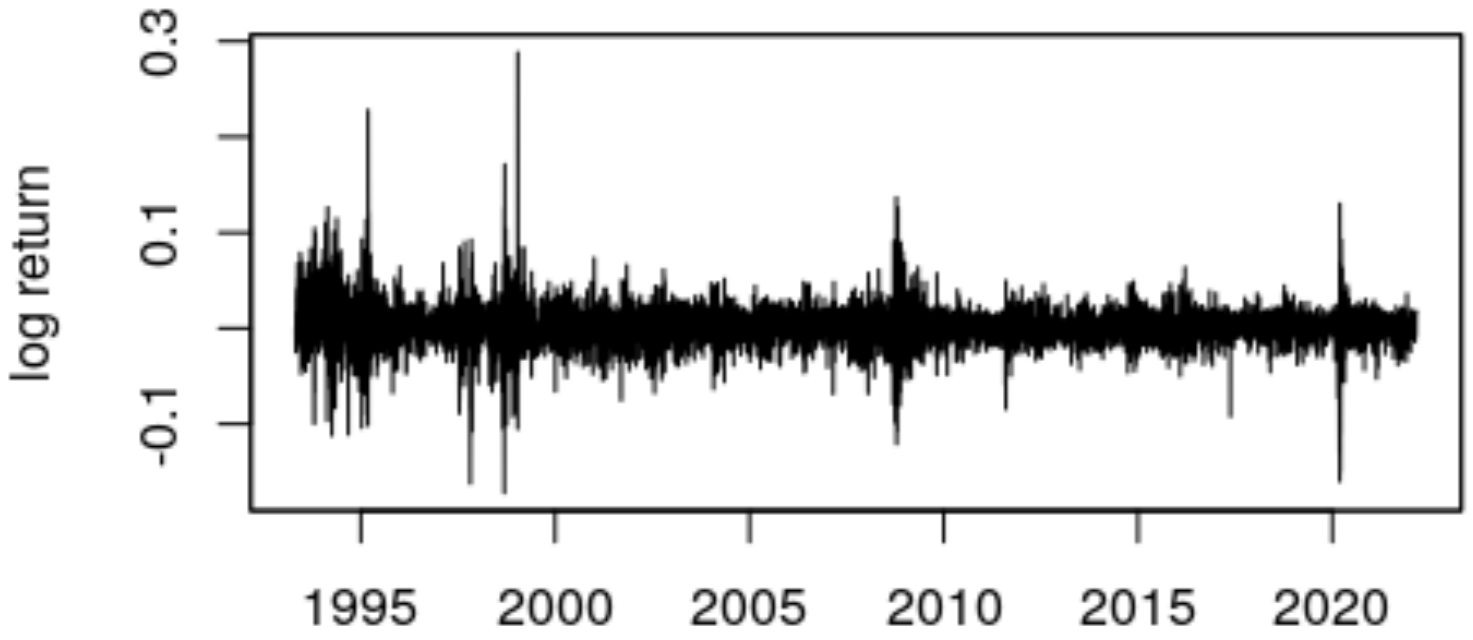
Volatility models

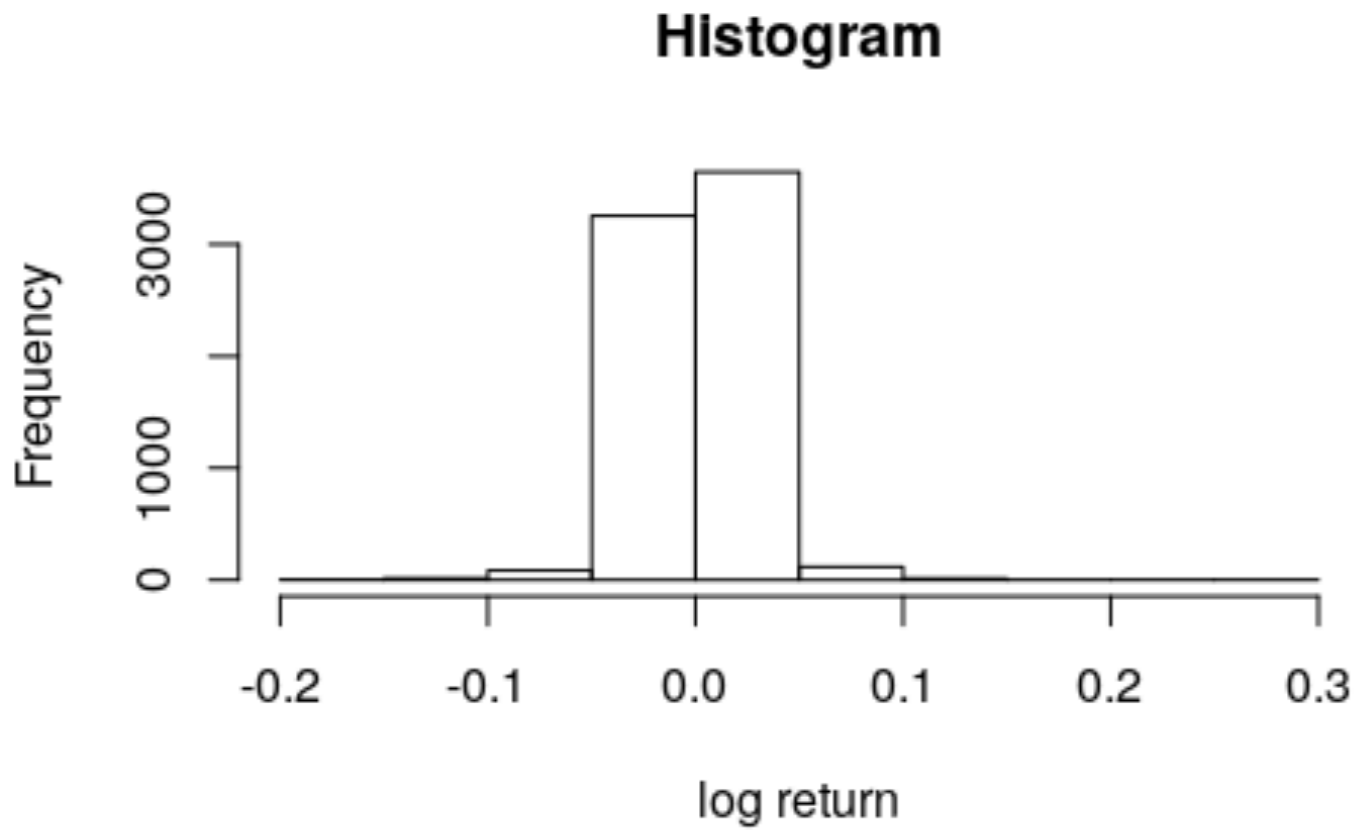
Lane Alencar – 2022

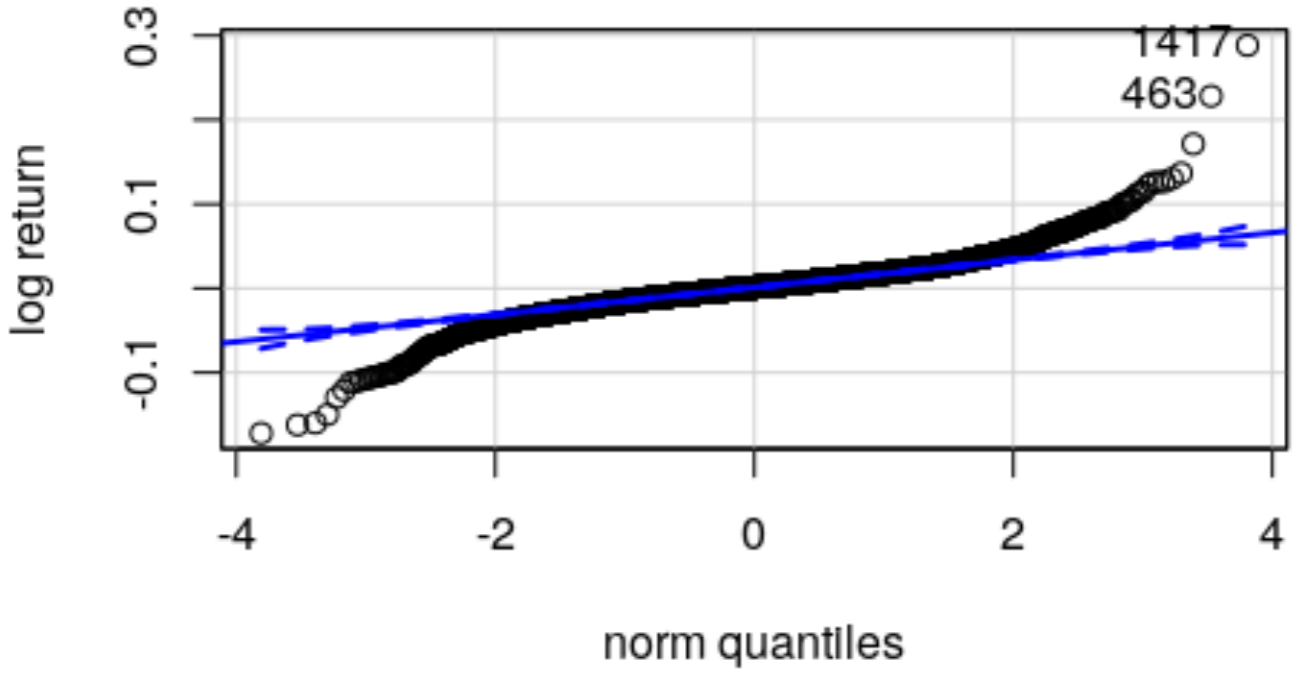
IBOVESPA - B3

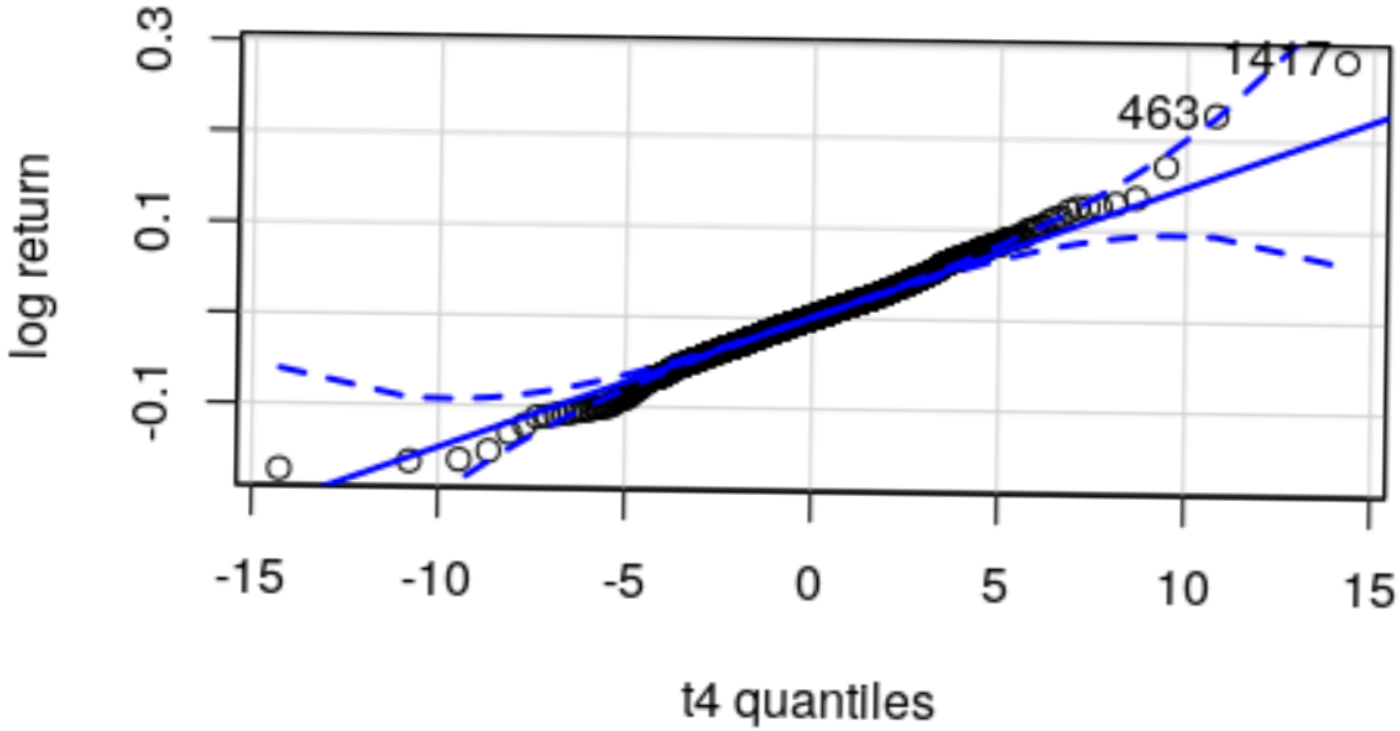
Log return = $\ln(P_t/P_{t-1})$



Log return




log return
0.3
0.1
-0.1
1417
463
-4
-2
0
2
4
norm quantiles


log return
0.3
0.1
-0.1
1417
463
-15
-10
-5
0
5
10
15
t4 quantiles

GARCH

- Model

$$y_t = \phi_0 + \phi_1 y_{t-1} + e_t$$

Uncorrelated e_t with $Var(e_t \mid y_1, \ldots, y_{t-1}) = \sigma_t^2$

