in this setting.

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Quantization on Riemannian Manifolds

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We formulate the covariant generalization of the Weyl ordering and study its properties. Then we apply such an ordering to the operatorial quantization of the non-relativistic particle, discussing the well-known problem of the quantum potential. The corresponding relativistic problem is discussed as well.

Towards a Socle for Lie Algebras II

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(joint work with A. Fernández López and E. Garcia)

After the introduction of the notion of Jordan socle in [1] and [2], we prove that any nondegenerate Lie algebra with essential Jordan socle is sandwiched, via the adjoint mapping, between the TKK-algebra TKK(V) of a nondegenerate Jordan pair V coinciding with its socle, and the algebra of derivations Der(TKK(V)). Moreover, in this case, $\mathrm{JSoc}(L) = \mathrm{ad}(\mathrm{TKK}(V)) = \bigoplus \mathrm{ad}(\mathrm{TKK}(V_i))$, where the V_i are simple Jordan pairs with minimal inner ideals.

Thus, to describe the Jordan socle of a nondegenerate Lie algebra, it suffices to compute the TKK-algebras of the simple Jordan pairs with minimal inner ideals. We prove that, up to two exceptional cases (types E_6 and E_7), simple Lie algebras with nonzero Jordan socle are finitary 3-graded Lie algebras; the 3-gradings are also described.

We complete the description of nondegenerate Lie algebras with essential Jordan socle by determining the algebra of derivations of the simple components of the Jordan socle (over an algebraically closed field of characteristic 0). Any Lie algebra which is sandwiched between an infinite dimensional finitary simple Lie algebra, say M, and its algebra of derivations, $\mathrm{Der}\, M$, is strongly prime and contains a reduced element. Conversely, any 3-graded Lie algebra which is strongly prime, infinite dimensional and whose associated Jordan pair contains a reduced element can be sandwiched between a finitary simple Lie algebra and its algebra of derivations.

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A new series of simple finite dimensional Lie algebras over a field of characteristic 2

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We give a general construction of simple finite dimensional Lie algebras over a field of characteristic 2. First we generalize the Lie algebra of $n \times n$ matrices and construct the corresponding series. Starting from this generalization we construct an analogy of special, ortogonal. sympletic and Cartan type Lie algebras.

A Lie algebra over a field of characteristic 2

Marinês Guerreiro and Alexandre N. Grishkov

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Lie algebras over fields of characteristic 0 or $p > 3$ were recently classified, but over field of characteristic 2 or 3 there are only partial results up to now. S. Skryabin proved that any finite dimensional simple Lie algebra over a field of characteristic 2 has toroidal rank ≥ 2.

The toroidal rank $t(L)$ of a Lie 2-algebra L over a field k of characteristic 2 is the maximal dimension of an abelian subalgebra with basis $\{t_1, ..., t_n\}$ such that $t_i^{[2]} = t_i, i = 1, ..., n$, where $n = t(L)$.

The simple Lie 2-algebras of finite dimension over a field k of characteristic 2 and toroidal rank 2 were classified by A. Premet and A. Grishkov. The toroidal rank 3 case is much more difficult and the classification of the simple Lie 2-algebras over a field k of characteristic 2 and toroidal rank 3 which contains a Cartan subalgebra of dimension 3 is still an open problem. The main obstacle is the lack of examples.

In our work we constructed an example of a simple Lie 2-algebra of dimension 31 and of toroidal rank 3. We expect that this example will be useful for the construction of other simple Lie 2-algebra of toroidal rank 3 containing a CSA of dimension 3.

On Commutative Power-Associative Nilalgebras

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I will show new result about the Albert Problem. This problem concerns the existence of finite-dimensional simple (or nuclear in the sense $A^2 = A$) algebras in the class of commutative (static or power-associative) nilalgebras.

The b-radical in Bernstein-Jordan algebras

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Let F be a field with $\text{char}(F) \neq 2$ and (A, ω) be a baric algebra over F. For $B \subseteq A$, we will denote by $\text{bar}(B)$ the set $B \cap \ker \omega$. When B is a subalgebra of A and $B \nsubseteq \text{bar}(A)$, then B is called a *baric subalgebra* of (A, ω). In this case (B, ω') is a baric algebra, where $\omega' = \omega|_B \colon B \to F$.

If B is a baric subalgebra of (A, ω) and $\text{bar}(B)$ is a two-sided ideal of $\text{bar}(A)$ then B is called a *normal baric subalgebra* of (A, ω).

A baric algebra (A, ω) is b-simple if for all normal baric subalgebras B of (A, ω) either $\text{bar}(B) = 0$ or $\text{bar}(B) = \text{bar}(A)$. The b-radical of (A, ω) is zero if (A, ω) is b-simple, otherwise the b-radical of (A, ω) is the intersection of all $\text{bar}(B)$, where B is a maximal normal baric subalgebra of (A,ω). We will denote by $\mathrm{rad}(A)$ the b-radical of a baric algebra (A,ω).

