

Chance & Complementarity, Analogy & Prototype, Homology & Archetype as Sources of Knowledge

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Richard Owen (1832) Charles Darwin (1859) Konrad Lorenz (1932) Karl v. Frisch (1923) Johannes Kepler (1604)

www.ime.usp.br/~jmstern/presentations/ v. 18/06/25

8th Congr. Square of Opposition; S. José, Costa Rica, 9-13/09/24

3rd RatioLog - Logic, Rationality, and Probability; 05-09/05/2025

XXI EBL - Brazilian Logic Conference; Serra Negra, 12-16/05/25

14th Principia Intl. Symposium; Florianópolis, 28-31/07/2025

Borromean Threads (presentation's title & summary)

(T) Chance & Complementarity, Homology & Archetype, Analogy & Prototype, as Sources of Knowledge .. in Evolution of Life

(II) Random evolution: Fortuity of Variation / Probability of Survival

• Probability: Discretization / Convexification Operator

(III) Homology & Archetype, Structure, Divergence ..

(IV) Analogy & Prototype, Function, Convergence ..

• Analogous Use but Different Intrinsic Mechanisms?

(V) Teleology & Fine-Tuning: Testing, Verification, Truth

• Defective (always) Prototypes w. Fine-Tuning conditions

(VI) Fallacies & Solutions in Logic & Statistical inference

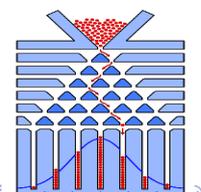
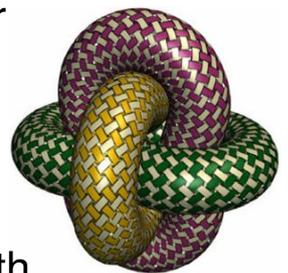
• Abstract Belief Calculi and Coherent Statistical Inference

(VII) Case study: "Symbolic Language" of honeybees

• Homology, structure, syntax; Analogy, function, semantics ..

(VIII) Language: Innovation, Conscience, Metaphysics ..

(IX) F. Remarks (T): 3 threads of 2++ conflating conceptual strands, interlocked in complex interactions



Key concepts: Im+Probability, Purpose, Organism

- *But chance and spontaneity are reckoned among causes... Some people even question whether they are real or not. They say that nothing happens by chance, but that everything... has some definite cause and it is always possible... to find something which is the cause; but not chance, for if chance were real, it would seem strange indeed.*

Aristotle, 350 BCE, Physics (II, 4, 195-196)

• Το γαρ μη τυξουτως αλλ' ενεκα τινος εν τοις της φυσικης εργασις εστι και μαλιστα: ου δ' ενεκα συεστακεν η γεγυνητε τελους, την του καλου ζοραν ειληφεν.

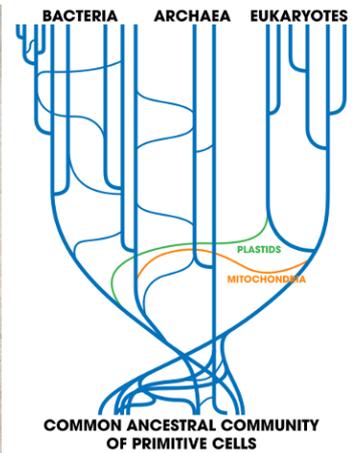
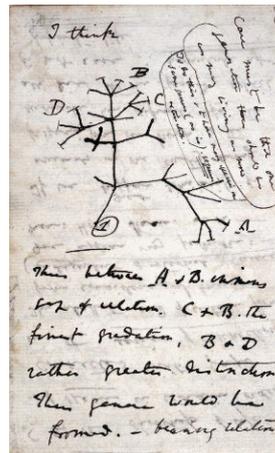
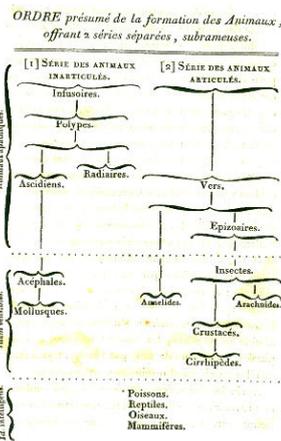
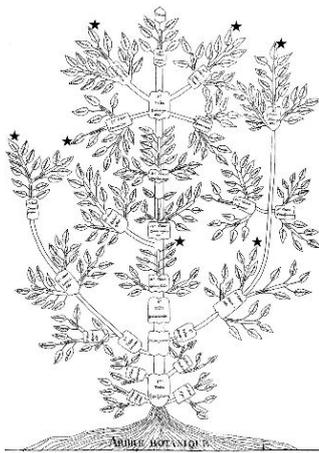
- *Absence of haphazard [chance] and conduciveness of everything for a purpose are to be found in nature's works in the highest degree, and the end for which those works are put together and produced is a form of the beautiful [good, virtuous].*

Aristotle, Parts of Animals (I, 5, 645)

Darwin: Probability, Organism, Purpose, Evolution

- Charles Darwin letter to Charles Lyell (Aug. 01, 1861):
- *The view that each variation has been providentially arranged seems to me to make natural selection entirely superfluous, & indeed takes the whole case of appearance of new species out of the range of science. [FV] It seems to me that variations in the wild and domestic conditions are due to unknown causes & are without purpose & insofar accidental; [PS] and that they become purposeful only when they are selected by man for his pleasure, or by what we call natural selection in the struggle for life under changing conditions.*
- *I do not wish to say that God did not foresee everything which would ensue; but here comes very nearly the same sort of wretched embroglio as between free-will and preordained necessity.*
- C. Johnson (2014,xxii) *Darwin's* (dual / complementary) *Dice*:
Fortuity of Variations: random haphazard generatn. mechansm;
Probablty of Survival: more likely conservation of best adapted.

Tree of Life: History of organisms' evolution process



- Family or Life trees, by proximity / transformation / evolution:
- Augustin Augier, Philosophie zoologique, 1809;
- Jean-Baptiste Lamarck, Histoire naturelle des animaux, 1815;
- Charles Darwin, Transmutation of Species, 1837* (1859); and
- Contemporary version with horizontal / lateral gene transfers: **Fortuitous branching**: by mutation, cross-over, hybridization.. **Probabilistic pruning**: elimination ill-adapted individ. / species
- ...“coagulated past, where the history of events is recorded.”

Evolution: Continuous probability → Discrete events

• *The moment 'now' is like a sieve passing steadily through time. In front of it is a probability future, a [continuous] 'wavy' future if you like, in which we can predict only how likely some result is to happen. As time streams through our sieve, it coagulates this wavy future into a [discrete] particle past, where the precise history of events is recorded.*

- William Lawrence Bragg (1969) *What makes a scientist?*

- I will focus on entangled² and Complementary nature of:
- Causal explanat: Structure / Homology vs. Function / Analogy;
- Improbability arguments for Identity (same) / Teleology / Truth: Small vs. Infinitesimal prob. in Discrete vs. Continuous spaces.

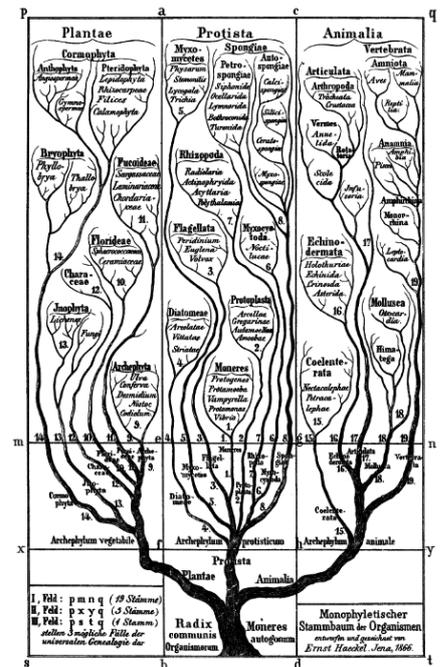
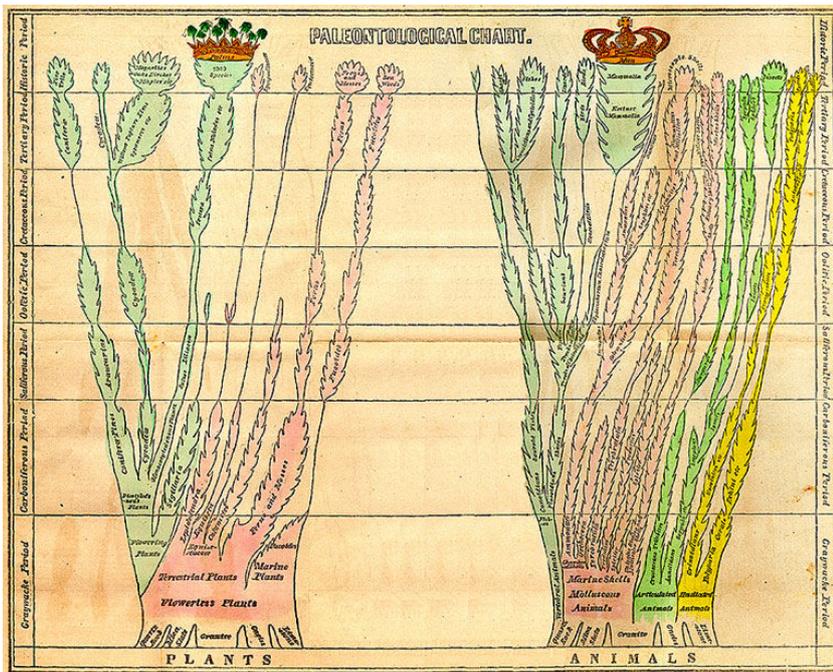
(A) Related Opposition / Duality / Complementarity Paradoxes:

- Probability as (De-)Convexification operator (Stern 2008, 6.8):
 - Continuum of (future) possible configurations of a system, vs.
 - Discrete set of (observed) states or structures of a system

Probability as Discrete- / Convex- -ification Operator

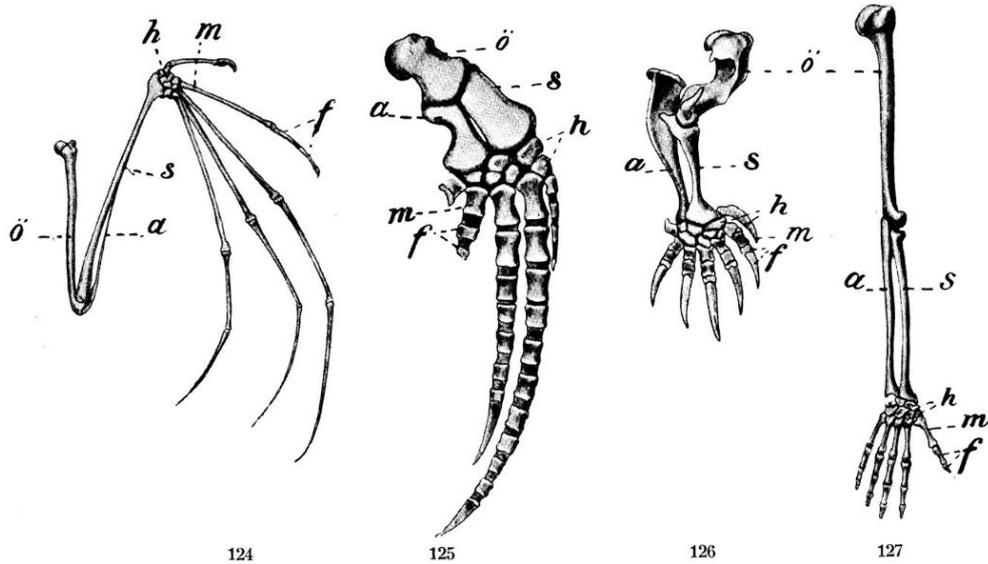
- *Matching Pennies* game, played by players Odd and Even:
- Each of the players has to show, simultaneously, a bit (0 or 1)
- If both bits disagree / agree (01, 10) / (00, 11) Odd / Even wins.
- An *eigen-solution* or *equilibrium-point* of a game is a set of strategies that leaves each player at a local optimum, that is, – each player, having full knowledge of all the other players' strategies, can gain nothing by unilaterally changing his own.
- In a Pure or Deterministic strategy, players must choose a (discrete) action to take; \Rightarrow MP game has No equilibrium point.
- In a Mixed or Randomized strategy, players must choose a continuous vector of probabilities for their possible actions; $\Rightarrow p_O = p_E = [0.5, 0.5]$ = only equilibrium point for MP game.
- Transformation of strategy space from Discrete to Continuous and Convex space allows the emergence of an eigen-solution.
- O.Morgenstern, J.Neuman (1947, 2008) *Th. of Games & Econ. Beh.*
- Kemeny, Snell (1977) *Finite Math.* • Rozanov (1977) *Probability Th.*
- Probability as (De-) Convexification operator, see Stern (2008, 6.8)

How to (re)assemble and structure a tree of life?



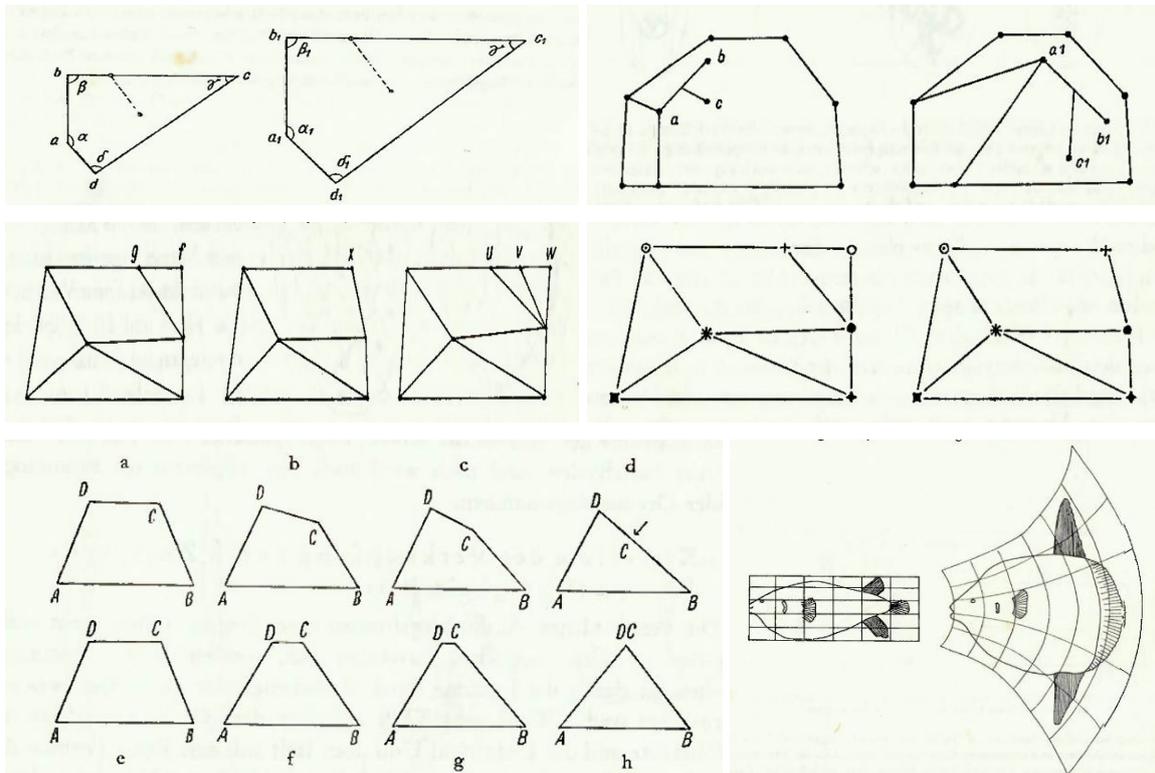
- Edward Hitchcock (1857) *Elementary Geograpy*, linking geological eras, fossil records & tree-structured grouping of species
- Ernst Haeckel (1866) *Generelle Morphologie der Organismen*
- E.Torrens, A.Barahona (2013) *Darwin's Muses Behind his 1859 Diagram*

Homology: Similar Topological or Connection Maps



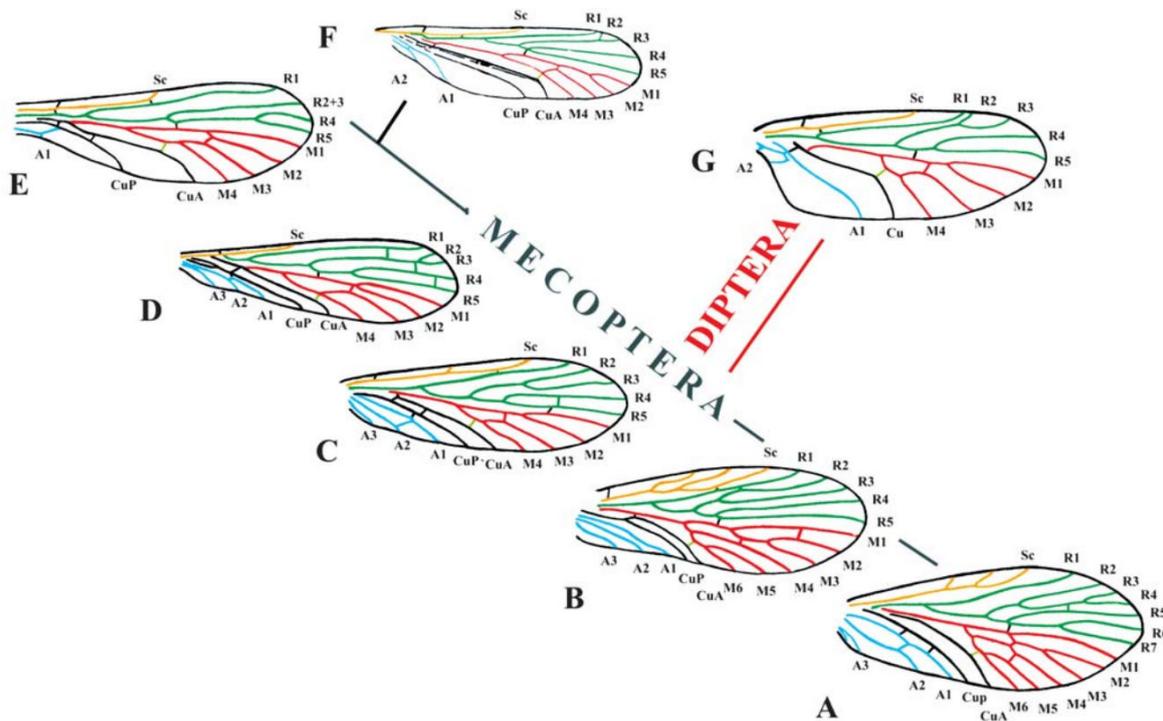
- Leche (1909): Homology between forelimbs' skeletons of some mammals having distinct functions /purposes: 124 Bat's wing for flying, 125 Whale's flipper for swimming, 126 Mole's arm for digging, 127 Human arm for object manipulation.
- Structural correspondnce between organisms established by topological coincidence / similarity between their parts^v, $v=1,2,\dots$

Discrete Coincidences & Gradual Transformations



- A. Remane (1952) schemata for structural evolution by transformation, inclusion or exclusion (fusion) of structural elements.