- Variance model

$$\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2$$

- $\alpha+\beta<1$ and $0<\alpha, \beta<1$ => stationarity

$$Var(y_t) = \frac{\omega}{1 - \alpha - \beta}$$

- Maximum likelihood fit under $e_t \sim N(0, \sigma_t^2)$

Likelihood

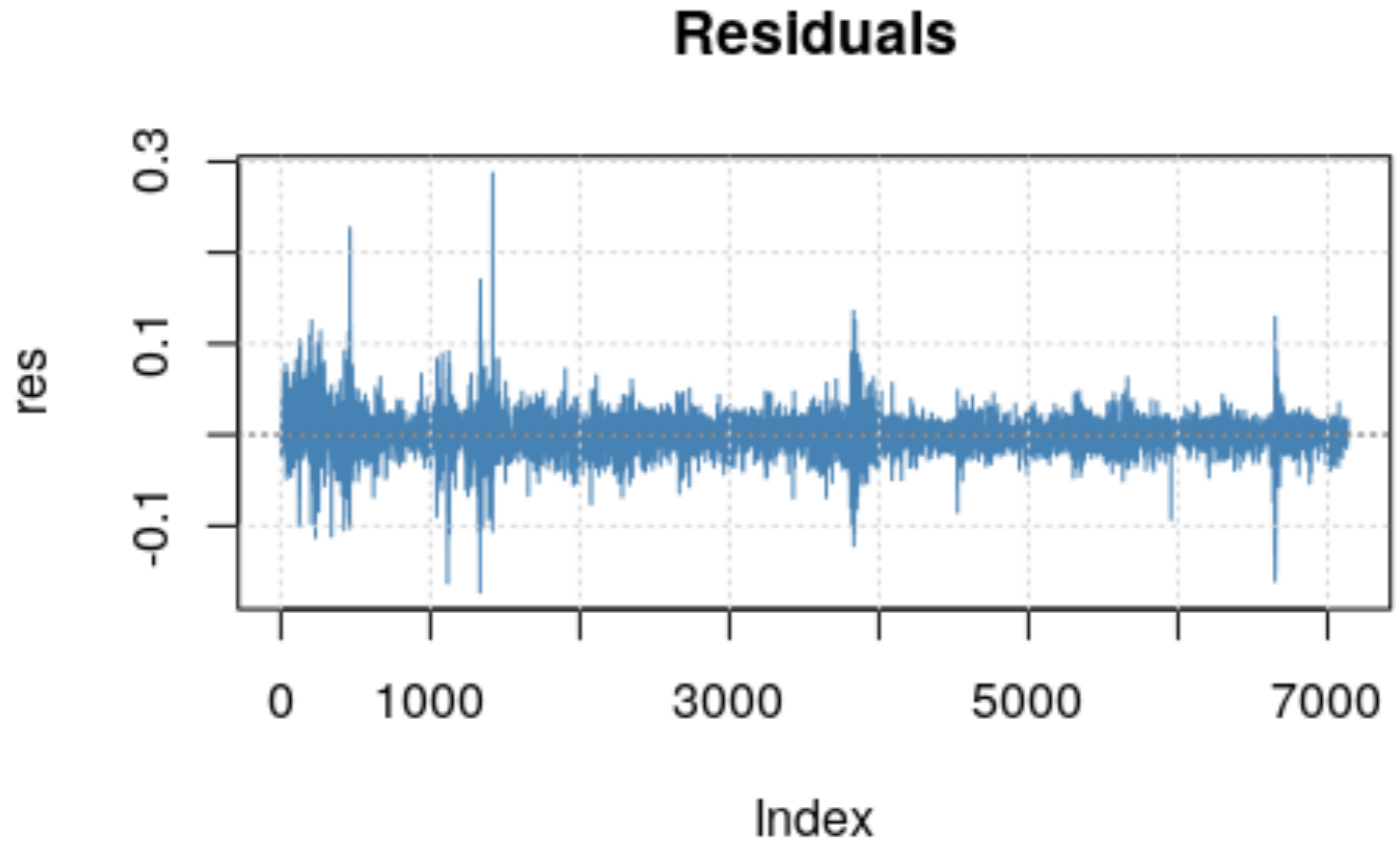
$$L(\theta) = f(x_1, x_2, \ldots, x_T) = f(x_1) f(x_2|x_1) \ldots f(x_T|x_{T-1})$$

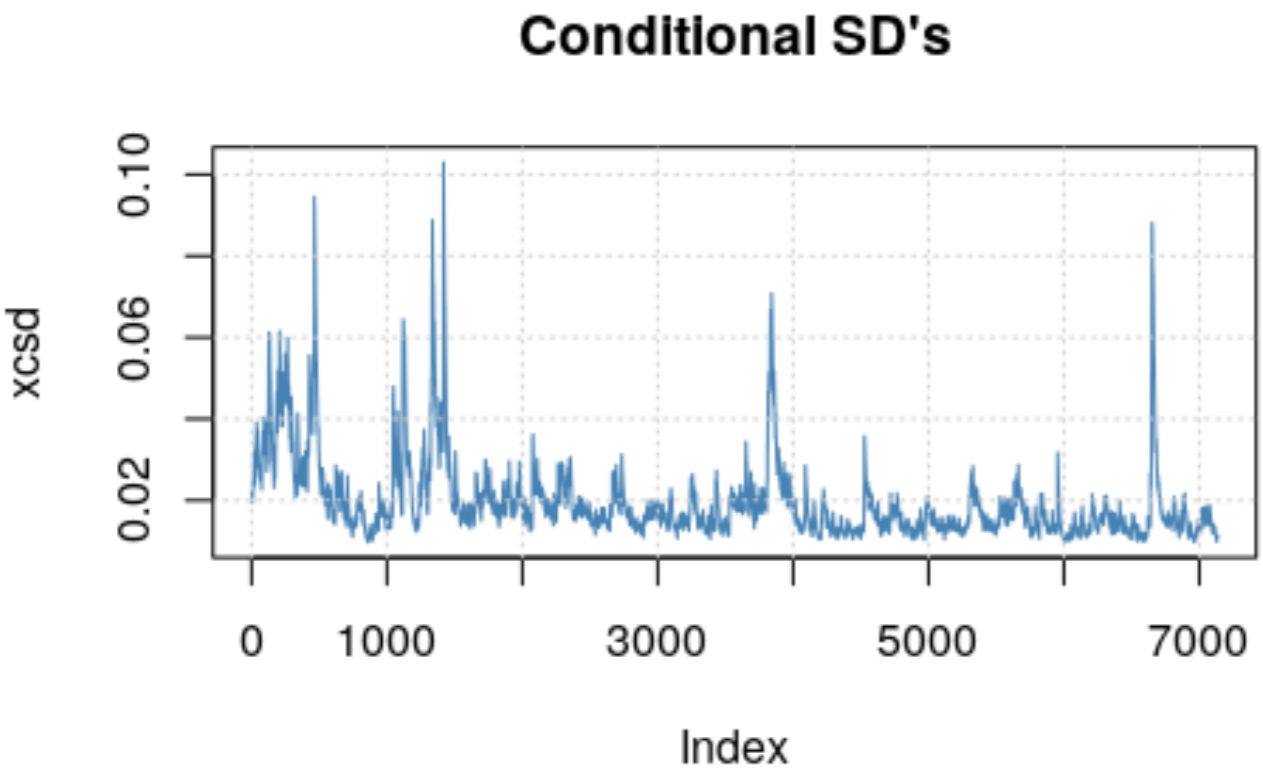
$$f(x_t|past_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left[-\frac{(x_t - \mu_t)^2}{2\sigma_t^2} \right]$$