A n^{th}-*order Bernstein algebra* is a commutative baric algebra (A,ω) satisfying, $x^{[n+2]} = (\omega(x))^{2^n} x^{[n+1]}$, for all $x \in A$ and n is the smallest one with such property, where $x^{[1]} = x$, $x^{[n+1]} = x^{[n]}x^{[n]}$, $n \geq 1$. If $n = 1$ then (A,ω) is called Bernstein algebra.

When (A,ω) is a Jordan and n^{th}-order Bernstein algebra, then $\mathrm{rad}(A) = (\mathrm{bar}(A))^2$. If (A,ω) is a power-associative and n^{th}-order Bernstein algebra, then $\mathrm{rad}(A) \subseteq (\mathrm{bar}(A))^2$.

Let (A,ω) be a train algebra of rank 3 with train equation $x^3 + \gamma_1\omega(x)x^2 + \gamma_2\omega(x)^2x = 0$. If $2\gamma_2 \neq 1$, we define a new multiplication on the vector space A by: $x * y = (1-2\gamma_2)^{-1}(xy - \gamma_2(x\omega(y) + y\omega(x)))$. Therefore, the new baric algebra $(\tilde{A},\omega)$ is a Bernstein-Jordan algebra, hence $\mathrm{rad}(A) = (\mathrm{bar}(A))^2$.

Non-linear algebras

Victor Kac

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I will discuss the notion of a non-linear algebra, introduced recently in a joint paper with A. De Sole, in the context of conformal algebras. I will explain how the classification of "non-degenerate" vertex algebras reduces to that of non-linear Lie conformal algebras.

Generalized representations of Jordan algebras

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Let V be a linear space. For $a \in End(V)$ we denote by $*_a$ a multiplication in $End(V)$, given by the formula: $x *_a y = [[a,x],y] = xay + yax - axy - yxa$.

Let π be a linear map of a Jordan algebra A in $End(V)$. The element $a \in End(V)$ is called *consistent* with π, if 1) $\pi(x) *_a \pi(y) = \pi(A(x,y))$ $\forall x,y \in A$, 2) if $b \in [a,[a,\pi(A)]$ satisfies the condition 1), then $b = a$.

The map $\pi : A \to End(V)$ is called a *generelazied representation* of the algebra A in a linear space V if there is $a \in End(V)$, which consistent with π.

We will say that π has an *order* l, if $(\pi(A))^{l+1} = 0$ and l is minimal with this property. It can be shown, that a generelized representation is a classical one iff it has the order 1.

It will be shown that the finite-dimensional generelized representations of finite order of a Jordan algebra A are in one-to-one correspondence with finite-dimensional representations of the 3 graded Lie algebra $L(A)$, corresponding to A. Moreover, it will be given a classification of irreducible generelized representations of finite order of semisimple Jordan algebras.

Representation type of Jordan algebras

Iryna Kashuba

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(joint work with S.Ovsienko and I.Shestakov)

This talk is devoted to the problem of classification of Jordan bimodules using the methods of the representatins theory of finite dimensional associative algebras. Due to properties of their module categories the finite dimensional algebras split into the following three classes by their *representation types*:

- algebras of finite type which posess finitely many isoclasses of indecomposable modules;
- algebras of tame type where almost all isoclasses of indecomposables in every dimension can be covered by finitely many one-parameter families;
- algebras of wild type when there are families of nonisomorphic indecomposable modules depending on any number of parameters.

Since (see [**Ja**]) for any Jordan algebra $\mathcal{J}$ the category of Jordan bimodules $\mathcal{J}-$ bimod is equivalent to the category of right modules over its universal multiplication envelope $\mathcal{U}(J)$ then the category $\mathcal{J}-$bimod is of one of the types above. Our work is a part of the program of classification of Jordan algebras of finite (respectively tame and wild) representation type. We will discuss some examples (mostly in the case $(\mathrm{Rad}\mathcal{J})^2 = 0$) which show that the categories of Jordan bimodules are rich enough.

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Quantum Lie Algebras via Friedrichs Criteria

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According to the Friedrichs criteria Lie polynomials are characterized as primitive elements of free associative algebra with the diagonal coproduct. Since every Lie polynomial may be considered as a multivariable derived operation on Lie algebras, this fact yields an idea to define quantum Lie operations as polynomials of free algebra that are skew primitive for all “admissible” values of variables. In line with this idea a quantum analog of a Lie algebra is the subspace of a Hopf algebra span by skew primitive elements and equipped by quantum Lie operations. We prove that the $(n-2)!$-dimensional space of generic quantum Lie operations has a basis of the symmetric ones. We propose a notion of a quantum universal enveloping algebra based on the quantum Lie operation concept. This enveloping algebra has PBW basis that admits a monomial crystallization. Every homogeneous character Hopf algebra over a field of zero characteristic is a quantum universal enveloping algebra of a suitable Lie algebra. We investigate in detail a right covariant first order differential calculus that naturally arises on each primitively generated braided Hopf algebra with a weak homogeneity restriction.

Nonzero bi-ideals of character Hopf algebras always have nonzero skew-primitive elements. Nevertheless, in contrary with the classical case of universal enveloping algebras, all skew-primitive elements of an arbitrary bi-ideal may not generate this bi-ideal. For this reason a notion of a combinatorial rank naturally arises. We show that the combinatorial rank of the finite dimensional Hopf algebras defined by Lusztig and related to the Lie algebra of type A_n equals $\lceil \log_2 n \rceil$.