Assembling the Tree of Life by Tracing Homologies



- Krzeminski (2003): Phylogenetic relations within Mecoptera (A-F) and origin of Diptera (G) at beginning of Triassic. Colour of veins fields: yellow, subcostal; green, radial; red, medial; black, cubital; blue, anal.

Homology, Identity & Archetype; by Richard Owen

- *Homologue: The same organ in different animals under every variety of form and function.* Richard Owen (1843, p.379)
- *These relationships (homologies) are mainly, if not wholly, determined by the relative position and connection of the parts, and may exist independently of form, proportion, substance, function, and similarity of development.* R. Owen (1848, p.6)
- *Archetype (Gr. αρχητυπος < αρχη, origin, τυπος, type): The original of that which is represented in a picture or statue. In the language of Plato, it means the world as it existed before creation in the mind of God.* R. Owen (1866, p.146)
- *Archetype in Anatomy: Is that ideal original or fundamental pattern on which a natural group of animals or system of organs has been constructed, and to modifications of which the various forms of such animals or organs may be referred. The archetypal figure has been most clearly recognised in the study of the modifications of the skeleton of the vertebrate animals.*

Archetype & Homology, as used in this article

(H1) *(structural) Archetype*: Schematic plan, or Topological map characterizing the structure of a complex system (organism, organs thereof) by its layout of organization from constituent parts, or by interconnection pattern of its basic components.

(H2) *Homologous*: Distinct organisms, organs, or parts thereof described by the same or similar structural archetypes.

(H3) Homologous systems may have different shapes, sizes, functions, constructions, geneses, material implementations, ..

(H4) Homology may be used to establish “Identity” (using the same name?) of corresponding parts in different systems.

(H5) In the context of modern theories of biological evolution, homology may be a useful tool for investigating and tracing phylogenetic paths of gradual development diverging from a common ancestor (root of corresponding branches / sub-trees)



Improbability of (discrete, $n \in \mathcal{N}$) coincidences

• “Argument” of Improbability : [These structures] *...are too numerous and regular in their shape to allow for a moment the supposition of their being accidental...*

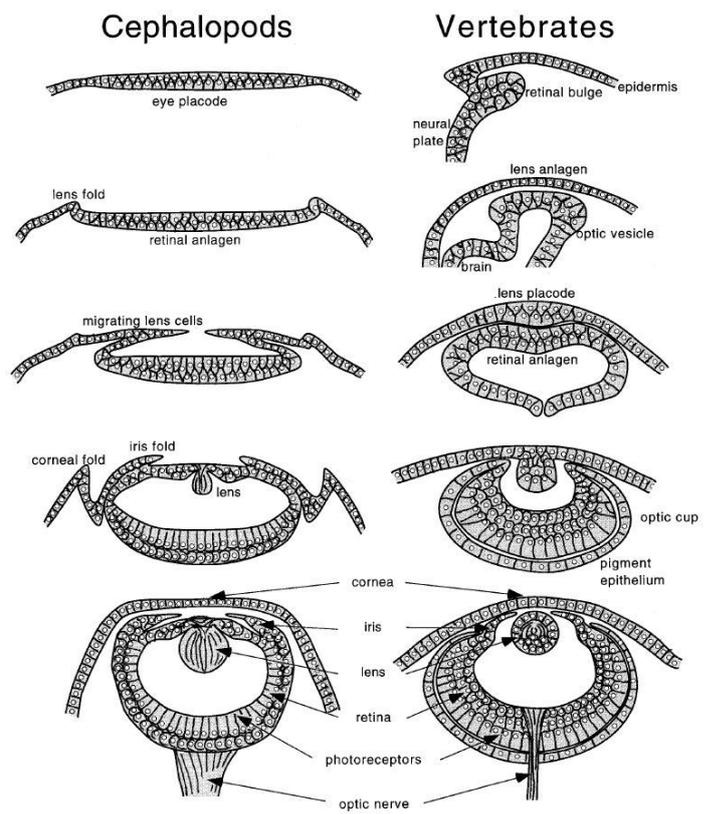
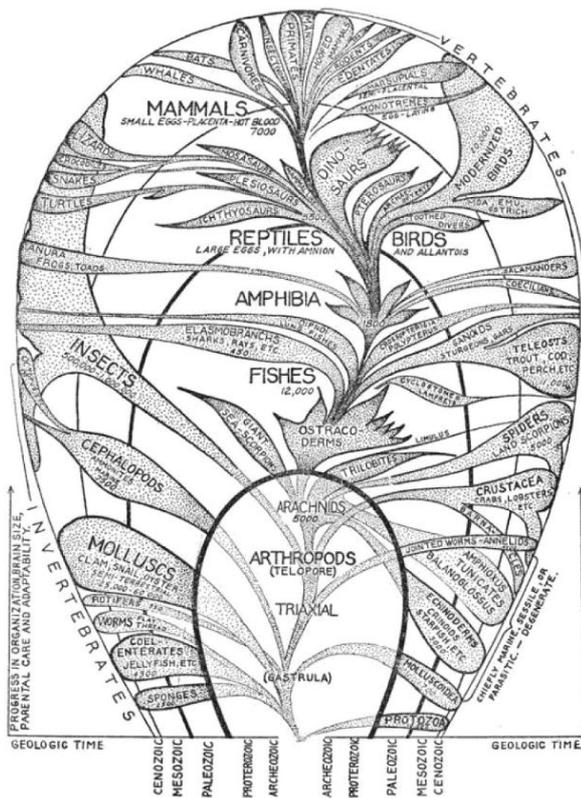
Richard Owen (1832, p.28)

• *The natural computer of our Gestalt perception can take in and evaluate a much greater number of data than our rational computation can. Many truths become falsified & many obvious facts become invisible if one restricts his methods to quantification alone. Quantification, however, has the last word in verification, and all that our perception tell us becomes "science" only when we succeed in confirming it by rational verification.*

• *The probability of two forms of life evolving, by sheer coincidence, a certain number of identical characteristics, can be calculated. It is equal to $1/2^{n-1}$, n being the number of similar or identical characteristics.* Konrad Lorenz (1981, p.95,88)

• Joseph Felsenstein (2004). *Inferring Phylogenies*. ◀ ◻ ▶ +Historic. Linguistics 🔍 ↻

Convergent & Independ./ Parallel evolution of eyes



- W.Patten (1923) *Evolution*. Th.Pietsch (2012) *Trees of Life: A Visual History of Evolution*.
- W.Harris (1997) Schematic diagrams of eye development.

Analogy as a Source of Knowledge

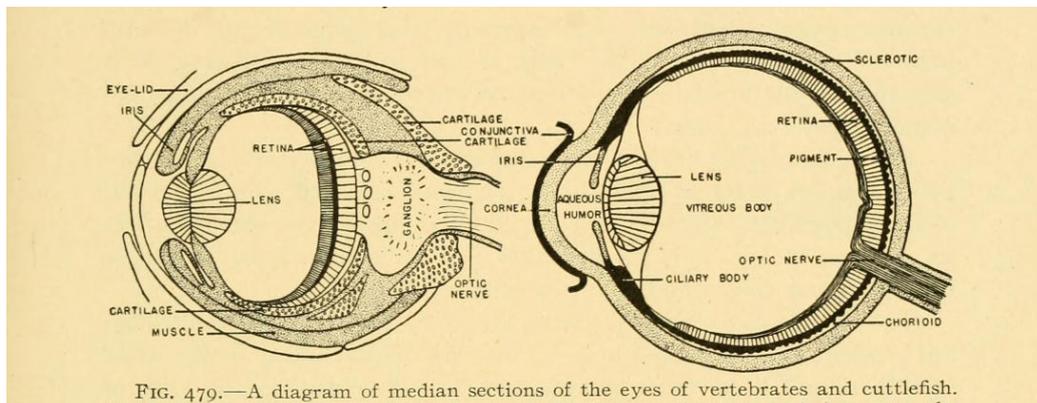


FIG. 479.—A diagram of median sections of the eyes of vertebrates and cuttlefish.