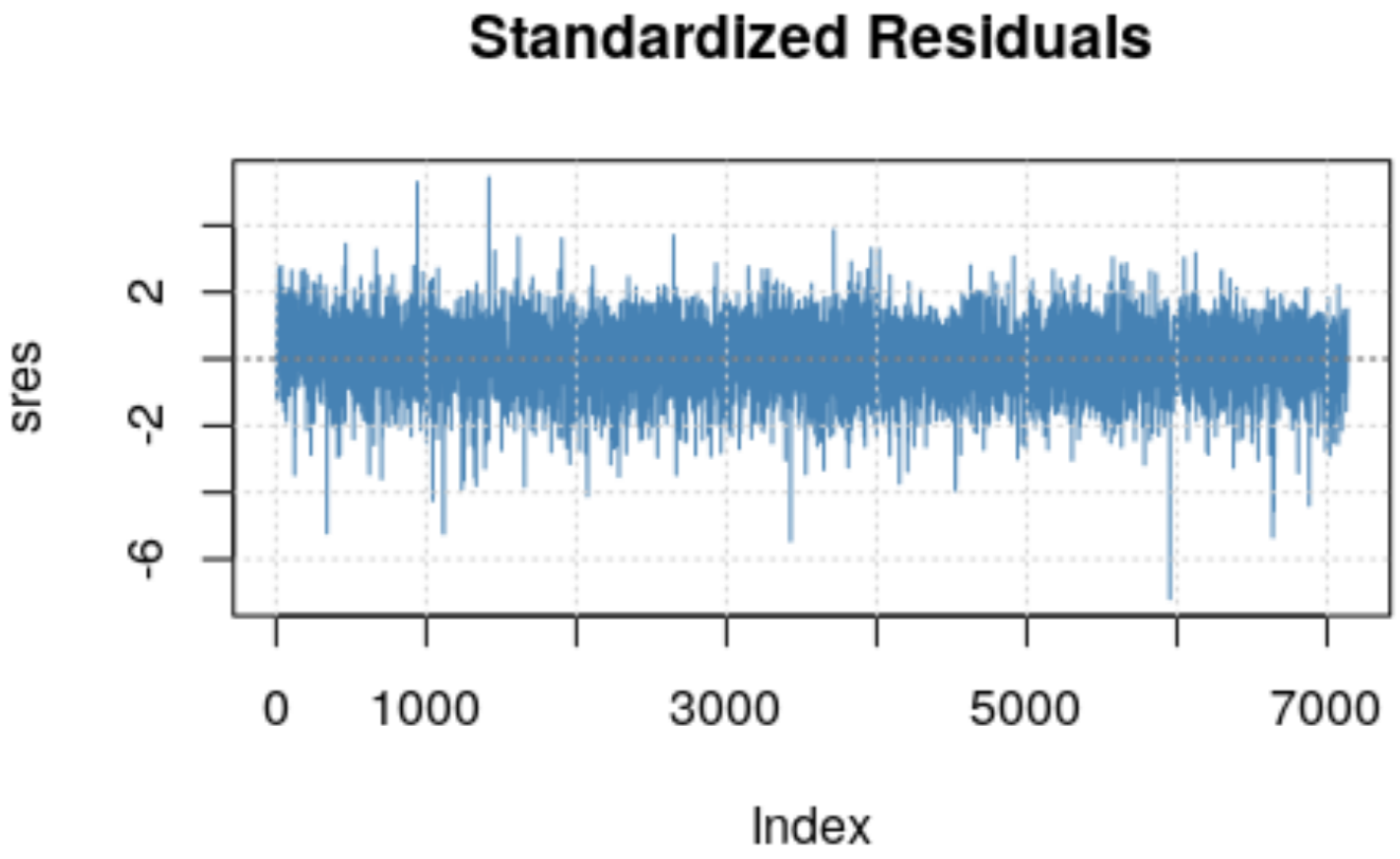
$$l(\theta) = \Sigma \ln f(x_t|past)$$

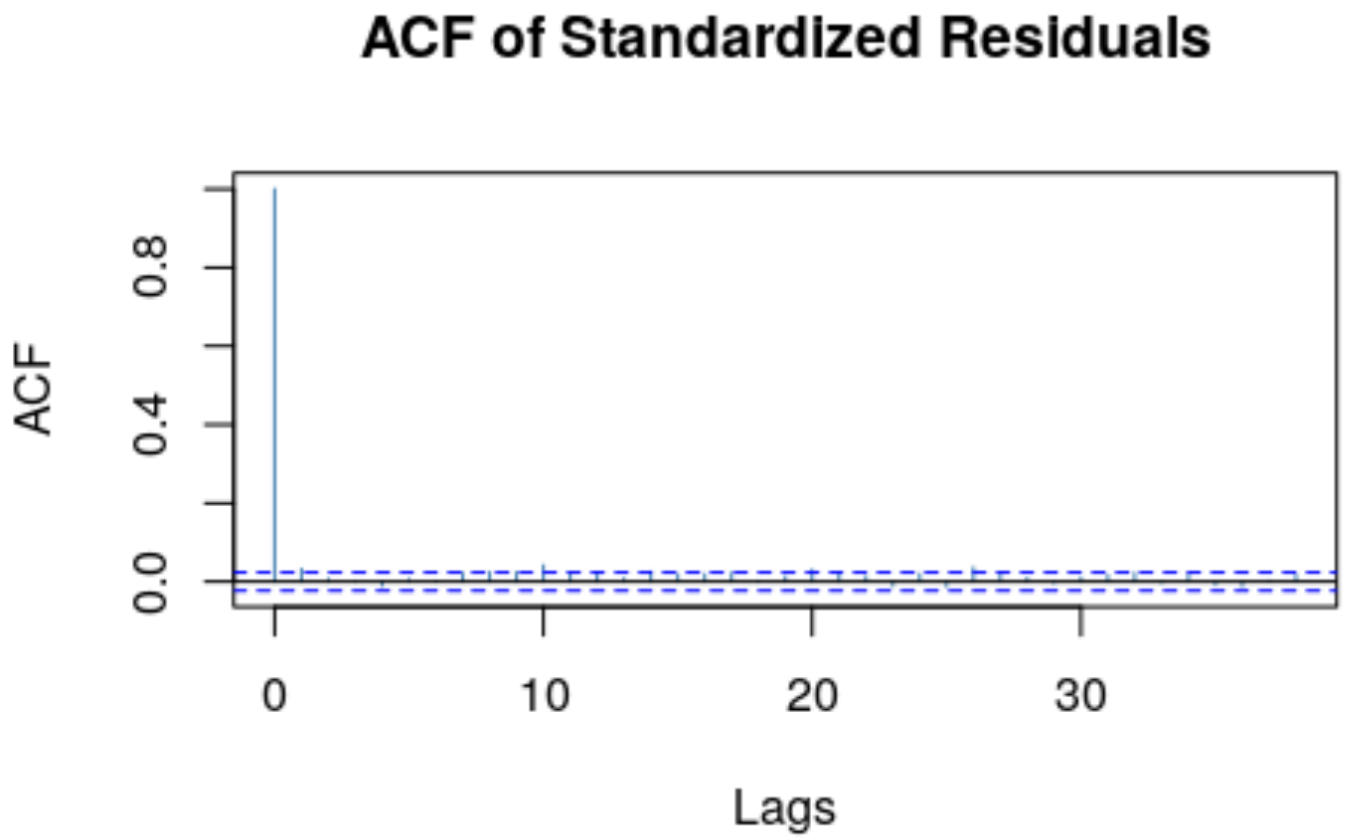
GARCH fit

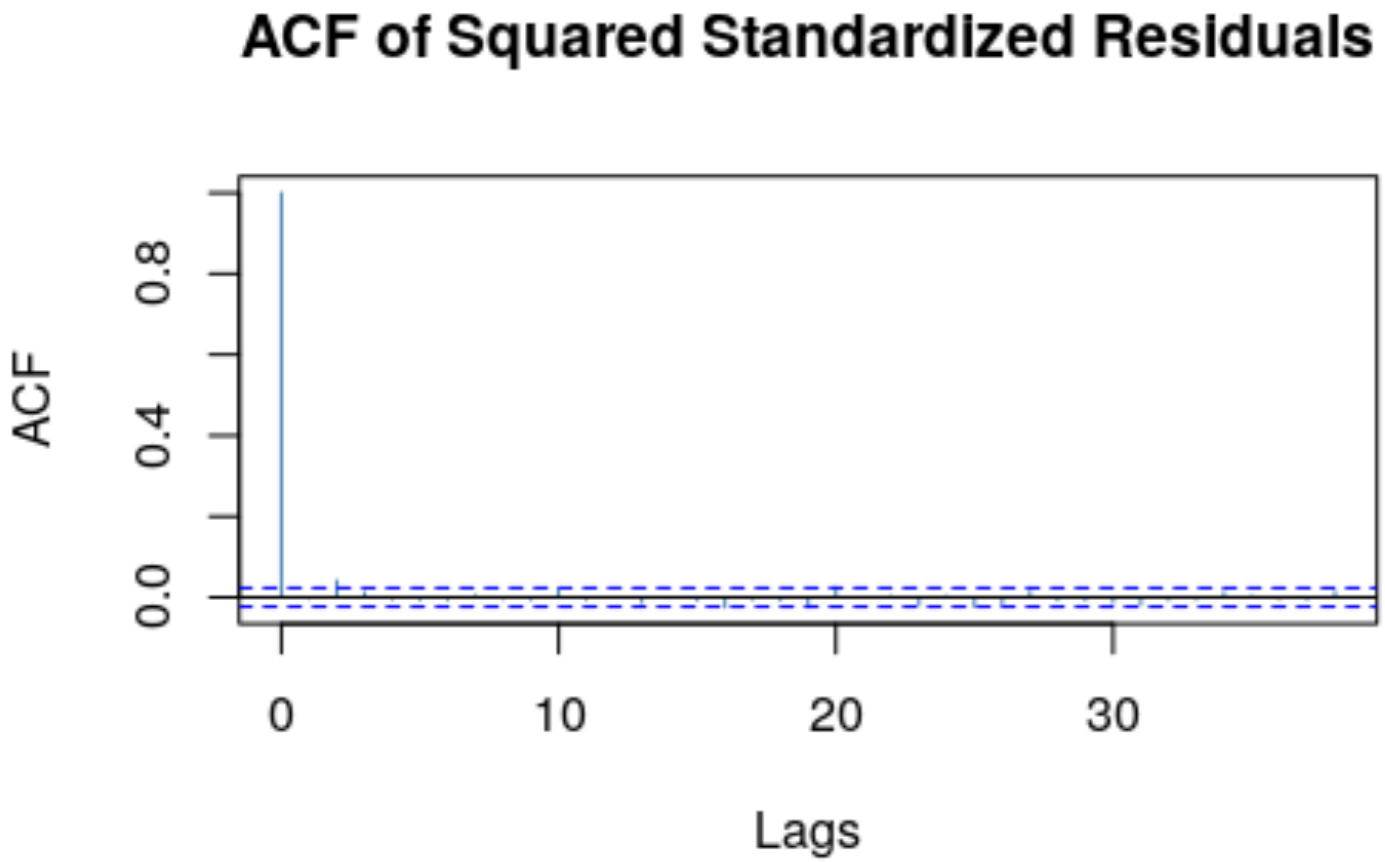
```
garchFit(formula = ~garch(1, 1), data = d$lr,
include.mean = TRUE,
    trace = F)

         Estimate  Std. Error  t value Pr(>|t|)
mu     9.839e-04   1.864e-04    5.278 1.31e-07 ***
omega  6.798e-06   1.018e-06    6.679 2.40e-11 ***
alpha1 9.822e-02   7.343e-03   13.375  < 2e-16 ***
beta1  8.864e-01   8.244e-03  107.525  < 2e-16 ***
```