Polynomial identities with involution in matrix alegbras

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We discuss the polynomial identities with an involution satisfied by the matrix algebra of order two over an infinite field of characteristic different from two. We consider the two types of involutions, namely the transpose and the symplectic, and exhibit bases of the polynomial identities in each one of the cases. When one deals with the transpose involution, one uses invariant theory in order to describe a basis of the identities. In the case of the symplectic involution, one needs the description of the weak polynomial identities for the pair (M_2, sl_2). Here M_2 stands for the matrix algebra of order two and sl_2 is the Lie algebra of the 2×2 traceless matrices.

This is a joint work with J. Colombo.

Homological properties of Lie algebras

Dessislava Kochloukova

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The results to be reported are joint work with John Groves (Melbourne). We study Lie algebras of type FP_∞. One of the main results is that every soluble Lie algebra of type FP_∞ is finite dimensional. Some refinements of this result, when the algebra is abelian-by-finite dimensional and only type FP_m is assumed, are obtained. It is also shown, using the complete cohomology of Vogel and Mislin, that for a wide class of Lie algebras, including all countable soluble ones, FP_∞ implies finite cohomological dimension.

For L a finitely generated metabelian Lie algebra over a field K and a natural number m we show that provided the characteristic p of K is 0 or $p > m$ L can be embedded in a metabelian Lie algebra of type FP_m, the case $m = 2$ has been already done by Gilbert Baumslag. This result is the best possible as for $0 < p \leq m$ every metabelian Lie algebra over K of type FP_m is finite dimensional as a vector space.

Irreducible conformal subalgebras of Cend_N and gc_N

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The theory of conformal algebras is closely related with conformal field theory in physics. On the one hand, the language of conformal algebras provides formalism for certain calculations related with the singular part of the operator product expansion (OPE), see [6]. On the other hand, conformal algebras are just the algebras in the pseudotensor category [2] related with polynomial Hopf algebra.

From now on, let F be an algebraically closed field of zero characteristic, and let $H = F[D]$ be the polynomial algebra over F (D is just a formal variable). We consider usual Hopf algebra structure on H, and the corresponding H-module structure on $H^{\otimes n}$, $n \geq 1$.

Definition [1], [6]. *Conformal algebra* is a (unital) left H-module C endowed with H-bilinear map $* : C \otimes C \to (H \otimes H) \otimes_H C$. Conformal algebra is said to be *finite* if it is finitely generated as H-module.

Conformal algebra C is associative, if the associativity identity holds: $a * (b * c) = (a * b) * c$. Analogously, Lie conformal algebra is defined via "pseudo-analogues" of anti-commutativity and Jacobi identity (see [1] or [8] for the explicit expressions). As in the case of usual algebras, associative conformal algebra C could be turned into Lie one $C^{(-)}$ under a new operation (pseudo-commutator) [1].

A representation of associative conformal algebra C on an H-module V is defined as an H-bilinear map (also denoted by $*$) $C \otimes V \to (H \otimes H) \otimes_H V$, such that $a * (b * u) = (a * b) * u$, $a, b \in C$, $u \in V$. If the corresponding H-module V is finitely generated then the representation is said to be *finite*. Representations of Lie conformal algebra could be defined analogously (see [1], [4]).

Structure theory of finite (associative and Lie) conformal algebras and pseudoalgebras has been developed in [1], [4], [7]. The next step is to describe the structure of conformal algebras with finite faithful representation. Let us denote the class of such algebras by $\mathcal{F}$; so, by $\mathcal{F}$-algebra (for short) we will mean conformal algebra with finite faithful representation. If a conformal algebra C has irreducible finite faithful representation then it said to be irreducible $\mathcal{F}$ algebra.

Studying of the structure theory of $\mathcal{F}$-algebras makes sense since there exist infinite $\mathcal{F}$-algebras. Some observations on conformal algebras allow to conclude that the class of irreducible $\mathcal{F}$-algebras is a more adequate analogue of simple finite-dimensional algebras than finite conformal algebras. For example, every finite semisimple conformal Lie algebra could be decomposed into a direct sum of finite $\mathcal{F}$-algebras, but it is not true that finite semisimple conformal Lie algebra is a direct sum of simple ones [4]. Also, the Virasoro conformal Lie algebra (it is finite and simple) could not be embedded into finite associative conformal algebra (c.f. classical Ado theorem), but it could be embedded into associative $\mathcal{F}$-algebra.

The general examples of $\mathcal{F}$-algebras are given by (associative) Cend_N and (Lie) $\mathrm{gc}_N = \mathrm{Cend}_N^{(-)}$.

The natural construction of the algebra Cend_N comes from the notion of conformal linear map (see [1], [7]): let V_N be the free N-generated H-module, then

$$\mathrm{Cend}_N = \{a : V_N \to H \otimes H \otimes_H V_N \mid a(fu) = (1 \otimes f \otimes_H 1)a(u),\ f \in H, u \in V_N\}.$$

By the construction, Cend_N has faithful finite representation on V_N, but it is not a finite conformal algebra. Namely, $\mathrm{Cend}_N = M_N(F[D, x]) = H \otimes M_N(F[x])$, where

$$A(x) * B(x) = \sum_{s\geq 0} (-D)^{(s)} \otimes 1 \otimes_H A(x)\partial_x^s B(x).$$

The algebras Cend_N and gc_N are direct conformal analogues of $M_N(F)$ and $gl_N(F)$, respectively.