- In an octopus eye, “like” in a photographic camera, focus is obtained by adjusting the image’s distance to the lens, x .
- Similarly, in a human eye, focus is obtained by adjusting the focal distance of a flexible lens that can be pushed or pulled by special muscles and hence increase or decrease its convexity.
- by -Analogy-, we name both biological structures an “eye”, and understand their physiology as an optical camera system.
- ex: retina plays a role analogous to the photographic plate.
- R.Neal (1936) *Comparative Anatomy*. • R.Owen (1832) *Memoir on the Nautilus..*

Same Thing / Name by Function (not structure)

- *Analogue: A part or organ in one animal which has the same function as another part or organ in a different animal.*

Richard Owen (1843, p.374)

- *But homologous parts may be, and often are, also analogous parts in a fuller sense, viz. as performing the same functions; [*..they are*] homologous inasmuch as [*..they are*] composed of the same or answerable parts... and they are analogues of each other, inasmuch as they have the same relation of subserviency to [*a function*].*

Owen, (1848, p.7)

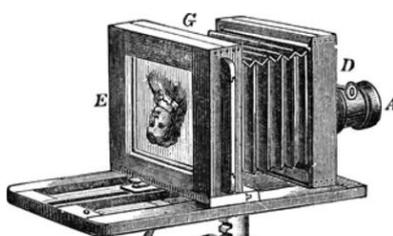
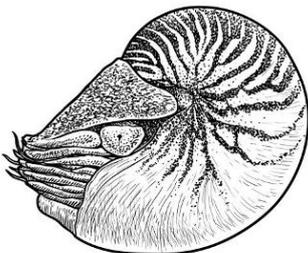
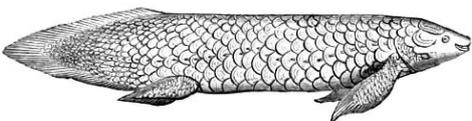
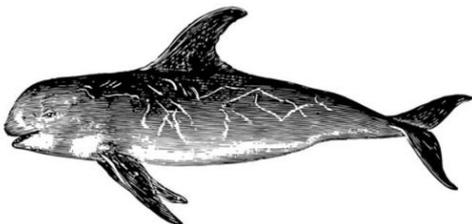
- *Owen distinguished two kinds of resemblance in the corresponding organs or parts of the bodies of different animals:*
 - (1) *essential structural agreements relating particularly to relative position and connections;*
 - (2) *similarities in the function or use to the organism.*

– *These are really different qualities & they have no necessary dependency upon each other.*

A. Boyden (1943, p.231)



Same Thing / Name by Function (not structure)



- *...thus the fin or pectoral limb of a Porpoise is homologous with that of a Fish, inasmuch as it is composed of the same or answerable parts: and they are analogues of each other, inasmuch as they have the same relation of subserviency to swimming.*

Owen, (1848, p.7)

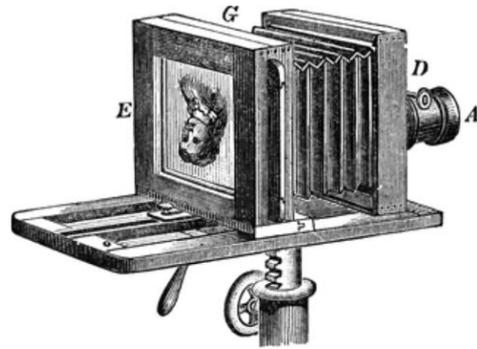
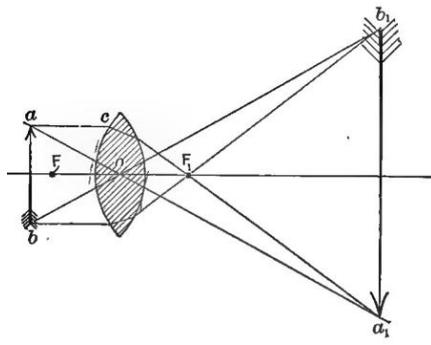
- *In the Nautilus the eyes [*present*] the simplest condition of an organ of vision, consisting only of a darkened globular cavity or camera obscura*, into which light was admitted by a single orifice, and a nerve expanded at the opposite side to receive the impression.*

Owen (1836, p.551)

* J. Kepler (1604, 1611) *Astr. Optica, Dioptrice*



The 'In Focus' *Fine-Tuning Condition* ($x \in \mathcal{R}$)



In focus condition for a camera with a single convex lens:
Lens equation relates the lens' focal distance, f , to
 the image's and the object's distances to the lens, x and x' ,

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{x'} \Rightarrow f = \left(\frac{1}{x} + \frac{1}{x'} \right)^{-1}, \quad x = \left(\frac{1}{f} - \frac{1}{x'} \right)^{-1}$$

This system can be brought into focus by adjusting either the
 lens' focal distance, f , or the image's distance to the lens, x .
 When in focus, system's magnification factor is $m = x/x'$.

- Fuller (1978) *Physics: Including Human Applications*.
- PSSC (1974) *Physics*.

Function ~ Purpose: What-for? How? (Whose* ?)

- ... *adaptations to a special end, are made comprehensible on a higher principle, and a final purpose is gained in relation to human intelligence in the instances where the analogy of humanly invented machines ... explain[s] the structure of a divinely* created organ...*
 - R.Owen (1854, p.263)

* (Spinoza) Dei sive Naturae? Life's, in its evolution? (sec.IV)

Of the nature of creative acts by which the successive races of animals were called into being, we are ignorant. But this we know, as the evidence of unity of plan testifies to the oneness of the Creator, so the modifications of the plan for different modes of existence illustrate the beneficence of the Designer. Those structures, moreover, which are at present incomprehensible as adaptations to a special end, are made comprehensible on a higher principle, and a final purpose is gained in relation to human intelligence; for in the instances where the analogy of humanly invented machines fails to explain the structure of a divinely created organ such organ does not exist in vain if its truer comprehension, in relation to the Divine idea, or prime exemplar, lead rational beings to a better conception of their own origin and Creator.

- Owen (1854) *Structure of skeleton and teeth*. Orr's Circle of the sciences.

(A1) *(functional) Prototype*: A Humanly invented machine or method, Proof-of-concept device, or Law-governed model, demonstrating key functional aspects of a system conceived or built to achieve a certain aim or to serve a specific purpose.

(A1') *Fine-Tuning Condition*: A Prototype's configuration or set-up well-calibrated for or precisely adjusted to its aim or purpose.

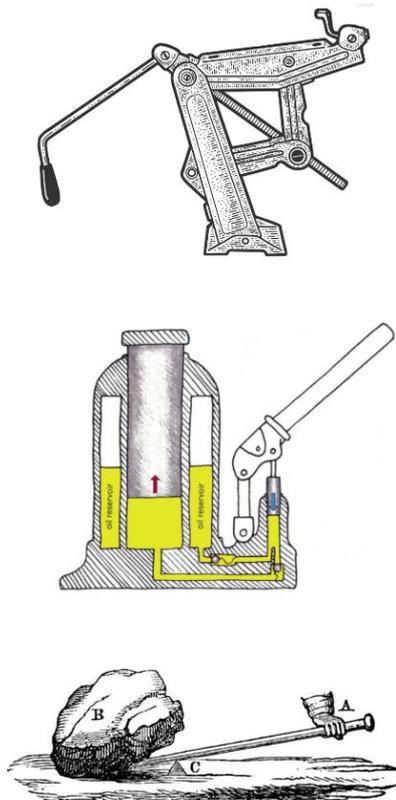
(A2) *Analogous*: Distinct organisms, organs, or parts thereof explained by the same or similar functional prototypes.

(A3) Analogous systems may have different structures, shapes, sizes, constructions / geneses / material implementations, ...

(A4) Analogy may be used to establish "Identity" (using the same name?) of corresponding parts in different systems.

(A5) In the context of theories of biological evolution, analogy may be a useful tool for investigating and tracing distinct phylogenetic paths of development converging to similar solutions.

Same Function with Different Prototypes?



$$F_1 a_1 = F_2 a_2$$

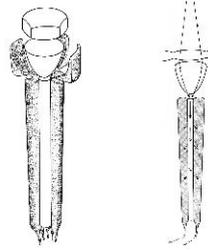
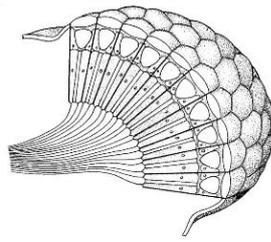
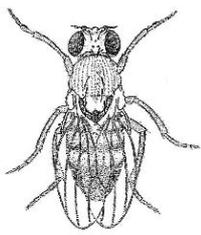
• *Similar Use or Function but Different Intrinsic Mechanism*: In asserting a functional difference [equivalence] between organs, one must distinguish clearly between function in the sense of:

- Use [What-for] (Function A), and
- Intrinsic mechanism (Function B).

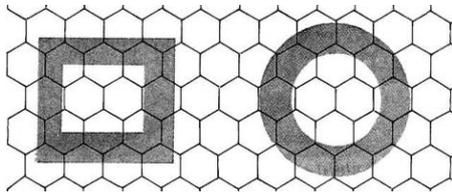
- John Tait (1928, p.155) Homology, Analogy and Plaxis. *Quart. Rev. Biol.*, 3, 2, 151-173.
- J.L. Shearer, A.T. Murphy, H.H. Richardson (1971). *Introduction to System Dynamics*.
- Bernd Ulmann (2022). *Analog Computing*.
- C.F. Herreid, Ch.R. Fournier eds. (1981) *Locomotion and Energetics in Arthropods*.
- M. Land, D.E. Nilsson (2012). *Animal Eyes*.

