MAT-111 - Cálculo Diferencial e Integral I

Bacharelado em Matemática - 2010

4ª Lista de exercícios

Integrais

1. Calcule a área da região compreendida entre os gráficos de

$$f(x) = x^3 - 2x + 1 \text{ e } g(x) = -x + 1,$$

com $-1 \leq x \leq 1$. Resposta: $\frac{1}{2}$

2. Desenhe a região $A = B \cap C \cap D$ e calcule a área de A, onde

$$B = \{(x,y) \in \mathbb{R}^2 | y \geq x^2 - 4\},\ C = \{(x,y) \in \mathbb{R}^2 | y \leq 12 - 3x^2\} \text{ e}$$

$$D = \{(x,y) \in \mathbb{R}^2 | y \leq 3x^2 + 12x + 12\}.$$

Resposta: $\frac{104}{3}$

3. Desenhe a região

$$A = \{(x,y) \in \mathbb{R}^2 | y \geq x^2 - 1,\ y \leq x + 1 \text{ e } y \geq -x^2 - 3x - 2\}$$

e calcule a sua área. Resposta: $\frac{107}{24}$

4. Desenhe a região do plano delimitada pela curva $y = x^3 - x$ e por sua reta tangente no ponto de abscissa $x = -1$. Calcule a área desta região. Resposta: $\frac{27}{4}$

5. Encontre as seguintes primitivas:

1. $\int \frac{x^7 + x^2 + 1}{x^2}\, dx$
2. $\int e^{2x}\, dx$
3. $\int \cos 7x\, dx$
4. $\int \text{tg}^2\, x\, dx$
5. $\int \frac{7}{x-2}\, dx$
6. $\int \text{tg}^3\, x\, \sec^2 x\, dx$
7. $\int \frac{\text{sen}^3\, x}{\sqrt{\cos x}}\, dx$
8. $\int \text{tg}\, x\, dx$
9. $\int \text{tg}^3\, x\, dx$
10. $\int \frac{x}{1+x^2}\, dx$
11. $\int \frac{x}{1+x^4}\, dx$
12. $\int \frac{x^2}{1+x^2}\, dx$
13. $\int x\sqrt{1-x^2}\, dx$
14. $\int \sec x\ dx$
15. $\int \frac{dx}{x\sqrt{1+\ln x}}$
16. $\int x^2 \sqrt[5]{x^3+1}\, dx$
17. $\int \frac{4x+8}{2x^2+8x+20}\, dx$
18. $\int \frac{\sqrt{\ln x}}{x}\, dx$
19. $\int \frac{dx}{(\text{arcsen}\, x)\ \sqrt{1-x^2}}$
20. $\int \frac{e^x}{1+e^x}\, dx$
21. $\int \frac{\text{sen}\, 2x}{1+\cos^2 x}\, dx$