The most important point of this problem is an analogue of Burnside theorem on irreducible subalgebras. In this context, the problem is to describe those subalgebras of Cend_N and gc_N which have no non-trivial submodules in V_N. The corresponding statement has been conjectured in [7] and partially proved in [3], [5], [9]. We prove this conjecture for associative conformal algebras:

Theorem 1. *Let $C \subseteq \mathrm{Cend}_N$ be an irreducible conformal subalgebra. Then either $C \simeq \mathrm{Cur}_N \equiv M_N(F[D])$ or $C = \mathrm{Cend}_{N,Q} \equiv M_N(F[D, x])Q(-D + x)$ for some matrix $Q(x)$,* $\det Q \neq 0$.

As a corollary, we obtain the description of simple and semisimple associative conformal algebras with finite faithful representation.

Theorem 2. *Let C be an associative conformal algebra with finite faithful representation.*

(i) *If C is simple then C is isomorphic to either* Curr_N *or* $\mathrm{Cend}_{N,Q}$ *as in Theorem 1.*

(ii) *If C is semisimple then $C = \bigoplus_{s=1}^{n} I_s$, where I_s, $s = 1, \dots, n$, are simple ideals of C described by* (i).

This result could be considered as a confirmation of the following conjecture on Lie conformal algebras with irreducible finite faithful representation (see also [3], [5], [9]).

Conjecture [3]**.** *Every infinite irreducible Lie conformal subalgebra of gc_N is a conjugate (by an automorphism of Cend_N) of either $\mathrm{gc}_{N,P}$, or $\mathrm{oc}_{N,P}$, or $\mathrm{spc}_{N,P}$, where* $\mathrm{gc}_{N,Q} = \mathrm{Cend}_{N,Q}^{(-)}$, $\det Q \neq 0$; $\mathrm{oc}_{N,Q} = \{a(Q_{(-1)} \otimes Q_{(2)}) \mid a \in \mathrm{Cend}_N,\ \sigma(a) = -a\}$,

$\det Q \neq 0$, $Q^t(-v) = Q(v)$; $\mathrm{spc}_{N,Q} = \{a(Q_{(-1)} \otimes Q_{(2)}) \mid a \in \mathrm{Cend}_N, \sigma(a) = a\}$, $\det Q \neq 0$, $Q^t(-v) = -Q(v)$.

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The example of Simple Lie Superalgebra with several independent invariant bilinear forms

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The associative superalgebra A of observables of 3-particle quantum Calogero model has two-dimensional space of supertraces. This algebra has an ideal if the coupling constant of Calogero model is equal to n+1/2, n+1/3 or n-1/3 for some integer n. The hypothesis arises that for other values of coupling constant the algebra A is simple. It is shown that if the coupling constant is zero, then (i) A is simple, (ii) its commutant [A, A] is a simple Lie superalgebra and (iii) this commutant has at least 2-dimensional space of nondegenerate bilinear invariant forms.

Composition algebras, exceptional groups, and higher composition laws

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We consider the Freudenthal construction of the exceptional group E_7 and its 56-dimensional representation. We classify (integer) orbits of the projective elements in this representation under the action of the integral form of this group.

We also consider other pairs (*group*, *module*) arising from this construction and show how they are related to spaces underlying higher composition laws in number theory.

A note on a class of commutative algebras

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We shall study commutative algebras A over fields F which satisfy the identity $\beta\{(x^2y)x - ((yx)x)x\} + \gamma\{x^3y - ((yx)x)x\} = 0, \ \ \beta, \gamma \in F$.

For $\beta = 3$ and $\gamma = -1$, they are called Lie triple algebras[5] (Petersson) or almost-Jordan algebras[3] (Hentzel and Peresi). These algebras were also studied by Osborn in [4].

If the identity $((xx)x)x = 0$ holds in the algebra A then the multiplication operator was shown to be nilpotent by Correa, Hentzel and Labra[1]. Here we prove that any algebra satisfying the identitities $((xx)x)x = 0$ and $\beta\{(x^2y)x - ((yx)x)x\} + \gamma\{x^3y - ((yx)x)x\} = 0$ for every $\beta, \gamma \in F$ are 3-Jordan algebras, that is commutative algebras satisfying the identity $x^3(yx) - (x^3y)x = 0$. This is a variety containing Jordan and Pseudo-composition algebras, studied by Hentzel and Peresi[2]. Our result requires characteristic of F to be different from 2 and 3.

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Generalised Euler Characteristics of Varieties of Tori in Lie Groups

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Let G be a complex Lie group with a real structure. The variety $\mathcal{T}$ of maximal tori of G then has real points and one may consider the topology of the space of real points of various real subvarieties of $\mathcal{T}$. We present results concerning the Euler characteristics of certain such varieties, weighted by a sign which is attached to each of their connected components. The results are adaptations to the real case of results concerning Fourier transforms of Steinberg functions of Lie algebras over finite fields. The Euler characteristics are realised as characteristic functions of certain complexes of sheaves, and the Brylinski-Kashiwara-Shapiro technology of Fourier transforms of conical complexes of sheaves is used in the proofs.