← Lever force & arms in lifting jacks by screw & parallelogram frame, or by hydraulic pistons

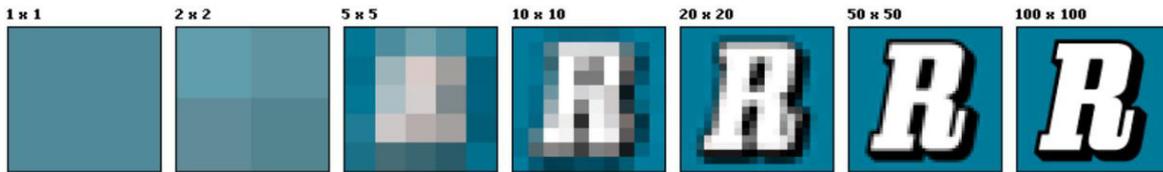
Composite Eyes of Arthropods (insects, spiders..)



E	1	20/200
F P	2	20/100
T O Z	3	20/70
L P E D	4	20/50
P E C F D	5	20/40
E D F C Z P	6	20/30
F E L O P Z D	7	20/25
D E F F O T E C	8	20/20
L E P O D F C T	9	
P E P L Y D F O	10	
.....	11	

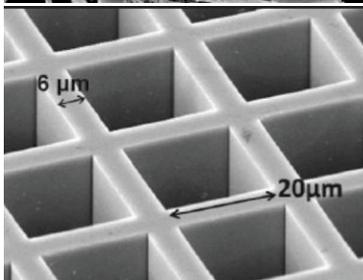
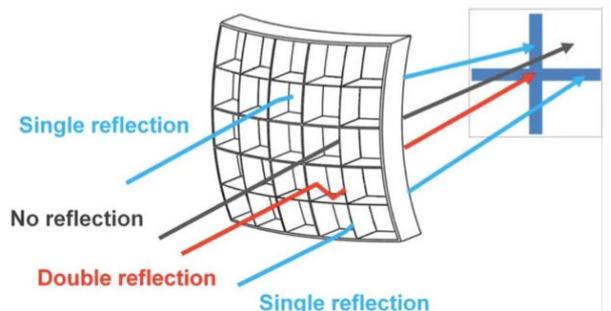
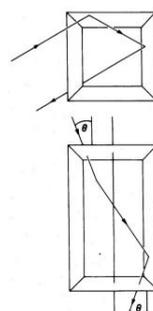
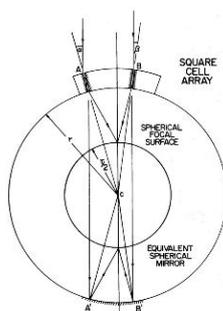
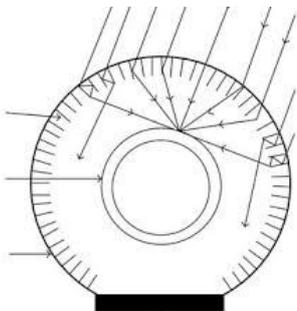


How / How-Well it works



- Final image formed by (neural) composition of many optical elements, each having focus at ∞ and very narrow field angle;
- Objectively evaluate function: Image formation + perception = effective resolution, amplification, field angle / depth, spectrum, etc

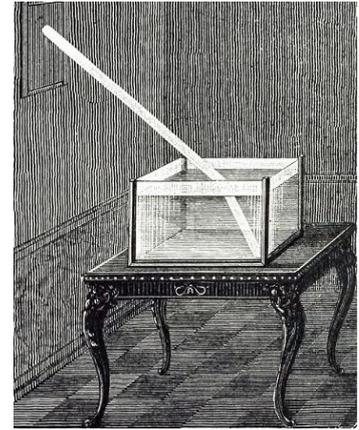
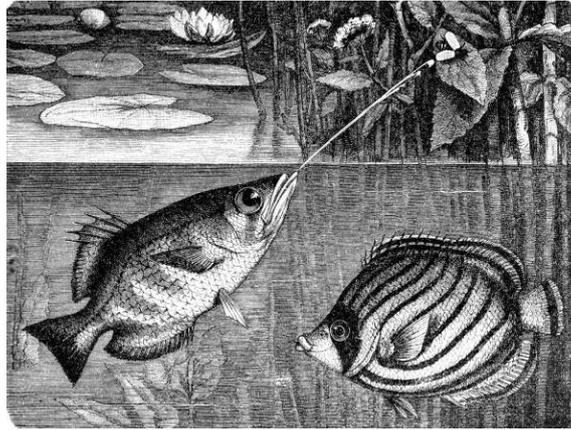
“Collimators”, “Lenses” & “Mirrors” in Lobsters’-Eyes



Lobster-eye optics. Fig.1,l,r, from Gorenstein (2010) and ESA (2024): General diagrams for lobster-eye reflective optics, consisting of an external spherical grid of square reflecting cells projecting an image over an internal spherical (convex) retina. Fig.1,cl, from Angel (1979): Schematic diagram explaining the optics of a focusing collimator, made with an internal convex spherical retina & an external grid of square cells with reflective walls; diagram also shows a virtual spherical mirror (at the opposite side) with similar geometry. Fig.1cr, from Angel (1979): shows top and side views of a double reflection at perpendicular walls of a cell.

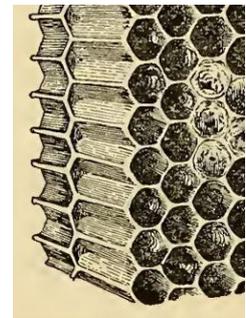
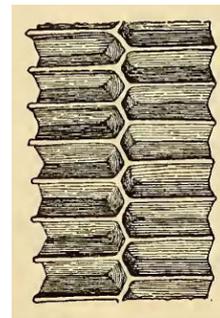
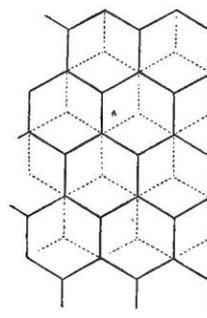
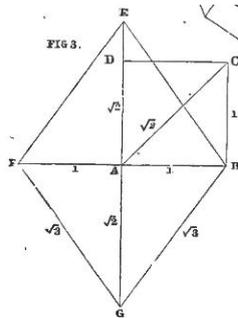
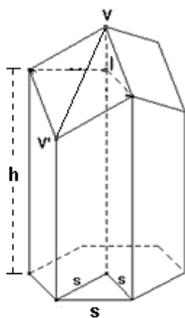
- Exner (1891) Schmidt (1975) Vogt (1977) Angel (1979)

Fine-Tuning Conditions & Teleological action by:



- Brazilian archer, fishing w. bow-&-arrow native *tekhné*;
- Australian archer-fish, *Toxotes jaculatrix*; (conscious intent?)
- (the non-anthropomorphic agent) *Natura Naturans*, explained according to Snell-Descartes' diffraction law or, alternatively, according to Fermat's teleological Principle of Least-Time, see
- J.M. Stern (2020) A Sharper Image: The Quest of Science and Recursive Production of Objective Realities. *Principia*, 24, 2, 255-297.

Fine-Tuning cell-lattice geometry for honeycombs



- Honeycomb's geometry optimizes (has the *goal* or *purpose* of maximizing) the amount (volume) of honey stored for a given use (wall surface) of wax, and other ancillary constraints.
- One of my final classes at high-school (1976), integrating introd. calculus, vector / solid geometry, and applied sciences.
- How were these angles / sizes actually measured?
- Edward Batschelet; *Introd. Mathematics for Life Scientists* (1975, prb.9.7.3)
- Giacomo Maraldi (1712), René A. Reaumur (1740), Collin Maclaurin (1743)
- Bernd Roling (2013): *Die Geometrie der Bienenwabe: Albertus Magnus, Karl von Baer und die Debatte über das Vorstellungsvermögen und die Seele der Insekten zwischen Mittelalter und Neuzeit.* (imagination & soul!)

Contrast *Sharp* vs. *Slack* Sub-Sets / Conditions

- Absence of haphazard [chance] and conduciveness of everything to an end are to be found in Nature's works in the highest degree, and the resultant end of her generations and combinations is a form of the beautiful. Aristotle (350 BCE)

- The more complex and more improbable a combination of characteristics is, with that more certainty can one conclude a relationship between function and selection, and that more easily answer the question, 'What for?'. Lorenz (1981, p.29)

- Contrasting Improbable hypotheses:
- Homology= a lucky but possible strike;
 - Slack, Small probability, Discrete space,
 - Finite Combinatorial calculus; vs.
- Analogy = miracle = infinitely lucky strk;
 - Sharp hypothesis, Infinitesimal volume sub-manifold in Continuous space $\subseteq \mathcal{R}^d$,



$$\Pr(\sqrt{2}) = \int_{\sqrt{2}-dx}^{\sqrt{2}+dx} x \, dx = 0$$

- L.G.Esteves .. (2019). Pragmatic Hypotheses in the Evolution of Science.