22. $\int e^{x^3} x^2\,dx$

23. $\int e^x \sqrt[3]{1+e^x}\,dx$

24. $\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx$

25. $\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx$

26. $\int 2x(x+1)^{2008}\,dx$

27. $\int x\ \operatorname{sen} x\ dx$

28. $\int e^x\ \cos x\ dx$

29. $\int x^r\ \ln x\ dx, r \in \mathbb{R}$

30. $\int (\ln x)^2\ dx$

31. $\int xe^{-x}\,dx$

32. $\int x \operatorname{arctg} x\ dx$

33. $\int \operatorname{arcsen} x\ dx$

34. $\int \sec^3 x\ dx$

35. $\int \cos^2 x\,dx$

36. $\int \operatorname{sen}^2 x\ \cos^3 x\ dx$

37. $\int \operatorname{sen}^2 x\ \cos^2 x\ dx$

38. $\int \frac{1-\operatorname{sen} x}{\cos x}\,dx$

39. $\int \frac{3x^2+4x+5}{(x-1)(x-2)(x-3)}\,dx$

40. $\int \frac{dx}{2x^2+8x+20}$

41. $\int \frac{3x^2+4x+5}{(x-1)^2(x-2)}\,dx$

42. $\int \frac{x^5+x+1}{x^3-8}\,dx$

43. $\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

44. $\int x^2\sqrt{1-x^2}\,dx$

45. $\int e^{\sqrt{x}}\,dx$

46. $\int \ln(x+\sqrt{1+x^2})\,dx$

47. $\int \frac{dx}{\sqrt{5-2x+x^2}}$

48. $\int \sqrt{x}\ \ln x\ dx$

49. $\int \operatorname{sen}(\ln x)\,dx$

50. $\int \frac{x}{x^2-4}\,dx$

51. $\int \frac{3x^2+5x+4}{x^3+x^2+x-3}\,dx$

52. $\int \sqrt{a^2+b^2x^2}\,dx$

53. $\int \frac{dx}{\sqrt{a^2+b^2x^2}}$

54. $\int \sqrt{x^2-2x+2}\,dx$

55. $\int \sqrt{3-2x-x^2}\,dx$

56. $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

57. $\int \cos^3 x\,dx$

58. $\int \operatorname{sen}^5 x\ dx$

59. $\int \frac{\cos^5 x}{\operatorname{sen}^3 x}\,dx$

60. $\int \operatorname{sen}^3\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)\ dx$

61. $\int \frac{dx}{\operatorname{sen}^5 x\ \cos^3 x}$

62. $\int \operatorname{sen}^4 x\ dx$

63. $\int \operatorname{sen}^2 x\ \cos^5 x\,dx$

Respostas:

1) $\frac{x^6}{6}+x-\frac{1}{x}+k$
2) $\frac{e^{2x}}{2}+k$
3) $\frac{1}{7}\operatorname{sen} 7x+k$
4) $\operatorname{tg} x - x + k$
5) $7\ln|x-2|+k$
6) $\frac{1}{4}\operatorname{tg}^4 x + k$
7) $2\sqrt{\cos x}(\frac{1}{5}\cos^2 x - 1) + k$
8) $-\ln|\cos x| + k$
9) $\frac{1}{2}\operatorname{tg}^2 x + \ln|\cos x| + k$
10) $\frac{1}{2}\ln(1+x^2)+k$
11) $\frac{1}{2}\operatorname{arctg} x^2 + k$
12) $x - \operatorname{arctg} x + k$
13) $-\frac{1}{3}\sqrt{(1-x^2)^3}+k$
14) $\ln|\sec x + \operatorname{tg} x| + k$
15) $2\sqrt{1+\ln x}+k$
16) $\frac{5}{18}\sqrt[5]{(x^3+1)^6}+k$
17) $\ln(2x^2+8x+20)+k$
18) $\frac{2}{3}\sqrt{(\ln x)^3}+k$
19) $\ln|\operatorname{arcsen} x| + k$
20) $\ln(1+e^x)+k$