The Terwilliger algebra of some association schemes

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The action of a group on a set can be generalized with the notion of associated schemes. Main examples are the Hamming schemes H(n,q) (the hypercube is the case q=2), partial geometries and the Johnson schemes J(n,k) that consist of k-subsets of an n-set. Associated to these combinatorial structures we have the Bose-Mesner algebra and the subconstituent algebra, also known as the Terwilliger algebra. The first one is commutative and the second one is an associative algebra containing the former. We give the structure of the Terwilliger algebra for the Hamming schemes and the the Johnson schemes with 3k¡ n.

Conformal subalgebras of $Cend_n$ and gc_n

Jose Liberati

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We present the classification of infinite irreducible associative conformal subalgebras of $Cend_n$. We also describe all finite irreduclible modules, automorphism and anti-involutions of these subalgebras, and we present a conjecture on the classification of infinite irreducible Lie conformal subalgebras of gc_n.

Ref. On the classification of subalgebras of $Cend_n$ and gc_n, J. of Algebra 260 (2003), 32-63, C. Boyallian, V. G. Kac and J. Liberati.

Representations of exceptional simple Jordan superalgebras of characteristic 3

M. C. López-Díaz and Ivan P. Shestakov

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Representations of simple Jordan superalgebras of Hermitian 3×3 matrices over the exceptional simple alternative superalgebras $B(1,2)$ and $B(4,2)$ of characteristic 3 are studied. Every irreducible bimodule over these superalgebras up to isomorphism is either a regular bimodule or its opposite. As a corollaries, some analogues of the

Kronecker factorization theorem are proved for Jordan superalgebras that contain $H_3(B(1,2))$ and $H_3(B(4,2))$.

Vertex algebras and the Calabi-Yau/Landau-Ginzburg correspondence

Fyodor Malikov

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In this talk, based on a joint work with V.Gorbounov, we will construct a spectral sequence which interpolates between two vertex algebras: a chiral algebra that Witten associated with a Landau-Ginzburg model (or rather its orbifolded version); and the chiral de Rham complex over a Calabi-Yau hypersurface in the projective space. Consideration of this spectral sequence allows one to reproduce rigorously much of what is known among string theorists as "the Landau-Ginzburg/Calabi-Yau correspondence." As an application, we will derive an orbifold formula for the elliptic genus of a Calabi-Yau hypersurface.

Irreducible Lie-Yamaguti algebras and L-projections

Fabian Martin-Herce

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(joint work with P. Benito and A. Elduque)

The classification of irreducible Lie-Yamaguti algebras over an algebraically closed field of characteristic 0 shows that a large number of these structures are either irreducible Lie triple systems (i.e. binary product identically 0) that can be easily described by means of Lie algebras, Jordan algebras, Freudenthal triple systems and

Cayley algebras, or orthogonal complements into classical simple Lie algebras of derivations of the previous systems , together with derivations of Jordan pairs (closely connected with simple non-irreducible Lie triple systems). These orthogonal complements have

a more complicated description. The goal of this talk is to determine them under the common pattern of an interesting tool: L-projections.

Prime Z-graded Lie algebras with finite growth

Consuelo Martínez López

Universidad de Oviedo, Spain

The structure of simple graded Z-graded Lie algebras, having polynomial growth, was given by O. Mathieu (*Classification of simple graded Lie algebras of finite growth*, Inventiones Math. 86, 1986), answering in this way to a conjecture posed by V. Kac. A similar result in the case of Jordan algebras was obtained by C. M. and E. Zelmanov

(*Simple and Prime graded Jordan algebras*, J. of Algebra 194, 1997). In the same paper the structure of prime Z-graded Jordan algebras with finite growth was also obtained. This structure played an important role in the study of the even part of superconformal algebras that come from a Jordan superalgebra (V. Kac, C.M. and E. Zelmanov : *Graded simple Jordan superalgebras of growth one*, Memoirs of the AMS 150, 2001.) The structure of prime Z-graded Lie algebras with finite growth is unknown and this problem is addressed in the present paper.

From Cayley Dickson Algebra to $G2(q)$

Ibrahim Mashhour

Mutah University, Jordan

The aim of this paper is give all the relevant properties of the exceptional group G2(q) using Caley Dickson algebra.

The Role of Identities in Jordan Algebras

Kevin McCrimmon

University of Virginia, USA

Born of quantum mechanics, but abandoned at birth by physicists, Jordan algebras recovered to lead a productive life in a variety of mathematical fields. Zel'manov's amazing classification of simple Jordan algebras of arbitrary dimension focused attention on the identities (identical relations satisfied by all elements of an algebra) – if clothes make the man, then identities make the algebra. We give a survey of the history of Jordan structure theory, stressing the roles of the s-identities (which separate special algebras, those with an associative parentage, from algebras of Albert type), the Clifford identities (which separate Jordan algebras of Clifford type with just two idempotent genes from the other special algebras), and Zel'manov's dread penteater identity (which heartily eats pentads and spits out the heart of a Jordan algebra of hermitian type).