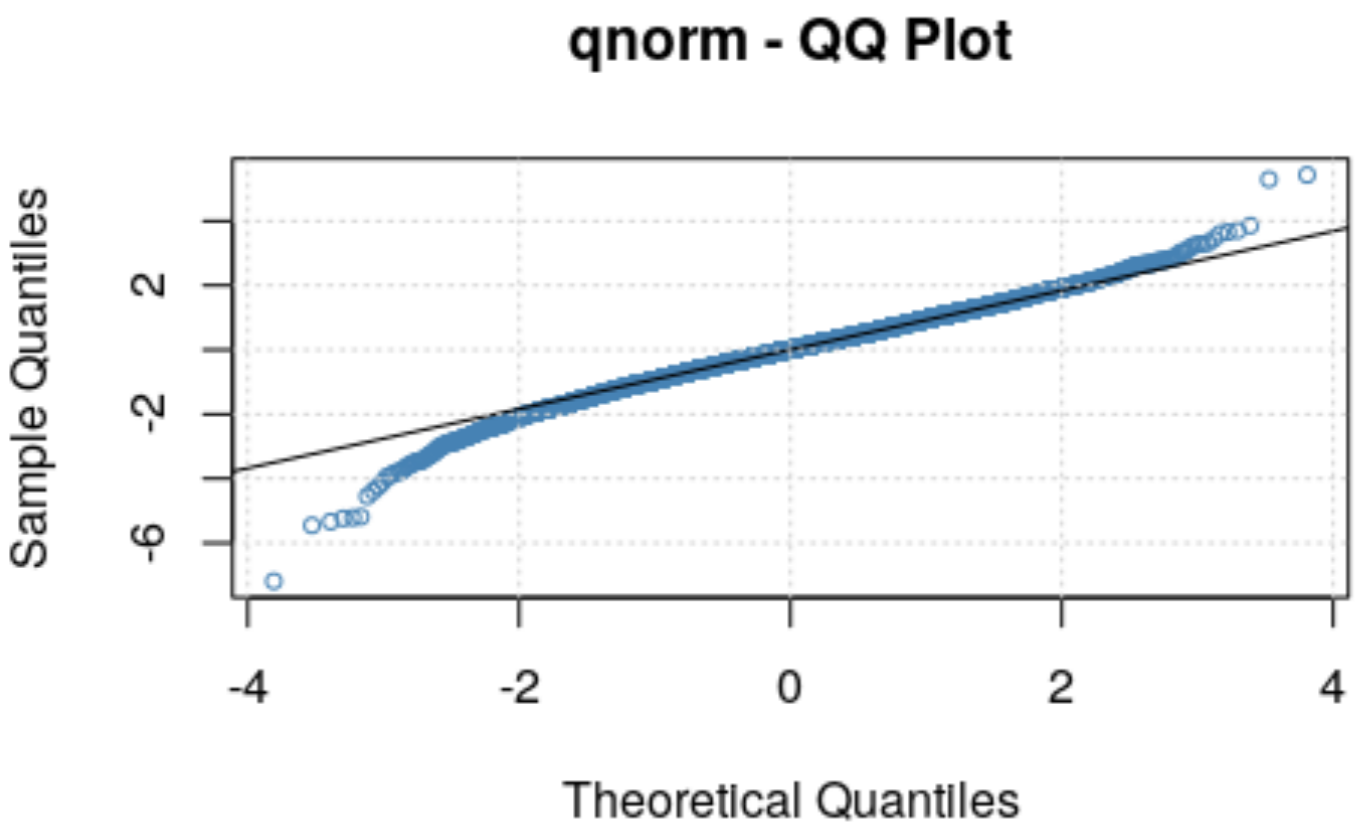

Residuals
res
0.3
0.1
-0.1
0
1000
3000
5000
7000
Index


Conditional SD's
xcsd
0.02
0.06
0.10
0
1000
3000
5000
7000
Index


Standardized Residuals
sres
2
-2
-6
0
1000
3000
5000
7000
Index

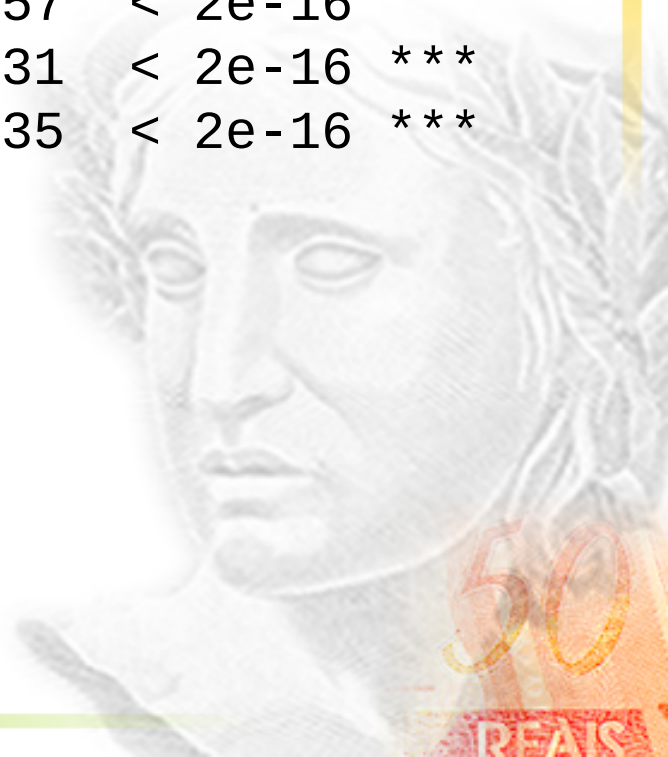

ACF of Standardized Residuals
ACF
0.0
0.4
0.8
0
10
20
30
Lags


ACF of Squared Standardized Residuals
ACF
0.0
0.4
0.8
0
10
20
30
Lags

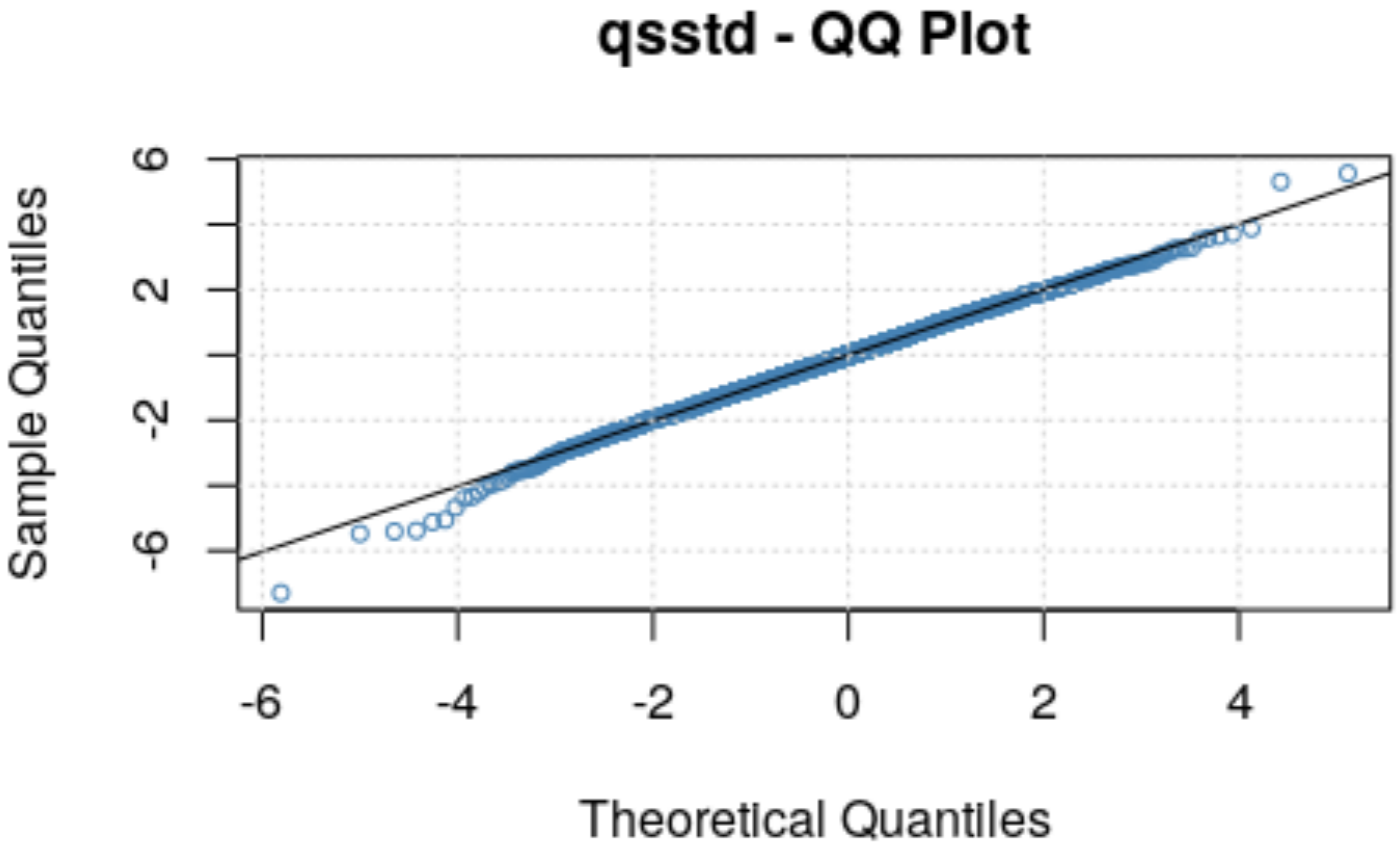

qnorm - QQ Plot
Sample Quantiles
-6
-2
2
-4
-2
0
2
4
Theoretical Quantiles

$e_t \sim t$

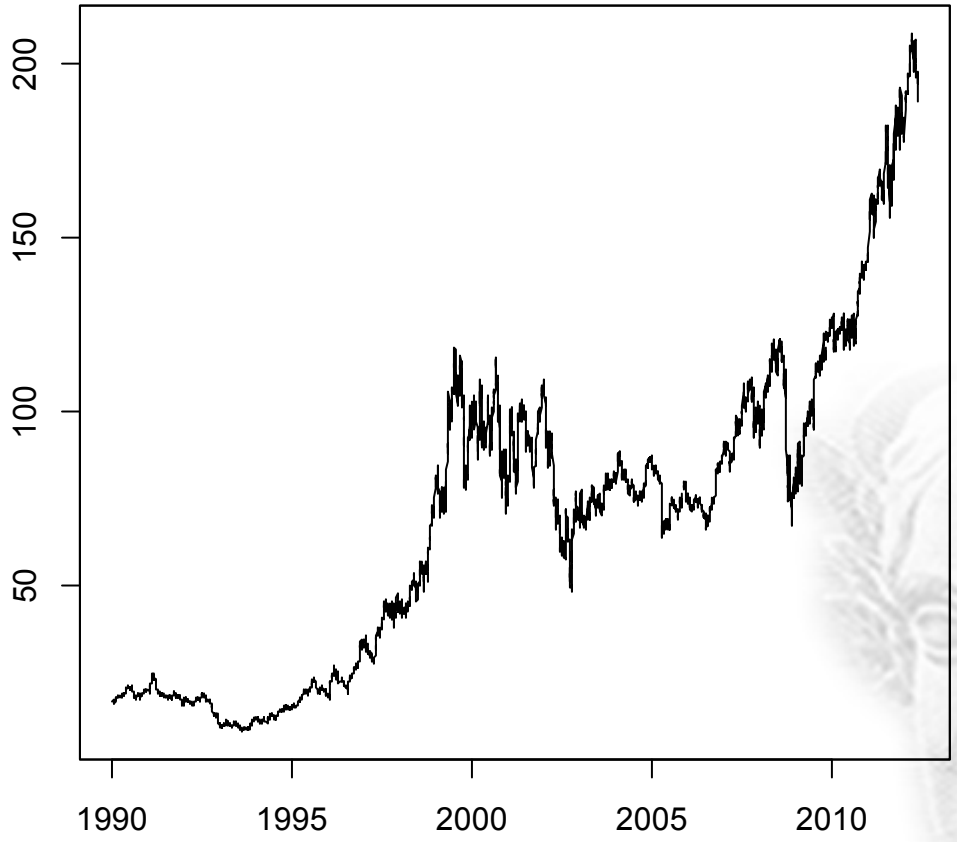
	Estimate	Std. Error	t value	Pr(>|t|)	
mu	9.128e-04	1.856e-04	4.919	8.71e-07	***
omega	5.259e-06	9.523e-07	5.523	3.34e-08	***
alpha1	8.809e-02	7.645e-03	11.522	< 2e-16	***
beta1	8.998e-01	8.266e-03	108.857	< 2e-16	***
skew	9.276e-01	1.593e-02	58.231	< 2e-16	***
shape	9.357e+00	9.054e-01	10.335	< 2e-16	***