21) $-\ln(1+\cos^2 x)+k$
22) $\frac{1}{3}e^{x^3}+k$
23) $\frac{3}{4}\sqrt[3]{(1+e^x)^4}+k$
24) $-2\cos\sqrt{x}+k$
25) $e^{\text{arctg}x}+k$
26) $2(x+1)^{2009}(\frac{x+1}{2010}-\frac{1}{2009})+k$
27) $-x\cos x+\text{sen}\,x+k$
28) $\frac{1}{2}e^x(\text{sen}\,x+\cos x)+k$
29) $\begin{cases}\frac{x^{r+1}}{r+1}\ln x-\frac{x^{r+1}}{(r+1)^2}+k, \text{ se } r\neq -1\\ \frac{1}{2}(\ln x)^2+k, \text{ se } r=-1\end{cases}$
30) $x(\ln x)^2-2(x\ln x-x)+k$
31) $(-x-1)e^{-x}+k$
32) $\frac{x^2}{2}\,\text{arctg}\,x-\frac{x}{2}+\frac{1}{2}\,\text{arctg}\,x+k$
33) $x\,\text{arcsen}\,x+\sqrt{1-x^2}+k$
34) $\frac{1}{2}\sec x\,\text{tg}\,x+\frac{1}{2}\ln|\sec x+\text{tg}\,x|+k$
35) $\frac{1}{2}(x+\text{sen}\,x\cos x)+k$
36) $\frac{1}{3}\text{sen}^3 x-\frac{1}{5}\text{sen}^5 x+k$
37) $\frac{1}{8}(x-\frac{1}{4}\text{sen}\,4x)+k$
38) $\ln|1+\text{sen}\,x|+k$
39) $6\ln|x-1|-25\ln|x-2|+22\ln|x-3|+k$
40) $\frac{\sqrt{6}}{12}\text{arctg}(\frac{x+2}{\sqrt{6}})+k$
41) $-22\ln|x-1|+\frac{12}{x-1}+25\ln|x-2|+k$
42) $\frac{x^3}{3}+\frac{35}{12}\ln|x-2|+\frac{61}{24}\ln(1+(\frac{x+1}{\sqrt{3}})^2)+\frac{\sqrt{3}}{12}\,\text{arctg}(\frac{x+1}{\sqrt{3}})+k$
43) $\frac{1}{2}\,\text{arcsen}\,x-\frac{1}{2}x\sqrt{1-x^2}+k$
44) $\frac{x}{8}(2x^2-1)\sqrt{1-x^2}+\frac{1}{8}\,\text{arcsen}\,x+k$
45) $2(\sqrt{x}-1)e^{\sqrt{x}}+k$
46) $x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}+k$
47) $\ln|\sqrt{5-2x+x^2}+x-1|+k$
48) $\frac{2}{3}x\sqrt{x}(\ln x-\frac{2}{3})+k$
49) $\frac{x}{2}(\text{sen}(\ln x)-\cos(\ln x))+k$
50) $\frac{1}{2}\ln|x^2-4|+k$
51) $2\ln|x-1|+\frac{1}{2}\ln(x^2+2x+3)+\frac{1}{\sqrt{2}}\text{arctg}(\frac{x+1}{\sqrt{2}})+k$
52) $x\sqrt{a^2+b^2x^2}+\frac{a^2}{2b}\ln(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a})+k$
53) $\frac{1}{b}\ln(\frac{bx}{a}+\frac{\sqrt{a^2+b^2x^2}}{a})+k$
54) $\frac{x-1}{2}\sqrt{x^2-2x+2}+\frac{1}{2}\ln(x-1+\sqrt{x^2-2x+2})$ +k
55) $\frac{x+1}{2}\sqrt{3-2x-x^2}+2\text{arcsen}(\frac{x+1}{2})+k$
56) $\frac{1}{\sqrt{2}}\text{arctg}(\frac{x\sqrt{2}}{\sqrt{1-x^2}})+k$
57) $\text{sen}\,x-\frac{1}{3}\,\text{sen}^3 x+k$
58) $-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+k$
59) $\frac{1}{2}\text{sen}^2 x-\frac{1}{2\,\text{sen}^2\,x}-2\ln|\,\text{sen}\,x|+k$
60) $\frac{1}{4}\cos^8(\frac{x}{2})-\frac{1}{3}\cos^6(\frac{x}{2})+k$
61) $\frac{1}{2}\text{tg}^2 x+3\ln|\text{tg}x|-\frac{3}{2\text{tg}^2 x}-\frac{1}{4\text{tg}^4 x}+k$
62) $\frac{3}{8}x-\frac{1}{4}\text{sen}(2x)+\frac{1}{32}\text{sen}(4x)+k$
63) $\frac{1}{3}\text{sen}^3 x-\frac{2}{5}\text{sen}^5 x+\frac{1}{7}\text{sen}^7 x+k$