Verma-type modules for affine Lie superalgebras

Duncan Melville

St. Lawrence University, USA

(joint work with V. Futorny)

It is well-known that the representation theory for finite-dimensional superalgebras is quite different from that of simple Lie algebras. However, in the affine case, along with numerous differences, there are some close similarities. In particular, it is possible to describe Verma-type modules. Verma-type modules are similar to Verma modules but can have both finite- and infinite-dimensional weight spaces, as they arise from

non-standard Borel subalgebras. In this talk, we discuss certain classes of Verma-type modules for some affine superalgebras.

Survey on R-algebras

Kurt Meyberg
Technische Universitaet Muenchen, Germany

R-algebras are a generalization of power associative algebras, they have their roots in Analysis. I will survey the (incomplete) structure theory of these algebras which has been developed by myself and several of my students.

Right alternative bimodules

Lucia S. I. Murakami
IME-USP, Brazil
(joint work with Ivan Shestakov)

The aim of this talk is to present the results concerning the classification of irreducible right alternative bimodules. In particular, low-dimensional bimodules over algebras with zero multiplication and unital bimodules over matrix algebras are classified. It will also be shown a possible relation between irreducible right alternative bimodules over algebras with zero multiplication and simple right alternative algebras.

Varieties of Nilpotent Matrices for Simple Lie Algebras: The Good, the Bad and the Support Varieties

Daniel Nakano
University of Georgia, USA

In this talk I will survey recent results on determining subvarieties of the nullcone of a semisimple Lie algebra g consisting of certain nilpotent matrices. One example of such a variety is the restricted nullcone which arises at the spectrum of the cohomology ring for the restricted Lie algebra cohomology of g. We have completed the determination of the restricted nullcone for all primes. If time permits, I will discuss the connections with computing support varieties of Weyl modules over arbitrary characteristic.

Applications of orbit functions of compact semisimple Lie groups

Jiří Patera
Univ. Montreal, Canada

An orbit function is the contribution to an irreducible character of a compact semisimple Lie group G, from one Weyl group orbit. Such functions are much simpler than the characters, but they carry most of the practically useful properties of the characters. Moreover, their verastile discrete orthogonality on certain finite Abelian subgroups of G, makes them particularly suitable for digital data processing.

Elements of Minimal Degree in the Center of The Free Alternative Algebra

Luiz Antonio Peresi
IME-USP, Brazil

It is unknown whether the center of the free alternative algebra has a finite number of generators, as a T-subalgebra. Therefore, it is of interest to find nonzero elements of minimal degree in the center of the free alternative algebra. This problem is related to the problem of finding nonzero elements in the annihilator of the free Malcev algebra.

Filippov (1999) conjectured that the minimal degree of nonzero elements in the center of the free alternative algebra is 7. Representing identities by matrices we prove that this conjecture is true. We find a basis for the elements of degree 7.

Dimension filtration on loops

José M. Pérez-Izquierdo
Universidad de La Rioja, Spain

In this join work with J. Mostovoy we show that over fields of characteristic zero the graded vector space constructed from the dimension filtration of a loop inherits the structure of a Sabinin algebra. We obtain the Shestakov-Umirbaev operations on this vector space from the multiplication on the loop by means of the linearizers introduced by J. Mostovoy to study lower central series for loops. The universal enveloping algebra of this Sabinin algebra is also recovered from the loop algebra in the natural way.

Twisted loop algebras

Arturo Pianzola

University of Alberta, Canada

I will describe how cohomological methods can be used to classify twisted loop algebras. The crucial ingredient is a version of Serre's conjecture I for Laurent polynomials. Examples will be given coming from Lie, Lie super, and superconformal algebras.

Higman's Theorem and f-unitary Moufang loops of units

Francisco Cesar Polcino Milies

IME-USP, Brazil

Let L be an RA loop, that is, a loop whose loop ring in any characteristic is an alternative, but not associative, ring. We shall show that Higman's Theorem describing RA loops such that ZL has only trivial units, which was known only for torsion loops, actually holds in the general case. This result will be applied to the following problem.

Let $f : L \to \pm 1$ be a homomorphism and, for an element $\alpha = \sum \alpha_l l \in ZL$, define $\alpha^f = \sum f(l)\alpha_l l^{-1}$. Call α *f-unitary* if $\alpha^f = \alpha^{-1}$ or $\alpha^f = -\alpha^{-1}$. We shall discuss necessary and sufficient conditions on the loop L so that all units in ZL are f-unitary.

Octonions and the Standard Model of Elementary Particles

Roldão da Rocha, Jr.