Teleology by Prototypes & Fine-Tuning Conditions

Fine* - Tuning* conditions:



(1) $H : f(\text{action} | \text{environment}) = 0$ ✓

(2) $\Pr(H : f(\text{act.} | \text{env.}) = 0) = 0$



(1) Successful action characterized by an abstract target, defined as an equation on system's *action/ /control* continuous variabs, given noisy *environment* variabs.

- ex: Optimization $\Rightarrow \partial f(\text{action} | \text{environment}) / \partial \text{action} = 0$

(2) Target is **exact** or **precise**, implying (almost) **Zero Probability** to achieve it by random action, i.e. by pure chance. (a **Paradox**)

✓ $\text{ev}(H | X)$ strongly supports **sharp** statistical hypotheses (H)

- G.Sommerhoff (1969) The Abstract Characteristics of Living Systems.
- J.Stern (2011) Constructive Verification, Empirical Induction & Falibillist Deduction. Threefold Contrast Interpretn. of e-values..
- J.M.Stern (2011) Symmetry, Invariance & Ontology in Physics & Statistics

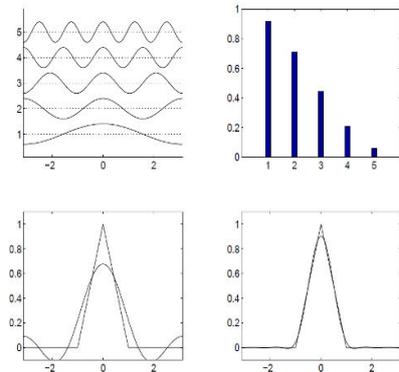


Defective (always) Prototypes or Fine-Tuning condtns

- Flying & Wing as Airfoil or as half a Venturi Tube
- H.Babinsky (2003). How do wings work? *Physics Education*, 38, 497-503.
- J.D.Anderson (1998) *A History of Aerodynamics & .. Flying Machines*
- G.A.Tokaty (1971) *A History and Philosophy of Fluid Mechanics*

- Hearing & Ear as an Inverted Piano

- Hermann v. Helmholtz (1895). *On Sensations of Tone as Physiological Basis for the Theory of Music*
- James Hudspeth (2018) There's an Inverse Piano in Your Head. *Scientific American*, 04/06/2018.
- Jess Josephs (1967). *Physics of Musical Sound*.
- T.Dau, V.Hohmann, B.Kollmeier (1999). *Psychophysics, Physiology and Models of Hearing*.



- A.B.Coffin, J.Sisneros (2024). *Handbook of Auditory Research*.
- D.R.Ketten, A.B.Coffin et al. (2024). *A History of Discoveries on Hearing*.
- S.Mandal, S.M.Zhak, R.Sarpeshkar (2009). A Bio-Inspired Active Radio-Frequency Silicon Cochlea. *IEEE J. Solid-State Circuits*, 44, 6, 1814-1828.

- Optimization Models for Reaction Networks

- J.Stern, F.Nakano (2014) Optimization Models for Reaction Networks: Information Divergence, Quadratic Program. & Kirchhoff Laws *Axioms*, 3, 109-118

Logical Inference Fallacies by Variables' Roles' Swap

- Erroneous definitions / inference schemata swap the roles of antecedent \leftrightarrow consequent or evidence \leftrightarrow conclusion:

- *Affirming a consequent / converse error* : $(A \rightarrow B), B \therefore A$

X Divergence \rightarrow Homology **X** Convergence \rightarrow Analogy

- *Affirming a disjunct / mutual exclusion error*: $(A \vee B), A \therefore \neg B$

X Analogy, Homology: Not Complementr. but Mutually exclusive

- What did Darwin himself say about homology & archetype?

Charles Darwin (1871, v.I, p.31), (1859, p.206):

- *The homological construction of the whole frame in the members of the same class is intelligible, if we admit their descent from a common progenitor.*

- *By unity of type, is meant that fundamental agreement in structure, which we see in organic beings of the same class, and which is quite independent of their habits of life.*

On my theory, unity of type is explained by descent.

Affirming the Consequent / Converse Fallacies

- *Structures which are genetically related, in so far as they have a single representative in a common ancestor, may be called homogenous.* [homologous]

Edwin Lankester (1870) apud Brower & Pinna (2012)

(L1) *Concept of Analogy: In the course of evolution it constantly happens that, independently of each other, two different forms of life take similar, parallel paths in adapting themselves to the same external circumstances.* Lorenz (1974, p.229)

(L2) *A homology can be defined as any resemblance between two species that can be explained by their common descent from an ancestor possessing the character in which they are similar to each other.* Lorenz (1974, p.230)

- E.Lankester (1870). On the use of the term homology in modern zoology.
- Konrad Zacharias Lorenz (1974). Analogy as a Source of Knowledge.
- Konrad Zacharias Lorenz (1981). *The Foundations of Ethology*.
- A.Brower, M.de Pinna (2012,p.531). Homology and Errors. *Cladistics*.

Mutually Exclusive / Non-Complementary explanations

- *Methodological criteria for analogies:*

(R1) *their relationship to function and way of life;*

(R2) *their contrast to homology in the sense of either-or.*

- *Methodological criteria for homology:*

(R3) *homology results from same location in comparable structural systems; (R4) structures can be declared homologous if intermediate forms between them can be proved.*

Adolf Remane (1952, p.83,58,45)

In due time, Darwinian evolutionary appropriation of the notion was so complete that homology not only became evidence of evolution, but came to be defined as a function of ancestry Rupke (2009)

There is an important and common misapprehension about the terms homologue and analogue. ... [People] who should know better treat them as antonyms and thus mutually exclusive. Owen was quite clear in saying that this was not the case. Panchen (1994, p.44)

- Adolf Remane (1952). *Die Grundlagen des Natürlichen Systems, der Vergleichenden Anatomie und der Phylogenetik.* reprt.1971

Yet, Complementarity lives in the ways of life ...

(L3) *Besides homology and analogy, no other explanation can be found for the appearance of similar-to-identical characteristics in different forms of life. (L4) There are, however, mixtures of the two; there exist similarities which are caused by both.*

$$L1+L2+L3 \Rightarrow \neg L4!$$

- *Reciprocal errors in the treatment of systems: Vitalists assume that it is exclusively the whole which is influencing its parts, while mechanists confine their attention to causalities leading from the part to the whole.*

Konrad Zacharias Lorenz (1981, p.93), (1950, p.226)

- *It is evident that the attitudes termed mechanistic and finalistic do not present contradictory views on biological problems, but rather stress the mutually exhaustive observational conditions equally indispensable in our search for an ever richer description of life.*

Niels Henrik David Bohr (1958, p.122) 

Statistical Inference Fallacies by Variables' Roles' Swap

- Concerning Sharp Hypotheses representing Prototypes:
- Frequentist p-values Swap *sample X parameter* spaces:
- Asymptotically Consistent:
 - Large number of observations yield coherent conclusions;
- Logically Inconsistent:
 - Small number of observations may yield incoherent concls.;
- Non-compliant w. Likelihood principle: *Ad Hoc* nuisance parameters elimination; • *Post-Hoc* sample space ordering..
- Bayes's Factors Swap *finite X infinitesimal* quantities:
- Asymptotically Inconsistent: – Lindley paradox, etc.;
- Logically Inconsistent:
 - Small number of observations may yield incoherent concl.;
- Non-compliant w. Likelihood principle: *Ad Hoc* integration measures on the hypothesis, *Ad Hoc* “artificial priors”, etc.
- Denial: Orthodox deFinettians regard Sharp hypotheses (all exact sciences' Laws) as representing ill-formulated 'Theories'.

e-value and FBST Calculi for Scientific Hypotheses

- A Bayesian model builds a posterior probability function, $p_n(\theta | X)$, for the parameter θ , given the observations X .
- $ev(H | X)$, e-value, Epistemic Value of hypothesis $H: \theta \in \Theta_H$, is a possibility function derived from the posterior $p_n(\theta)$, $\theta \in \Theta$!
- Full Bayesian Significance Test is a procedure that yields a 3-valent decision, $\{0, \frac{1}{2}, 1\}$, Reject, stay Agnostic, or Accept H .
- e-values & FBST allow para-consistent sensitivity analyses.
- Complementary & Concomitant use of several ABC's allows:
 - Simultaneous and Coherent Statistical and Logical Inference;
 - Homogeneous and Coherent treatment of Slack and Sharp Hypotheses in models with discrete or continuous probabilities;
 - Epistemological Induction Problem solved in a way amenable to contemporary use of Statistics supporting scientific theories.
 - Strong support for sharp hypotheses provides firm epistemic grounding for Ontology, Metaphysics, & Analogy arguments. 

ABC - Abstract Belief Calculus - definition

$\langle \Phi, \oplus, \oslash \rangle$, Support Structure;

Φ , Support Function, for statements in Universe \mathcal{U} ;

\oplus , Support Summation operator;

\oslash , Support Scaling or Conditionalization operator;

\otimes , Support Unscaling operator, inverse of \oslash .