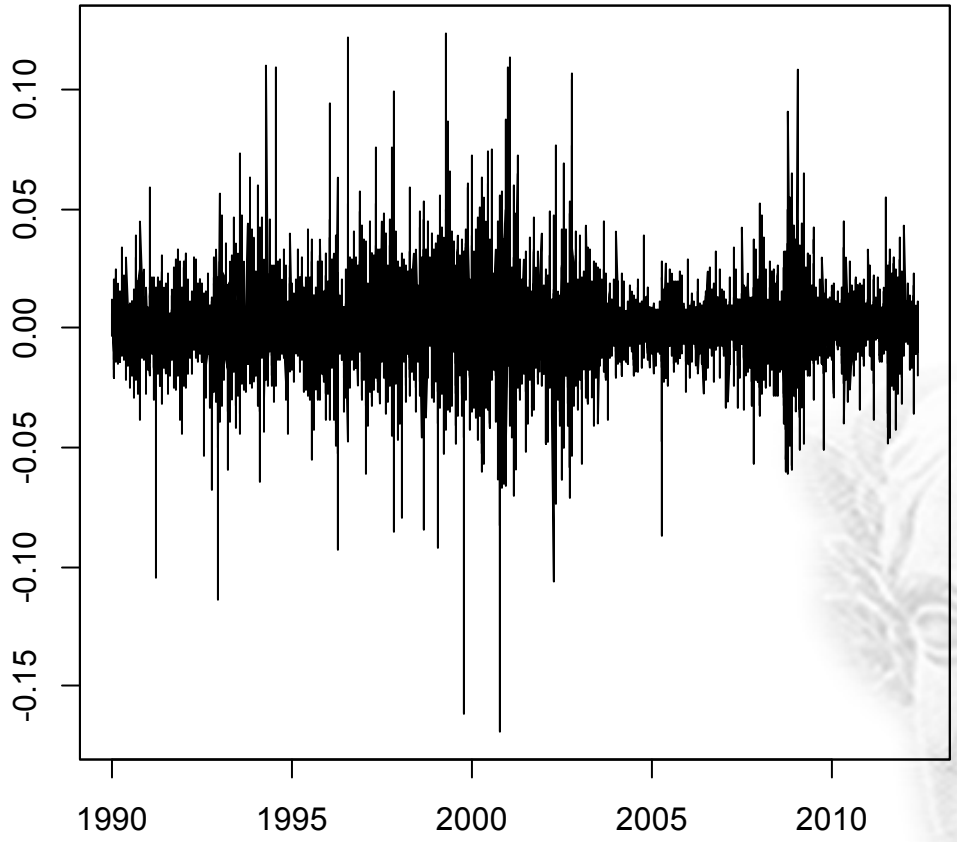
Asymmetric t distribution

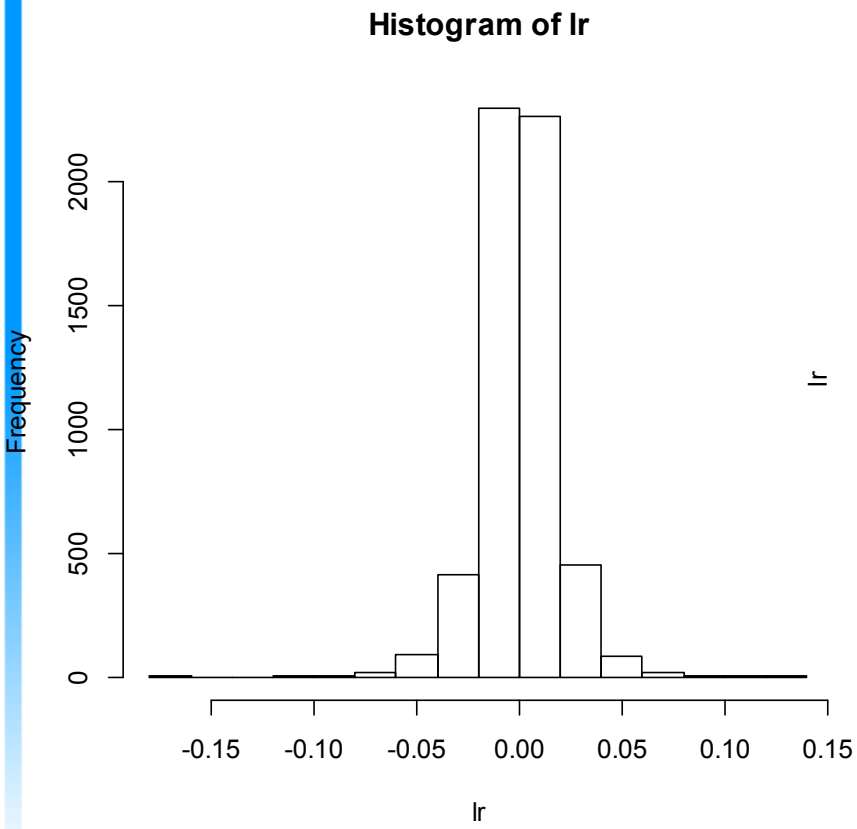


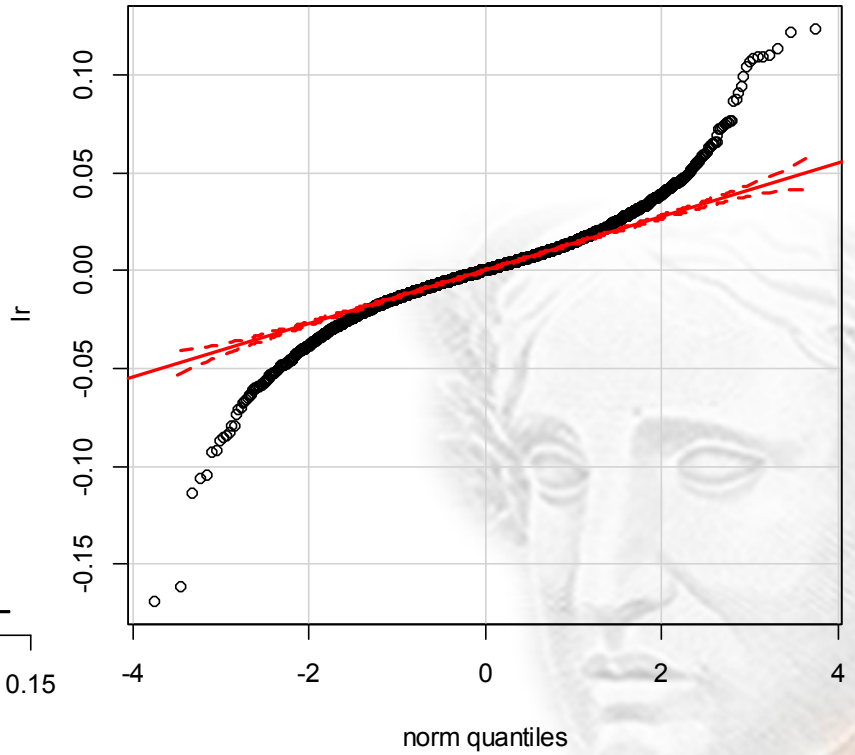
IBM



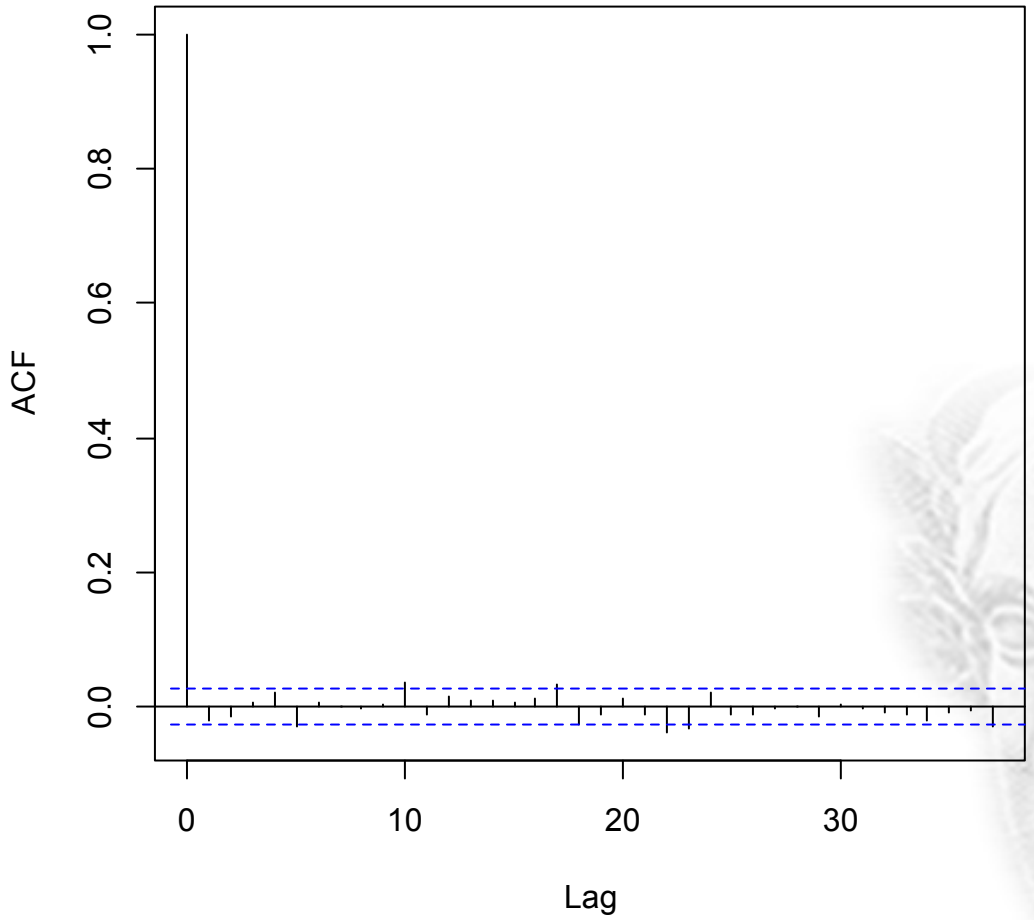
Log retornos



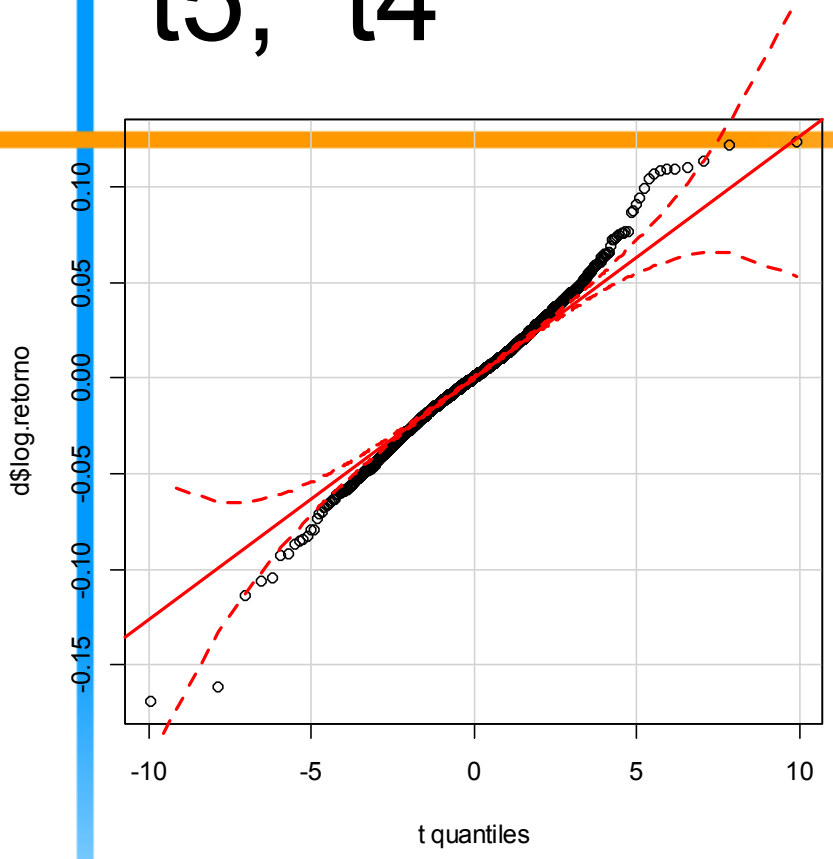

Histogram of lr
Frequency
0
500
1000
1500
2000
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
lr