Instituto de Física Gleb Wataghin - Unicamp, Brazil

The Cayley algebra of the octonions are associated with the five exceptional groups G_2, F_4, E_6, E_7 and E_8. From the division algebras it can be formulated the algebraic basis of the standard model of elementary particles. It is known that the imaginary element of $\mathbb{C}$ generates the group U(1) (via exponentiation) and the unit elements of the quaternions $\mathbb{Q}$ generate SU(2). Since SU(3) is realized as the stability group of a fixed imaginary direction of the octonions, the standard symmetry U(1)×SU(2)×SU(3) of quark and lepton theory is conceived over the division algebras. Since octonions give the spinor representation of $\mathfrak{so}(8)$, they are fundamental in particle physics because other than the Higgs boson, all particles transform as vectors (bosons) or spinors (vectors) and it is known that interactions between forces and matter is described by a trilinear map involving two spinors and one vector. It also can be proved that pairs of octonions give the spinor representation of $\mathfrak{so}(1,9)$ and the reduction of Dirac spinors of $\mathbb{R}^{1,9}$ to the ones of Minkowski spacetime is simultaneous [1] with the reduction $\mathfrak{so}(1,9) \times \mathfrak{su}(2)$ to $\mathfrak{so}(1,3) \times \mathfrak{su}(3) \times \mathfrak{su}(2) \times \mathfrak{u}(1)$. All this formalism is based on the tensor product of the division algebras and the Clifford algebra $\mathcal{C}\ell_{1,9}$ and the formalism over $\mathbb{R}^{1,9}$ is one of the largest arenas of theoretical physics and also plays a role in string theory.

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Tensor products of irreducible representations of quantum groups at a root of 1

Marc Rosso

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(joint work with C. De Concini, C. Procesi, N. Reshetikhin)

We consider the tensor product of two irreducible representations of maximal dimension of a quantized enveloping algebra at a root of 1, in the "De Concini-Kac-Procesi" version. If the couple of representations is "generic", we prove that this tensor product is semi-simple and splits as a direct sum of some irreducible representations of maximal dimension, eauch with the same multiplicity. This uses methods from the theory of representations of finite dimensional algebras (we introduce the class of "Cayley-Hamilton Hopf algebras"), and tools from the geometry of algebraic semi-simple groups.

Lie superalgebras based on gl(n)

Gil Salgado

CIMAT, Mexico

(joint work with O.A. Sanchez-Valenzuela)

Finite-dimensional real and complex Lie superalgebras whose underlying Lie algebra is gl(n) and whose odd module is gl(n) itself under the adjoint representation are classified up to isomorphism. It is shown that for $n \geq 3$ there are one-parameter families of non-isomorphic shuch Lie superalgebras, plus another set of finitely many different isomorphism classes. For $n = 2$ there are 10 different isomorphism classes over the real field, and 8 different over the complex numbers. For $n = 1$ there are 2 different isomorphism classes over either ground field. Representatives on each isomorphism class are given, and their automorphism groups are determinated. The question as to which representatives admit Z_2 graded, ad-invariant geometric structures (of orthogonal or symplectic type) is also addressed, and a precise list of which fo such geometric structures can be defined on each isomorphism class is given.

A combinatorial description of the syzygies of certain Weyl modules

Mari Sano
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A basis for the syzygies of the resolution over the Schur algebra of 3-rowed Weyl modules with at most one triple overlap presented by Buchsbaum and Rota [Proc. Nat. Acad. Sci. 90 (1993), 2448-2450.] is constructed using Letter-Place techniques. This basis is explicitly given by computing the image under the differential of a conveniently chosen subset of the canonical Letter-Place basis of each term in the resolution.

Extended Affine Lie Algebra of Type A_1

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In this work we classify the cores of an extended affine Lie algebra of type A_1 and we study the imaginary Verma modules for the extended affine Lie algebra $sl_2(\mathbb{C}_q)$.

Subalgebras and automorphisms of polynomial rings

Ivan Shestakov and Ualbai U. Umirbaev
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Let $A = F[x_1, x_2, \ldots, x_n]$ be the polynomial ring in the variables $x_1, x_2, \ldots, x_n$ over a field F, and let $Aut\, A$ be the group of automorphisms of A as an algebra over F. An automorphism $\tau \in Aut\, A$ is called *elementary* if it has a form

$$\tau : (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) \mapsto (x_1, \ldots, x_{i-1}, \alpha x_i + f, x_{i+1}, \ldots, x_n),$$

where $0 \neq \alpha \in F,\ f \in F[x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$. The subgroup of $Aut\, A$ generated by all the elementary automorphisms is called the *tame subgroup*, and the elements from this subgroup are called *tame automorphisms* of A.

It is well-known that the automorphisms of polynomial rings and free associative algebras in two variables are tame. At present, a few new proofs of these results have been found. However, in the case of three or more variables the similar question was open and known as "The generation gap problem" or "Tame generators problem". The general belief was that the answer is negative, and there were several candidate counterexamples. The most known of them is the following automorphism $\sigma \in Aut(F[x, y, z])$, constructed by Nagata in 1972:

$$\begin{aligned} \sigma(x) &= x + (x^2 - yz)z, \\ \sigma(y) &= y + 2(x^2 - yz)x + (x^2 - yz)^2 z, \\ \sigma(z) &= z. \end{aligned}$$

We give a negative answer to the above question. Our main result states that the tame automorphisms of the polynomial ring $A = F[x, y, z]$ over a field F of characteristic 0 are algorithmically recognizable. In particular, the Nagata automorphism σ is "wild", that is, not tame.