\oplus , gives the support value of the disjunction of any two logically disjoint statements from their individual support values,

$$\neg(A \wedge B) \Rightarrow \Phi(A \vee B) = \Phi(A) \oplus \Phi(B) .$$

\oslash , gives the conditional support value of B given A from the unconditional support values for A & conjunction $C = (A \wedge B)$,

$$\Phi_A(B) = \Phi(A \wedge B) \oslash \Phi(A) .$$

\otimes , unscaling: If Φ does not reject A ,

$$\Phi(A \wedge B) = \Phi_A(B) \otimes \Phi(A) .$$

ABC - Abstract Belief Calculi - examples

$$a = \Phi(A), \quad b = \Phi(B), \quad a \oplus b = \Phi(A \vee B), \quad a \otimes b = \Phi(C = A \wedge B)$$

Null and Full support values, **0** and **1**, and operators for ABCs: Probability, Possibility, 3-Valent Logic, ImProbability, & DisBelief

ABC	$\Phi(\mathcal{U})$	$a \oplus b$	0	1	$a \leq b$	$c \oslash a$	$a \otimes b$
Pr	$[0, 1]$	$a + b$	0	1	$a \leq b$	c/a	$a \times b$
Ps	$[0, 1]$	$\max(a, b)$	0	1	$a \leq b$	c/a	$a \times b$
3VL	$\{0, \frac{1}{2}, 1\}$	$\max(a, b)$	0	1	$a \leq b$	$\min(c, a)$	$\min(a, b)$
ImPr	$[0, 1]$	$a + b - 1$	1	0	$b \leq a$	$\frac{c-a}{1-a}$	$a + b - ab$
DB	$\{0.. \infty\}$	$\min(a, b)$	∞	0	$b \leq a$	$c - a$	$a + b$

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- G.J. Klir, T. Folger (1988). *Fuzzy Sets, Uncertainty, and Information*.

e-value & FBST Calculi - formal properties

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- J.M. Stern (2004). Paraconsistent Sensitivity Analysis for Bayesian Significance Tests. *Lecture Notes in Artificial Intelligence*, 3171, 134-143.
- W.S. Borges, J.M. Stern (2007). The Rules of Logic Composition for the Bayesian Epistemic e-Values. *Logic Journal of the IGPL*, 15, 5/6, 401-420.
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- J.M. Stern, C.A.B. Pereira (2014). Bayesian Epistemic Values: Focus on Surprise, Measure Probability! *Logic Journal of the IGPL*, 22, 236-254.
- R. Izbicki, L.G. Esteves (2015). LogicalConsistency in Simultaneous Statistical Test Procedures. *Logic Journal of the IGPL*, 23, 5, 732-758.
- L.G.Esteves, R.Izbicki, J.M. Stern, R.B. Stern (2016). The Logical Consistency of Simultaneous Agnostic Hypothesis Tests. *Entropy*, 18, 256, 1-22.
- J.M. Stern, R. Izbicki, L.G. Esteves, R.B. Stern (2017). Logically Consistent Hypothesis Testing and the Hexagon of Oppositions. *L. J. IGPL*, 25, 741-757.
- L.G. Esteves, R. Izbicki, J.M. Stern, R.B. Stern (2019). Pragmatic Hypotheses in Evolution of Science. *Entropy*, 21, 9, 883, 1-17.
- J.M. Stern, C.A.B.Pereira, M.S.Lauretto, L.G.Esteves, R.Izbicki, R.B.Stern, M.A. Diniz (2022). The e-value and the Full Bayesian Significance Test: Logical Properties and Philosophical Consequences. [arXiv:2205.08010](https://arxiv.org/abs/2205.08010)

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- (2007b) Language and the Self-Reference Paradox.
- (2008) Decoupling, Sparsity, Randomization, Objective Bayesian Inference. – *Cybernetics & Human Knowing*, 14, 1, 9-36; 14, 4, 71-92; 15, 2, 49-68.
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Honeybee's "Symbolic Language" – a Metaphor



- ...a good metaphor implies an intuitive perception of the similarity in dissimilars.
Aristotle, Poetics, (335 BCE)

- *συν βαλλω* - *syn-ballo* - *I put together*: A token used by one or more individuals to recognize, infer or imply a credit (\$\$), pledge, creed, or underlying truth.

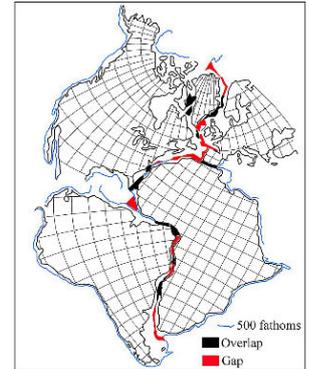
- Two parts that are distinct and different (or complementary) but that, somehow,...

- ... (almost) perfectly match or exactly fit together, forming or restoring a unity.

- A symbol, then, brings together two separate pieces of an originally single and whole reality, [like] matter and spirit.

Murray Stein (2022).

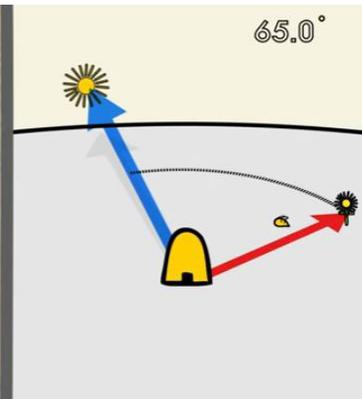
Literal & Metaphorical Symbolization in Science



- Continental drift/ tectonic plate maps by Snider-Pellegrini (1858), Alfred Wegener (1929), and Edward Bullard (1965).
- In (never perfectly exact) sciences, this kind of puzzle matching, or fitting of data to **exact** patterns, **precise** laws, or **sharp** hypotheses, relies on supporting statistical models.
- Although each separate continent retains its distinct identity, Symbolon is very **good**, i.e., the pieces fit together **very well!**
- **Homology** and **Analogy** are excellent tools for metaphorical symbolization in the development/context of scientific theories.



Bee's Dance "as a Metaphorical Language" Metaphor



- Equality / identity / similarity axis: (**good** symbol / metaphor)
- Waggle dance run / source distance = 1s / km (approx.)
- Angle: Up - Dance axis = Angle: Sun - Food source (approx.)
- Statistical model for equations (=) holds **very well!**
- Inequality / difference / disparity axis:
 - Hive is dark → Information is tactile / vibration sensing (?);
 - Flight direction info. is visual, given by polarized sun light (?);
 - Distance measured by flight's effort / spent-energy (?)

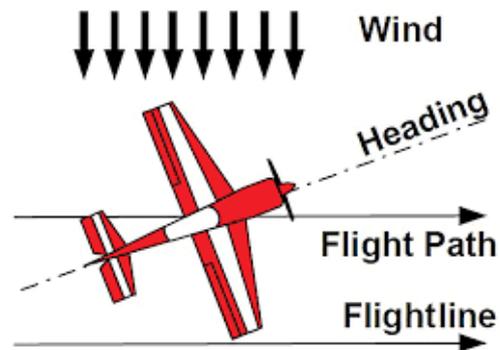
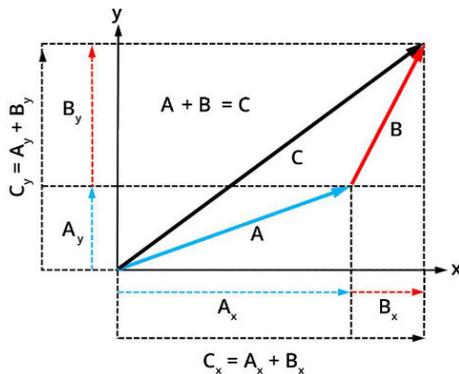


- In the case of the honeybee, the elaborate dance manoeuver with its striking correlations provides a persuasive teleological argument for the existence of communication. In this example, the teleological hypothesis is based on the improbability of the dance correlations having occurred by chance, without evolution's having somehow acted to fashion a symbolic system of communication. James Gould (1976, p.237)

- Five questions may be asked: (1) How accurately is direction indicated? (2) How accurately are “instructions” as to direction followed? (3) Can we state a simple manner the relation between the distance of the food (or effort required to reach it), and the rhythm of the dance; as v.Frisch has stated the relation between the direction of the food and direction of the dance? (4) How accurately is distance indicated? (5) How accurately are “instructions” as to distance followed? Cov=[?]

Ⓜ $n = 4.76(3.95 - \log_{10}(d))$ Haldane (1954, p.258)

Vector Algebra *Prototype* for the Bee's Language



- Language communicates *precise*¹ and *stable*² instructions for flying to a specific source, accommodating adjustments for wind drift, sun's daily movement, and other disturbances.
- *Separate*³ elements of the language (durations, angles, etc.) refer to *distinct*³ basic operations of flying and its control.
- *Articulation*⁴ rules of language (grammar, semantics) correspond to the *compositional*⁴ properties of the (real) operations.
- **1,2:** Prototype sharp conditions → good Analogy arguments;
- **3,4:** Structural properties ~ archetype for syntax / grammar.