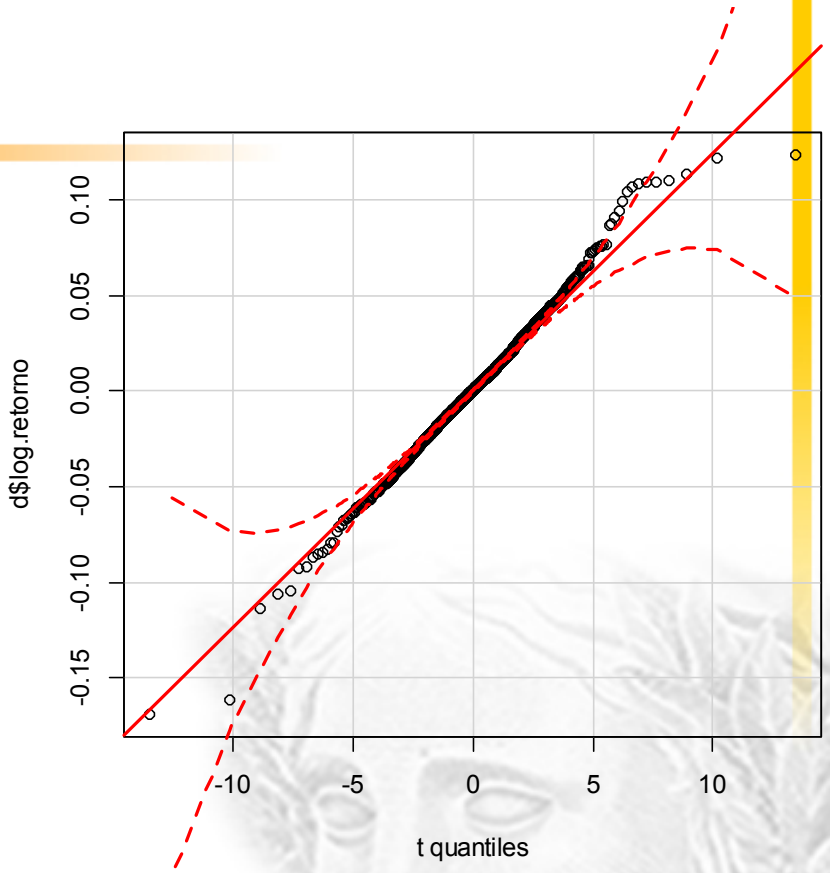

lr
0.10
0.05
0.00
-0.05
-0.10
-0.15
-4
-2
0
2
4
norm quantiles

Correlograma dos log retornos

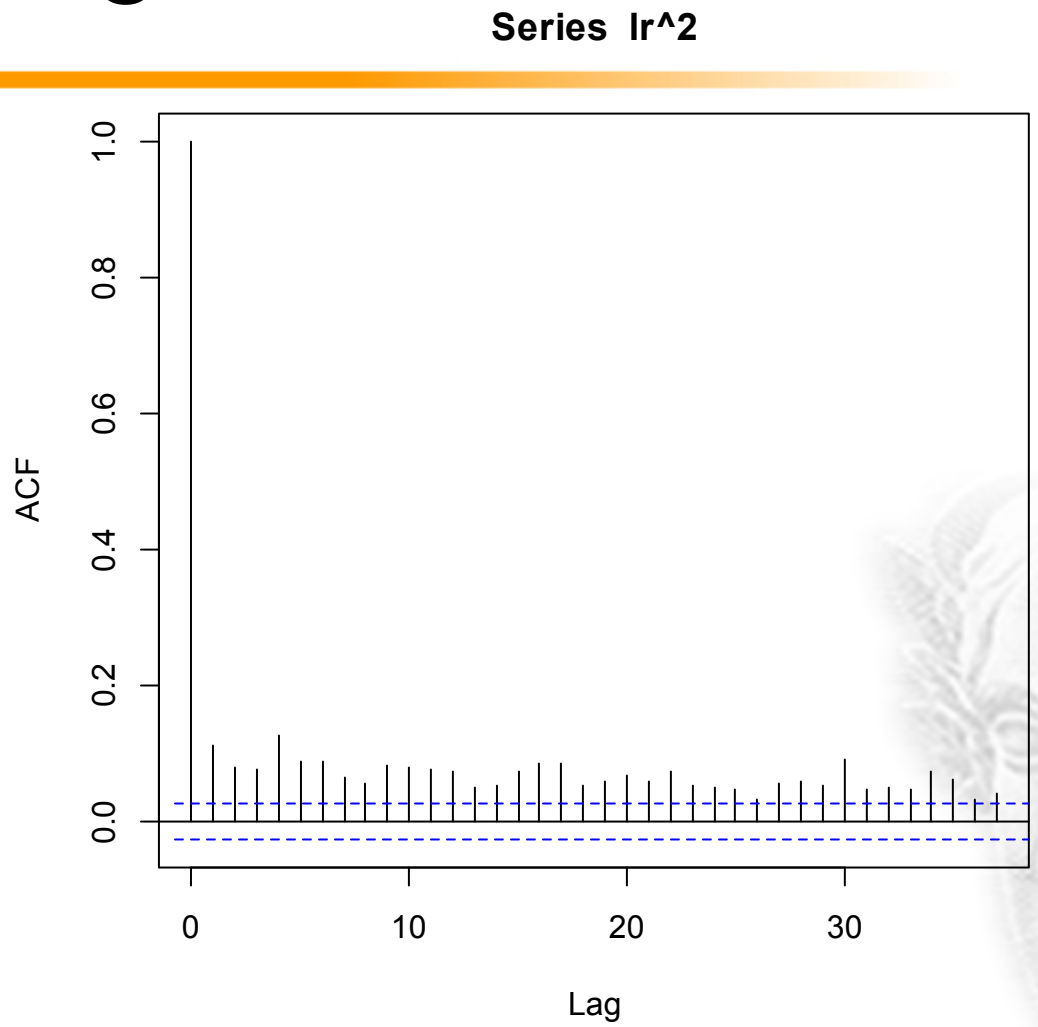


t5, t4


d$log.retorno
0.10
0.05
0.00
-0.05
-0.10
-0.15
-10
-5
0
5
10
t quantiles


d$log.retorno
0.10
0.05
0.00
-0.05
-0.10
-0.15
-10
-5
0
5
10
t quantiles

Correlograma



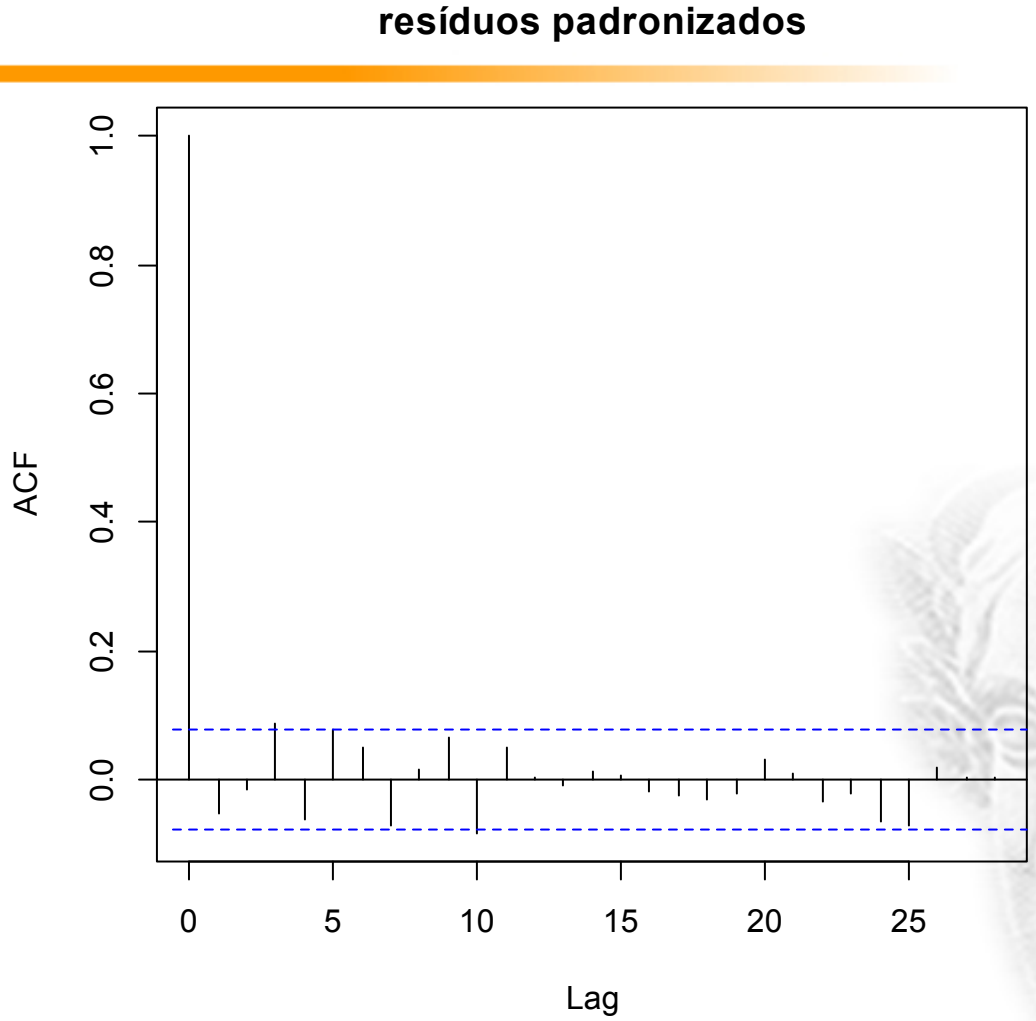
Modelo APARCH

- $\log(r_t) \sim \text{ARMA}(p,q)$
- $e_t = z_t \sigma_t$
- $z_t \sim D_v(0,1)$ não correlacionados
- $\sigma_t^{\delta} = \omega + \sum_{i=1}^{p} \alpha_i \left(|e_{t-i}| - \gamma e_{t-i} \right)^{\delta} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^{\delta}$

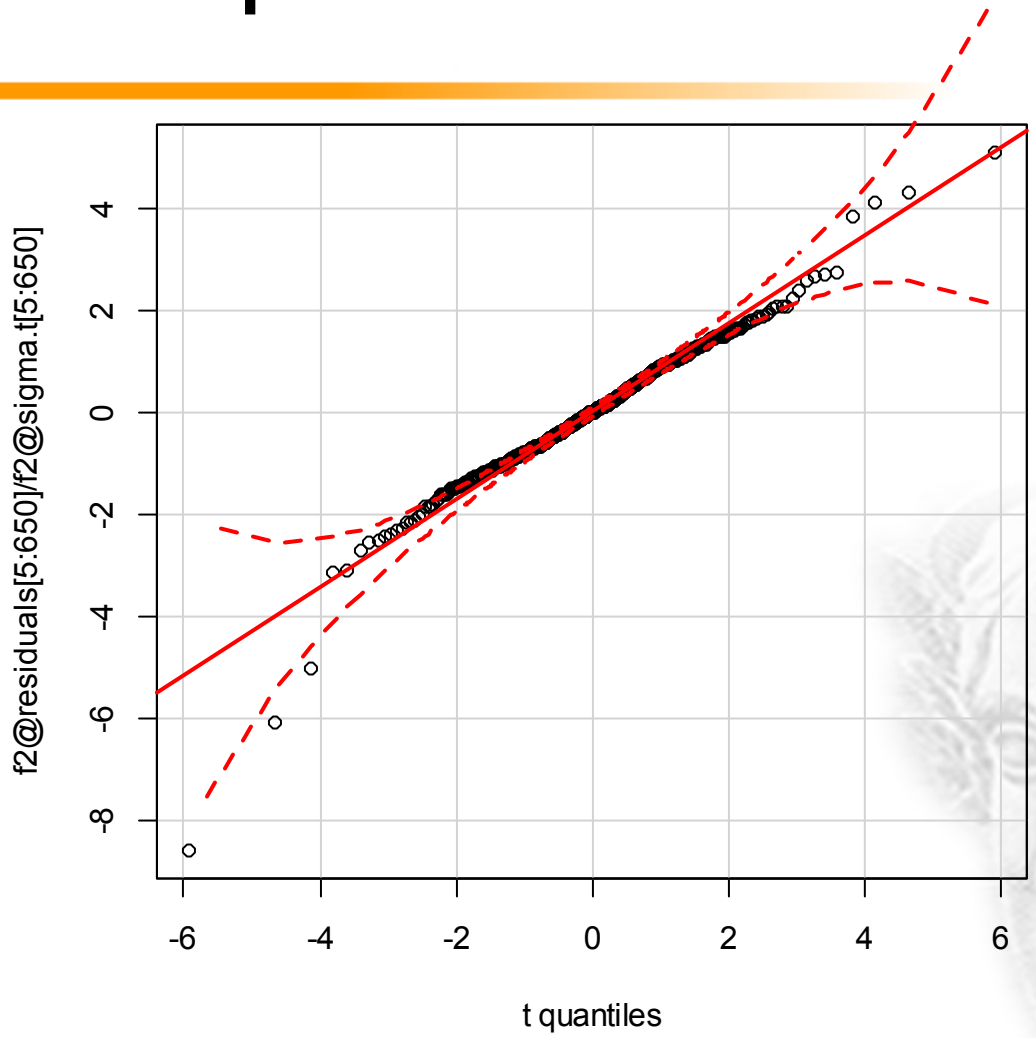
Modelo APARCH

	Estimate	Std. Error	t value	Pr(>|t|)	
ar1	0.0069670	0.5266210	0.013	0.989	
ma1	-0.0235647	0.5295123	-0.045	0.965	
omega	0.0004361	0.0001020	4.276	1.90e-05	***
alpha1	0.0732742	0.0075874	9.657	< 2e-16	***
gamma1	0.4347649	0.0642770	6.764	1.34e-11	***
beta1	0.9363072	0.0069555	134.613	< 2e-16	***
delta	0.7788780	0.1164535	6.688	2.26e-11	***
skew	1.0057449	0.0179652	55.983	< 2e-16	***
shape	5.3617965	0.3588884	14.940	< 2e-16	***