The approach we use is different from the traditional ones. The novelty consists of the imbedding of the polynomial ring A into the free Poisson algebra (or the algebra of universal Poisson brackets) on the same set of generatots and of systematical using of brackets as an additional tool. The crucial role in the proof is played by the description of the structure of two-generated subalgebras of polynomial rings. In particular, we obtain a lower estimate for degrees of elements of these subalgebras, which is essentially used in most of the proofs. Observe that this estimate provides one more proof of the classical Jung theorem on the tameness of the automorphisms of polynomials in two variables.

Self-adjoint extensions of the Dirac operator with the Aharonov-Bohm potential

Andrei Smirnov

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We studied the self-adjoint extensions of the radial Dirac Hamiltonian in the background combined of a uniform magnetic field and a flux string for 2+1 and 3+1 dimensions. It was shown that the case of 3+1 dimensions for any momentum aligned with the string can be formally reduced to the 2+1 dimensional case by the proper choosing of spin operator. A one-parameter family of allowed boundary conditions for 2+1 dimensions was established. A two-parameter family for 3+1 dimensions was established on the condition of conservation of the spin operator.

Complete sets of solutions and spectra of the self-adjoint extensions as the functions of the extension parameter Θ were found. Green functions of the Dirac equation for natuaral extensions were found.

The ideal of the Lesieur-Croisot elements of a Jordan algebra

Maribel Tocon Barroso

University of Malaga, Spain

Goldies theorem for Jordan algebras characterizes orders in nondegenerate artinian Jordan algebras. But the natural finiteness condition for Jordan algebras is having finite capacity. In [1] it is proved that a Jordan algebra is an order in a non degenerate Jordan algebra of finite capacity if and only if it is Lesieur-Croisot, that is, it has the property that an ideal is essential iff it contains an inyective element.

Our main task will be showing that the set of the Lesieur-Croisot elements of a strongly prime Jordan algebra J, that is, the set of the elements of J at which the local algebra is Lesieur-Croisot, is an ideal of J.

This communication is part of a research in process in collaboration with Professor Montaner from the University of Zaragoza (Spain).

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Irreducible representations of a certain Jordan superalgebra

Maria Trushina

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We obtain a classification of unital finite-dimensional irreducible Jordan representations of the superalgebra D(l,m) in the case of characteristic 0. As corollaries we obtain a classification of unital finite-dimensional irreducible modules over the simple Jordan superalgebra D(t) and a classification of all finite-dimensional irreducible bimodules over the simple Kaplansky superalgebra.

Canonical states on the Lie group, cohomology and isometric imbedding

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Canonical state on the semisimple Lie groups was defined in 70-th by Gel'fand-Graev-Vershik and was uzed for the representation theory of current groups. This notion uzed to be most important for the studying of cohomologies of the group with values in representations and for the question when there is an covarinat imbedding of the homogerneous spaces to the representation space.

Quantization on bounded manifolds

Boris Voronov

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We consider the Dirac equation in the magnetic-solenoid field (the field of a solenoid and a collinear uniform magnetic field). For the case of Aharonov-Bohm solenoid, we construct self-adjoint extensions of the Dirac Hamiltonian using von Neumann's theory of deficiency indices. We find self-adjoint extensions of the Dirac Hamiltonian and boundary conditions at the AB solenoid. Besides, for the first time, solutions of the Dirac equation in the magnetic-solenoid field with a finite radius solenoid were found. We study the structure of these solutions and their dependence on the behavior of the magnetic field inside the solenoid. Then we exploit the latter solutions to specify boundary conditions for the magnetic-solenoid field with Aharonov-Bohm solenoid.

Maximal subloops of finite simple Moufang loops

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We continue the study started in [1] of the properties of Moufang loops using their relation to groups with triality. We give a classification of the maximal subloops of the unique finite simple non-associative Moufang loops $M(q)$. It was shown in [1] that there exists a correspondence between the subloops of $M(q)$ and certain subgroups of the simple group with triality $P\Omega_8^+(q)$. This correspondence becomes more natural when we bring into consideration the simple alternative algebra $\mathbb{O}(q)$ and its automorphism group $G_2(q)$. As a corollary to our results, we have the following description:

Theorem The maximal subloops of the simple Moufang loop $M(q)$, $q = p^n$, are as follows:
(i) $q^2 : PSL_2(q)$, maximal parabolic;
(ii) $(PSL_2(q), 2)$, $q \neq 3$;
(iii) $M(q_0)$, $q = q_0^k$, k prime, $(q, k) \neq (odd, 2)$;
(iv) $PGL(\mathbb{O}(q_0))$, $q = q_0^2$ odd;
(v) $M(2)$, $q = p$ odd.
Moreover, all isomorphic maximal subloops of $M(q)$ are conjugate in $Aut(M(q))$.

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Universal multiplicative envelope of free Malcev superalgebra on one odd generator

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(joint work with I. Shestakov)

A basis of the universal multiplicative envelope for free Malcev superalgebra on one odd generator or, equivalently, the basis of the free one-generated module over this superalgebra is constructed. Some corollaries for skew-symmetric functions and central elements in free Malcev and free alternative algebras are obtained.