Dissimilarity(?) -1: Inherited vs. Developed Language



- Dance's Archetype & Prototype (algebra & geometry) are invariant, innate, and inherited.
- *The traces of geometry are expressed in the world so that geometry is, to speak, a kind of archetype of the world.* J. Kepler (1606) *Stella Nova*; apud C.G. Jung, W. Pauli (1955, p.163-4)
- Social learning by observing experienced sisters perfects communication – i.e. makes it more reliable and precise
- Innovation / adaptation (for the species) by genetic mutation
- Haeckel's law (1866): *Ontogeny recapitulates phylogeny.*

-
- K. v. Frisch (1993). *The Dance Language and Orientation of Bees.*
 - Martin Lindauer (1971). *Communication among social bees.*
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 - L. Chittka (2023). Bees learn to dance: Experience yields precision..
 - Dixon (1994) *Division Algebras.* • Hanson (2006) *Visualizing Quaternions.*
 - B. Goertzel al. (2008). Mirrorhouses & the Algebraic Structure of the Self.

Dissimilarity(?) -2: Individual vs. Swarm Intelligence

• *The [bees'] Round Dance as a Means of Communication: The swift understanding of a message by an interested group [of bees] is further facilitated by a peculiar behavior, one could almost say, by a caste or class spirit of the collecting groups.*

Karl von Frisch (1965, sec.3.9).

• *[a well-established skeptical argument] refuses to accept the bee dances as a true language, primarily because there is no evidence of conscious intent on the part of the bees. ...*

A reluctance to become embroiled in metaphysics should not anesthetize our perceptions. Heretical as it may seem to many behavioral scientists, I am willing to entertain the thought that perhaps the bees know what they are doing.

Donald R. Griffin foreword in Frisch (2014, p.xiii)

- E. Bonabeau, M. Dorigo, G. Theraulaz (1999) *Swarm Intelligence.* (algorithm)
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Dissimilarity(!)-3: Human Ontology & Metaphysics

- *For man, who endeavors to master his habitat and its phenomena through insight into causal relations, the correct aggregation of the stimuli emanating from the things in his surroundings into objects [named] of his environment is the basis of all knowledge and the highest requirement of life.*
- *For the animal, however, especially for the lower animal, who is essentially fitted to his habitat through inherited instinctual behavior, and to whom insight plays no role at all in his reaction to the stimuli of the environment, an objective comprehension of the environment is not an absolute biological necessity.*

Konrad Lorenz (1935, p.117)

- *The a priori and axiomatic character of causal thinking finds its equally convincing expression in the insatiable 'why' of intelligent children.* Konrad Lorenz (1978, sec.6.6)

- Lorenz (1935) *Companionship in Bird Life*. pp.83-128 in C.H. Schiller, K.S Lashley (1957), *Instinctive Behavior: The Development of a Modern Concept*.
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Metaphysics as a Source of Knowledge (& Ethics?)

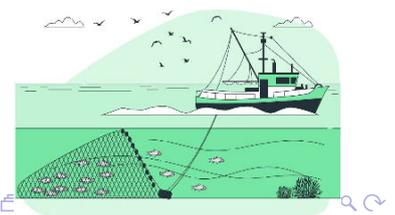
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הַסִּיד טוֹ(ב)מְשׁוּב הוּא הַשׁוֹאֵל עֲצָמוּ בְּכֹל דָּבָר לְמָה
וְמָה אֲנִי רוֹצֶה בָּזֶה וְכֵן מְצִינִי בְּנִמְרָא מֵאֵי טַעְמָא

*A kind and responsible person asks himself about everything:
Why? and what can I conclude about its purpose and reasons?*

Menachem Mendel Morgenstern (1787-1859), *Pillar of Truth* (2000)

- (2018) Verstehen (causal / interpretative understanding), Erklären (law-governed description / prediction), & Empirical Legal Studies. *J. Inst. Theor. Econ.* 174, 105-114.
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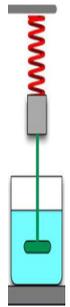


Complementarity Symbolic Meanings (& Religion?)

In materia igitur quae est quatuor elementa operatur summa Trinitas ipsam materiam creando in hoc quod est efficiens causa; creatam informando et disponendo in eo quod est formalis causa; informatam et dispositam diligendo et gubernando in eo quod est finalis causa. Nam Pater est efficiens causa, Filius vero formalis, Spiritus sanctus finalis, quatuor vero elementa materialis.

Therefore, in matter, which is the four elements, the supreme Trinity operates by creating matter itself in that it is the efficient cause; by informing & arranging the created matter in that it is the formal cause; and by loving and governing the informed & arranged matter in that it is the final cause. For the Father is the efficient cause, the Son the formal cause, the Holy Spirit the final cause, and the four elements the material cause. Theodoricus of Chartres (1140), *Tractatus de sex dierum operibus*

- Complementr. explanations provide super-abundant meaning(s)!
- Descartes / Fermat Optics or Newtonian / Analytical Mechanics yield complementary formal theories w. efficient / final causality;
- Complex exponential representation of *Vibrations & Waves*;
- apud p.689 in Silvia Fazzo, Mauro Zonta (2022) Aristotle's Theory of Causes & the Holy Trinity. • Anthony French (1971) MIT Physics.

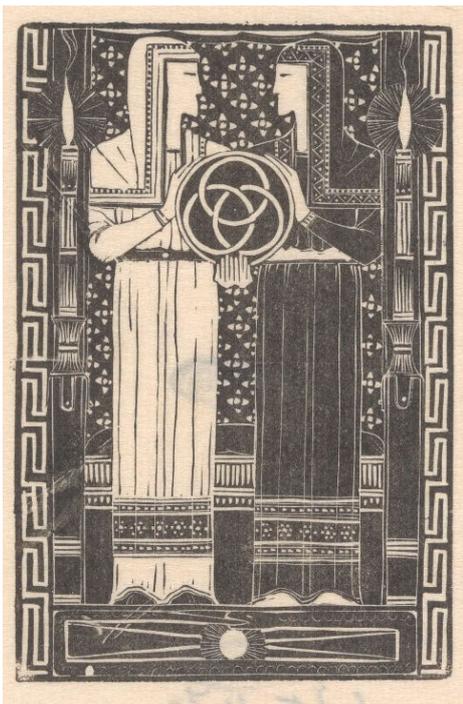


magic

$$R \alpha e^{(\beta+i\omega)t}$$



Thank you! Grato! Gracias! Bedankt! ευχαριστω



Mathieu Lauweriks (1895)
De geest des tijds
The Spirit of times



IME-USP



SBL - Sociedade Brasileira de Lógica
Brazilian Logic Society



UFRJ



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- Nancy Haynes et al. (1973) *Biological Science: An Ecological Approach*. BSCS, Biological Sci. Curriculum Study.
- Claude Welch et al. (1968) *Biological Science: Molecules to Man*. BSCS.
- O.T. Benfey et al. (1964) *Chemical Systems. CBA, Chemical Bond Approach*.
- George Pimentel (1963) *Chemistry: An Experimental Science*. ChEMS, Chemical Education Material Study.
- Harold, Richard & Robert Fuller (1978) *Physics: Including Human Applications*.
- Tracy Storer, Robert Usinger (1957) *General Zoology*.
- Jules Basin (1901) *Leçons de Chimie: 1.Metalloides, 2.Metaux, 3.Organique*
- Bruno de Finetti (1957) *Matematica Logico-Intuitiva*.
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- (1970) *Linear Algebra*.
- Lev Tarasov (1979) *Calculus for High School*.
- (1984) *The World is Built on Probability*
- Serge Lang (1968) *A 1st/2nd Course in Calculus*.
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- (1979) *A Second Course in Calculus*.
- Nikolai Piskunov (1969) *Differential and Integral Calculus*.
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- George Hadley (1962) *Linear Programming*.
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- John Hammersley, David Handscomb (1964) *Monte Carlo Meth.*
- A. Mood, F. Graybill, D. Boes (1974) *Introd. to Theory of Statistics*.
- Morris Herman DeGroot, Mark Schervish (1975, 1986, 2012) *Probability & Statistics*.
- Fred Attneave (1959) / Aleksandr Khinchin (1957) *Applic. / Found. Information Theory*

Further entry level books, paths and thoughts

- PSSC, BSCS, CBA, ChEMS, translated by IBECC / FUNBEC / USAID + Lab. kits ++
- Fuller (1978) Intuitive & illustrative applications using pre-calculus mathematics.
- Storer & Usinger (1957) Beauty! Diagrammatic drawings reveal essential structures.
- Basin (1901) Teaches real applied chemistry; Includes alchemic. / archaic names.
- John Free (1977) *Social Organization of Honeybees*.
- Albert Glenn Richards (1968) *The Complementarity of Structure and Function*; BSCS. Good activities for sci. clubs.
- + Entry level high school math; Bayesian statistics; History of math. language
- Richard Courant, H. Robbins (1961) *What Is Mathematics? An Elementary Approach to Ideas & Methods*.
- Roger Nelsen (1993, 2000, 2015) *Proofs without Words, I, II, III*.
- George Pólya (1945) *How to Solve It*. ~ • (1963) *Mathematical Methods in Science*.
- K.O. Friedrichs (1965) *From Pythagoras to Einstein*.
- Menahem Schiffer, L. Bowden (1984) *The Role of Mathematics in Science*.
- Leo Zippin (1962) *Uses of Infinity*.
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