Diagnóstico



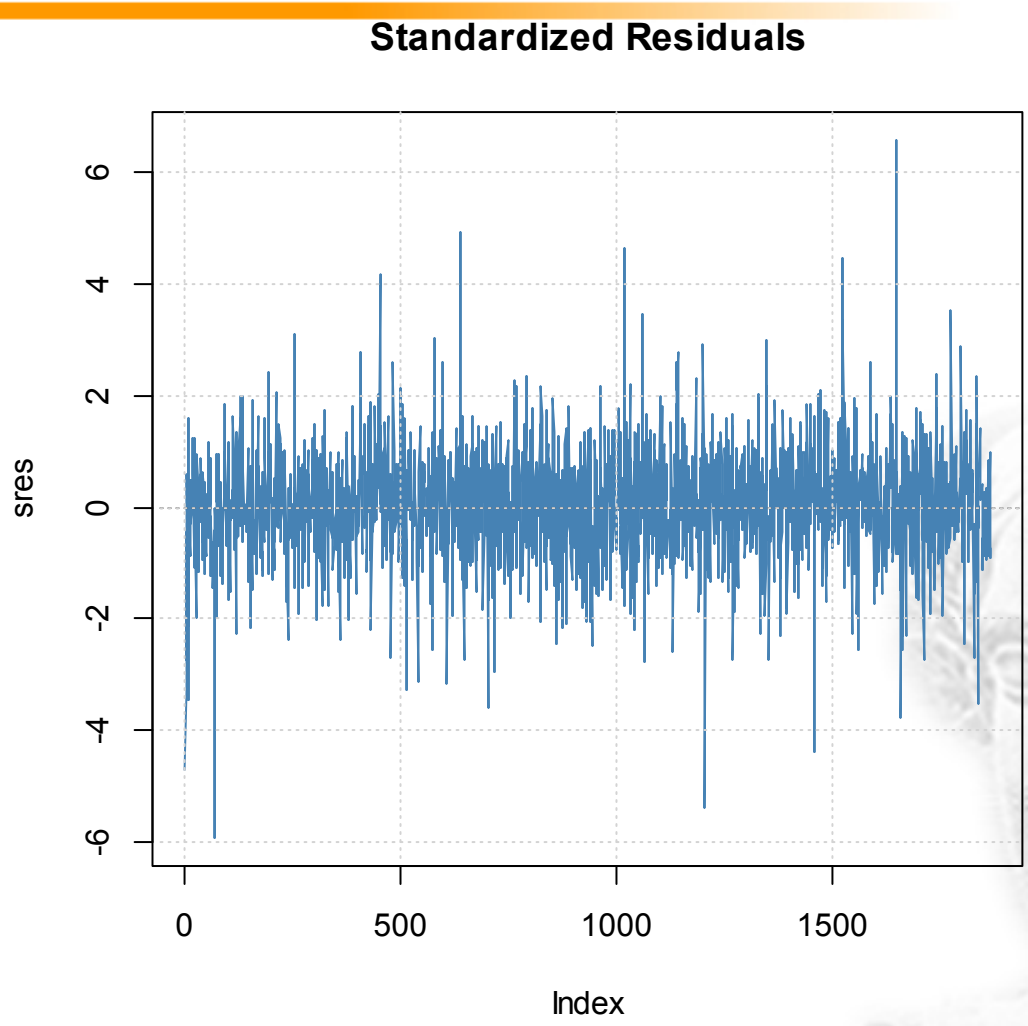
Resíduos padronizados ~ t5



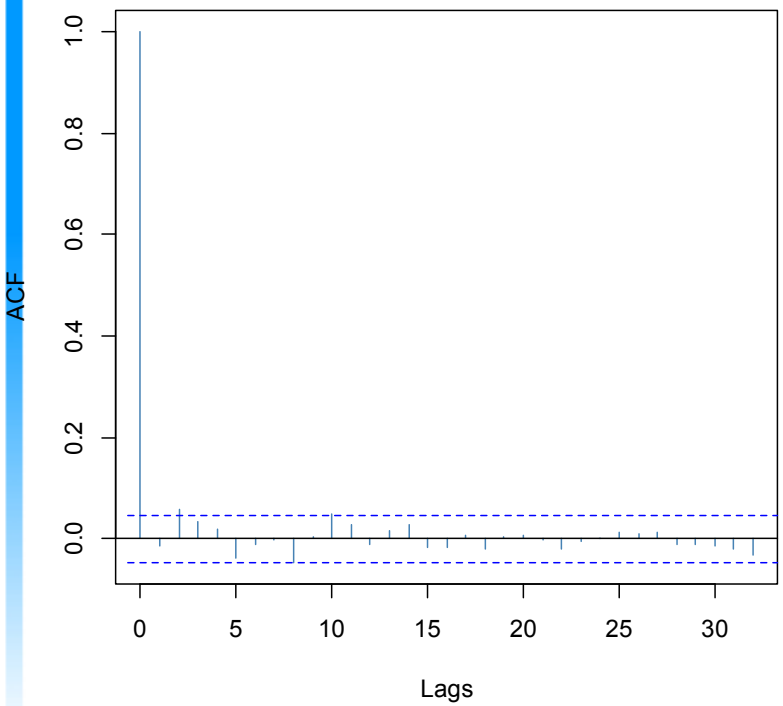
A partir de 2005

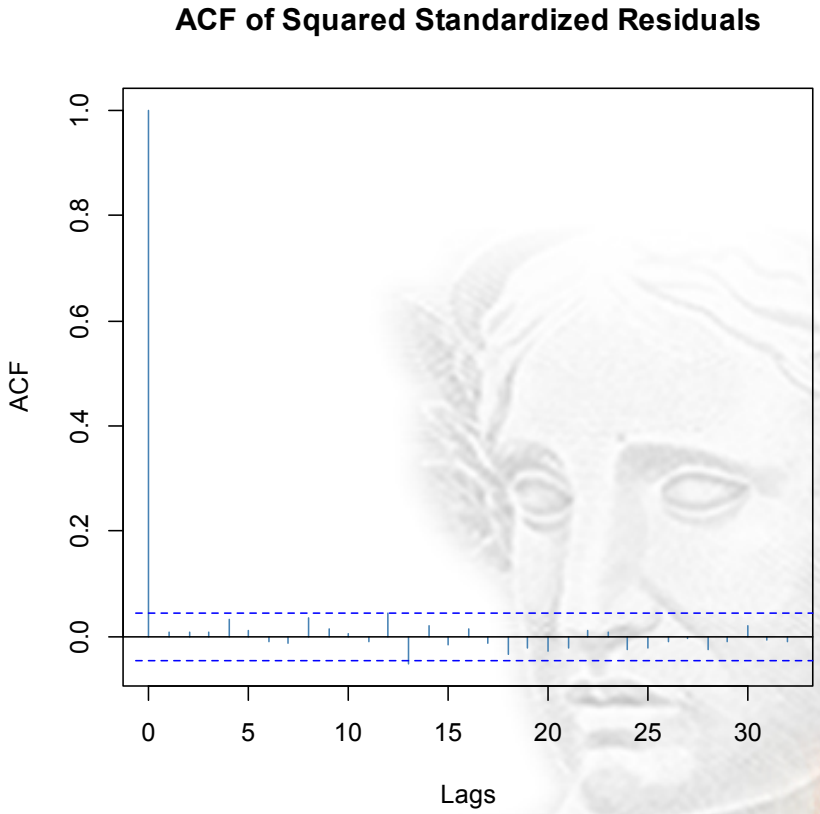
	Estimate	Std. Error	t value	Pr(>|t|)	
omega	0.0004113	0.0001196	3.440	0.000583	***
alpha1	0.0913509	0.0170443	5.360	8.34e-08	***
gamma1	0.5659135	0.1292614	4.378	1.20e-05	***
beta1	0.9092892	0.0173069	52.539	< 2e-16	***
delta	0.9492969	0.2072975	4.579	4.66e-06	***
skew	0.9622662	0.0298683	32.217	< 2e-16	***
shape	5.4743794	0.6748891	8.112	4.44e-16	***

