A Weibull Wearout Test: Full Bayesian Approach

Irony, T.Z., Laureto, M.,
Pereira, C.A.B., Stern, J.M.

In Hayabawa,Y., Irony,T., Xie,M. (editors)
Systems and Bayesian Reliability, 287-300
Singapore: World Scientifc, 2002

The two parameter Weibull density, reliability (survival) and hazard functions for a failure time $t \geq 0$ shape, and characteristic life (or scale) parameters $\beta > 0$, and $\gamma > 0$, are:

$$
\begin{aligned}
w(t\,|\,\beta,\gamma) &= (\beta\, t^{\beta-1}/\gamma^{\beta})\, exp(-(t/\gamma)^{\beta}) \\
r(t\,|\,\beta,\gamma) &= exp(-(t/\gamma)^{\beta}) \\
z(t\,|\,\beta,\gamma) &\equiv w(\ )/r(\ ) = \beta\, t^{\beta-1}/\gamma^{\beta} \\
\mu(W) &= \gamma\, \Gamma(1+1/\beta) \\
\sigma^2(W) &= \gamma^2(\Gamma(1+2/\beta) + \Gamma^2(1+1/\beta))
\end{aligned}
$$

$\beta = 1$, W is the exponential distribution
$\beta = 2$, W is the Rayleigh distribution
$\beta = 2.5$, W approximates the lognormal
$\beta = 3.6$, W approximates the normal
$\beta = 5.0$, W approximates the peaked normal
Regions $\beta < 1$, $\beta = 1$, $\beta > 1$
decreasing, constant, increasing hazard rates
infant mortality, memoryless, wearout failures

$$X_i \sim w(t \,|\, \beta, \gamma) \;\Rightarrow\; X_{[1,n]} \sim w(t \,|\, \beta, \gamma/n^{1/\beta})$$

Afine transformation $t = t' + \alpha$
three parameter truncated Weibull distribution
location (or threshold) parameter, $\alpha > 0$

$$w(t \,|\, \alpha, \beta, \gamma) = (\beta\,(t+\alpha)^{\beta-1}/\gamma^{\beta}) \\ exp(-((t+\alpha)/\gamma)^{\beta})/r(\alpha \,|\, \beta, \gamma)$$

$$r(t \,|\, \alpha, \beta, \gamma) = exp(-((t+\alpha)/\gamma)^{\beta})/r(\alpha \,|\, \beta, \gamma)$$

A panel displays 12 to 18 characters. Each character is displayed as a 5×8 matrix of pixels, and each pixel is made of 2 (RG) or 3 (RGB) individual color elements, (like a light emitting diode or gas plasma device). A panel fails when the first individual color element fails. The color elements are “burned in” so they are not at the infant mortality region, i.e. we assume $\beta > 1$.

A lot of panels were purchased as used components, taken from surplus machines. The dealer informed the machines had been operated for a given time, and also informed the mean life of the panels at those machines. Only working panels were acquired. The acquired panels were installed as components on machines of a different type. The use intensity of the panels at each type of machine corresponds to a different time scale, so mean lives are not directly comparable. The shape parameter however is an intrinsic characteristic of the panel. The used time over mean life ratio, $\rho = \alpha/\mu$, is adimensional, and can therefore be used as an intrinsic measure of wearout. We have recorded the time to failure, or times of withdrawal with no failure, of the panels at the new machines, and want to use this data to corroborate (or not) the wearout information provided by the surplus equipment dealer.

Model:

$$\Theta = \{(\alpha, \beta, \gamma) \ \in \ ]0, \infty] \times [1, \infty] \times [0, \infty[\ \}$$
$$\Theta_0 = \{(\alpha, \beta, \gamma) \in \Theta \,|\, \alpha = \rho\mu(\beta, \gamma) \,\}$$
$$f(\alpha, \beta, \gamma \,|\, D) \propto \prod_{i=1}^{n} w(t_i \,|\, \alpha, \beta, \gamma) \prod_{j=1}^{m} r(t_j \,|\, \alpha, \beta, \gamma)$$

data D are failure times, $t_i > 0$,
and the times of withdrawal $t_j > 0$.
Prior $\quad \beta \in [3.0, \ 4.0]$

At the optimization step it is better, for numerical stability, to maximize the log-likelihood:

$$wl_i = \log(\beta) + (\beta - 1) \log(t_i + \alpha) - \beta \log(\gamma) - ((t_i + \alpha)/\gamma)^{\beta} + (\alpha/\gamma)^{\beta}$$
$$rl_j = -((t_j + \alpha)/\gamma)^{\beta} + (\alpha/\gamma)^{\beta}$$
$$fl = \sum_{i=1}^{n} wl_i + \sum_{j=1}^{m} rl_j$$
$$h(\alpha, \beta, \gamma) = \rho\,\gamma\,\Gamma(1 + 1/\beta) - \alpha = 0$$

$n = 45$ failure times, in years,

0.01	0.19	0.51	0.57	0.70
0.73	0.75	0.75	1.11	1.16
1.21	1.22	1.24	1.48	1.54
1.59	1.61	1.61	1.62	1.62
1.71	1.75	1.77	1.79	1.88
1.90	1.93	2.01	2.16	2.18
2.30	2.30	2.41	2.44	2.57
2.61	2.62	2.72	2.76	2.84
2.96	2.98	3.19	3.25	3.31

and $m = 5$ withdrawals

1.19	3.50	3.50	3.50	3.50

Evidence for some values of ρ:

ρ	0.05	0.10	0.20	0.30	0.40
e	0.04	0.14	0.46	0.98	1.00

ρ	0.50	0.60	0.70	0.80	0.90
e	0.98	0.84	0.47	0.21	0.01