Resíduos padronizados

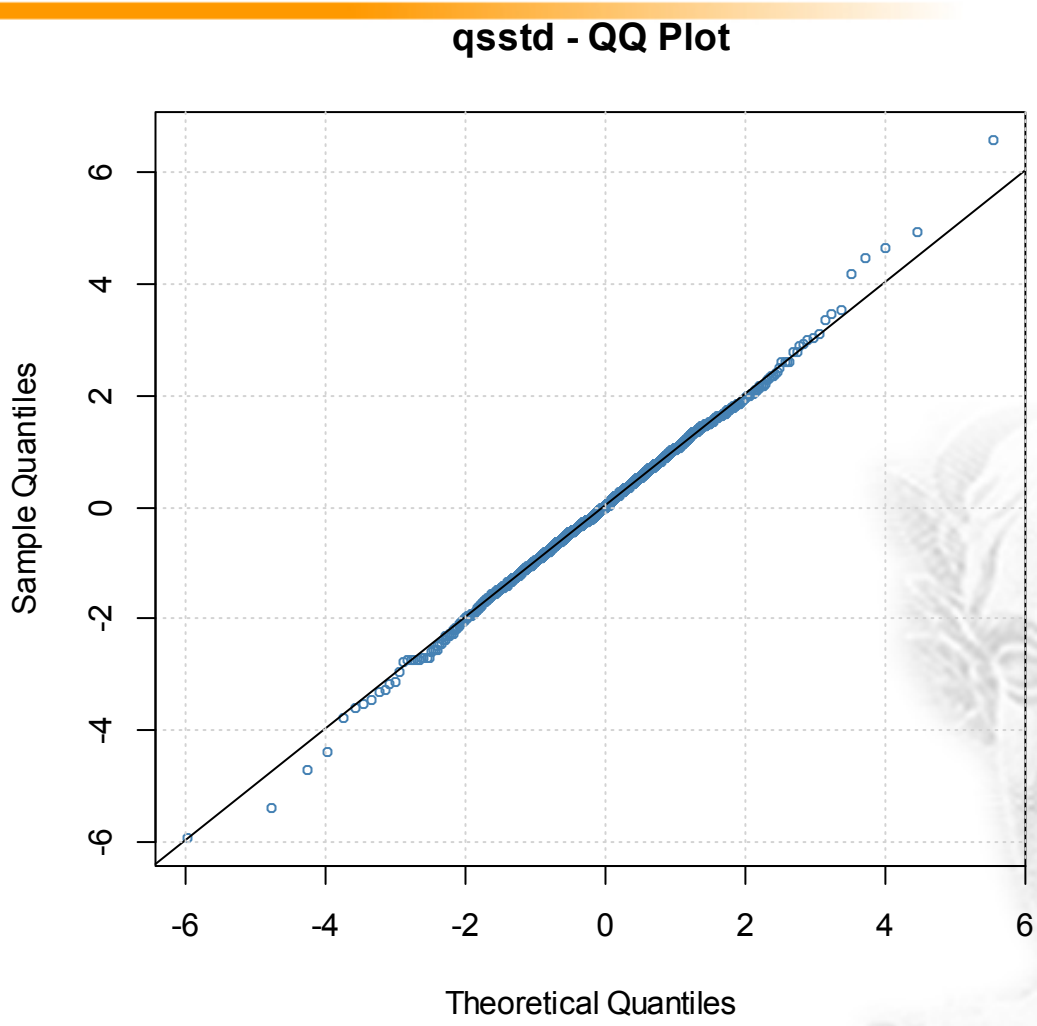


Resíduos padronizados

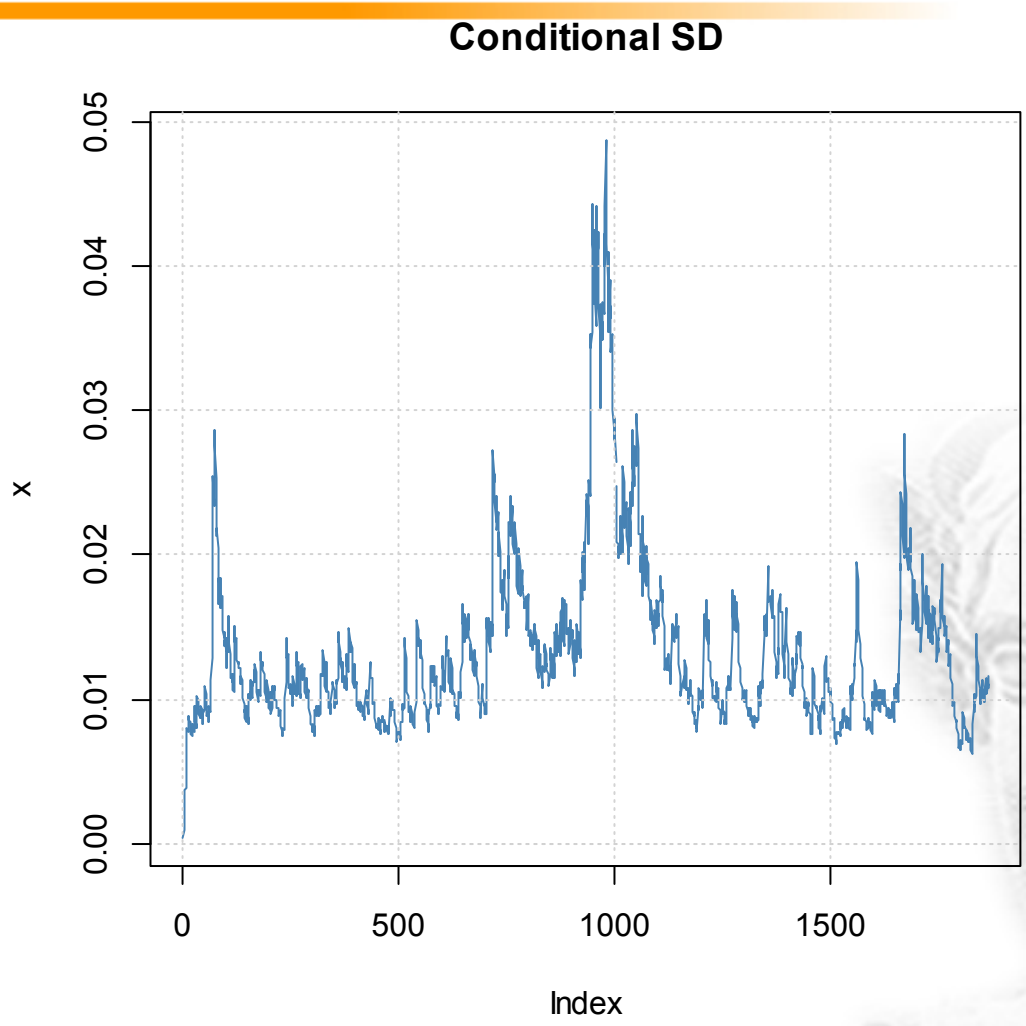




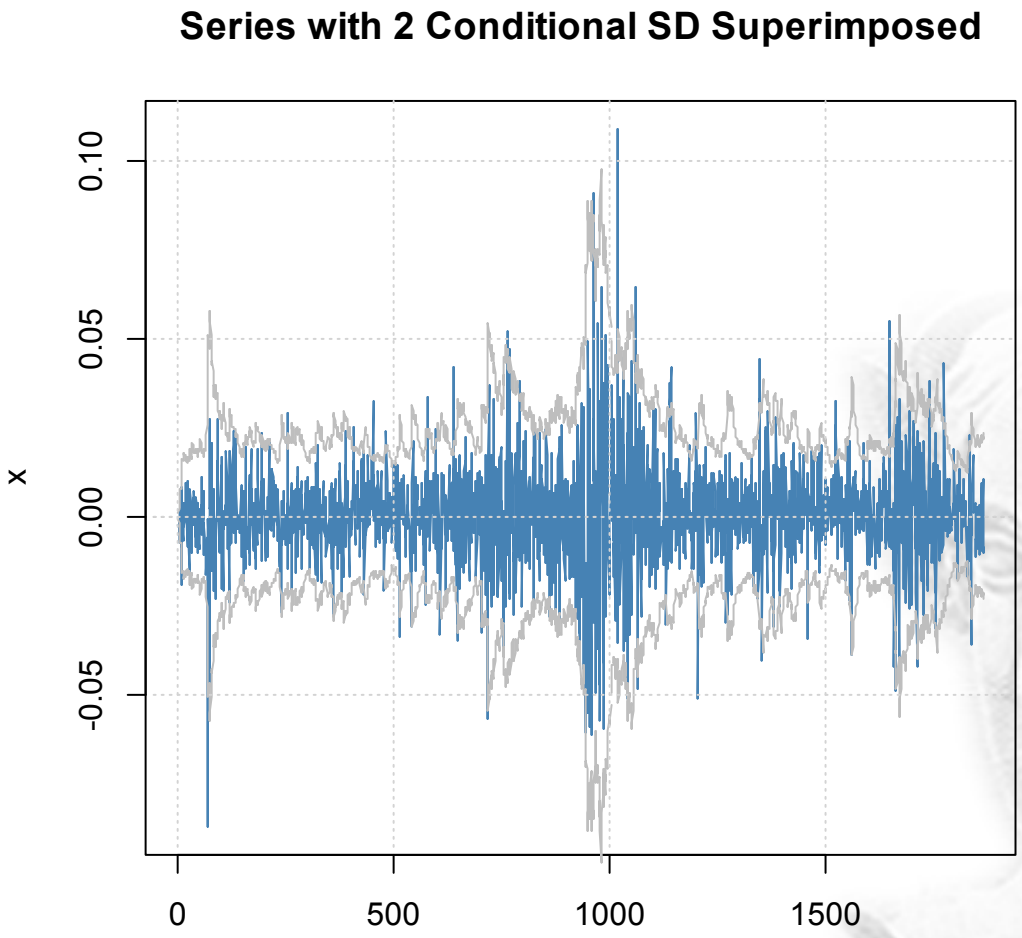
Resíduos padronizados



Desvio padrão condicional



Log retorno com 2 desvios



GARCH

- Modelo para a média

$$y_t = \phi_0 + \phi_1 y_{t-1} + e_t$$

com e_t não correl. e com $Var(e_t \mid y_1, \ldots, y_{t-1}) = \sigma_t^2$

- Modelo para a variância

$$\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2$$

- Se $\alpha+\beta<1$ e $0<\alpha, \beta<1$, temos estacionar.

$$Var(y_t) = \frac{\omega}{1 - \alpha - \beta}$$

- Ajuste pelo método de máxima verossimilhança supondo normalidade

TARCH

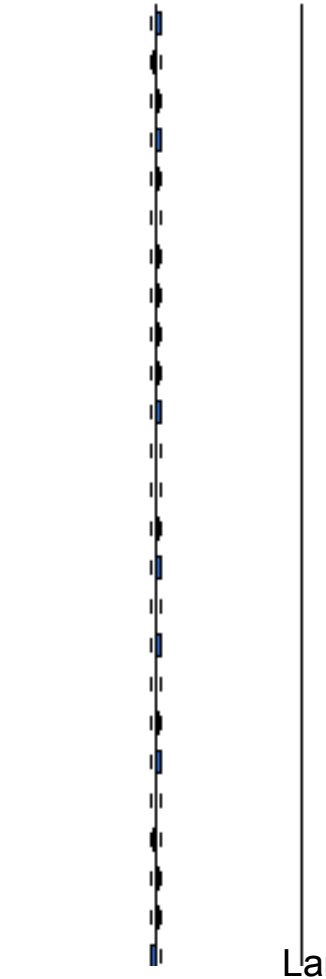
Dependent Variable: LR
Method: ML - ARCH (Marquardt)
Date: 06/12/09 Time: 16:14
Sample(adjusted): 1148 3747
Included observations: 2600 after adjusting endpoints
Convergence achieved after 13 iterations
Variance backcast: ON

	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000188	0.000132	-1.424904	0.1542
LR(-1)	0.124755	0.020713	6.022932	0.0000
Variance Equation				
C	1.73E-06	2.50E-07	6.924617	0.0000
ARCH(1)	0.251183	0.019309	13.00865	0.0000
(RESID<0)*ARCH(1)	-0.153676	0.021103	-7.282277	0.0000
GARCH(1)	0.811751	0.012966	62.60828	0.0000

R-squared	0.000665	Mean dependent var	-3.23E-06
Adjusted R-squared	-0.001262	S.D. dependent var	0.010717
S.E. of regression	0.010724	Akaike info criterion	-6.868071
Sum squared resid	0.298333	Schwarz criterion	-6.854541
Log likelihood	8934.493	F-statistic	0.345019
Durbin-Watson stat	2.084049	Prob(F-statistic)	0.885669

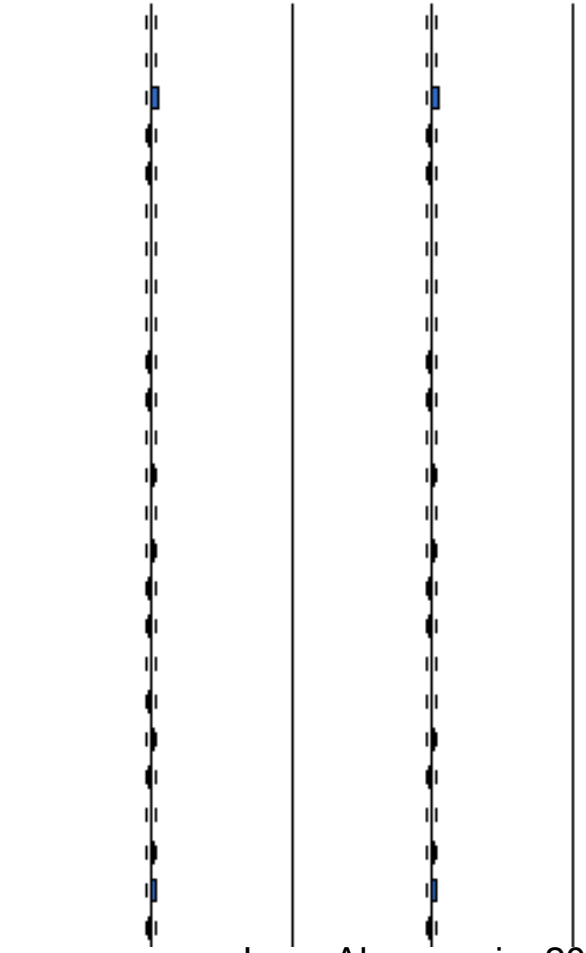
O impacto de log retornos<0 é (0,25-0,15) menor do que para log retornos>0.

Correlograma do res. pad.



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.034	0.034	3.0151	0.082
		2	-0.019	-0.020	3.9841	0.136
		3	0.020	0.021	5.0093	0.171
		4	0.029	0.027	7.2198	0.125
		5	0.017	0.016	7.9472	0.159
		6	0.005	0.004	8.0036	0.238
		7	0.014	0.013	8.4970	0.291
		8	0.011	0.008	8.7875	0.361
		9	0.010	0.009	9.0716	0.431
		10	0.021	0.020	10.203	0.423
		11	0.033	0.031	13.019	0.292
		12	0.000	-0.003	13.019	0.368
		13	-0.007	-0.007	13.137	0.437
		14	0.014	0.011	13.628	0.478
		15	0.040	0.037	17.836	0.271
		16	0.004	0.001	17.880	0.331
		17	0.035	0.036	21.070	0.223
		18	-0.006	-0.012	21.175	0.271
		19	0.009	0.008	21.398	0.315
		20	0.031	0.027	23.919	0.246
		21	-0.002	-0.007	23.928	0.297
		22	-0.009	-0.010	24.117	0.341
		23	0.009	0.007	24.323	0.386
		24	0.010	0.006	24.585	0.429
		25	0.037	-0.040	28.129	0.302

Correlograma dos res.pad.^2



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.003	-0.003	0.0213	0.884
		2	-0.006	-0.006	0.1096	0.947
		3	0.044	0.044	5.1197	0.163
		4	-0.013	-0.013	5.5441	0.236
		5	-0.016	-0.016	6.2521	0.282
		6	-0.003	-0.005	6.2753	0.393
		7	0.000	0.001	6.2756	0.508
		8	0.003	0.004	6.2946	0.614
		9	0.001	0.001	6.3005	0.710
		10	-0.012	-0.013	6.6864	0.755
		11	-0.015	-0.015	7.2419	0.779
		12	-0.007	-0.008	7.3872	0.831
		13	0.011	0.012	7.7271	0.861
		14	-0.005	-0.004	7.8013	0.899
		15	0.017	0.017	8.5850	0.898
		16	-0.022	-0.024	9.8501	0.874
		17	-0.011	-0.011	10.188	0.896
		18	-0.006	-0.007	10.273	0.923
		19	-0.010	-0.008	10.531	0.939
		20	0.013	0.014	10.968	0.947
		21	-0.020	-0.021	12.069	0.938
		22	0.007	0.007	12.201	0.953
		23	0.016	0.014	12.880	0.955
		24	0.030	0.032	15.231	0.914
		25	-0.018	-0.018	16.098	0.912

Referências

- Tsay. Analysis of Financial time series.
- Würtz. APARCH no R
- Alencar e Sáfadi. 2012.
- https://faculty.chicagobooth.edu/ruey-s-tsay/research/analysis-of-financial-time-series-2nd-edition