Color-Coded Epistemic Modes in a Jungian Hexagon of Opposition

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Dedicated to Eva Leonor Fanny Stern, my mother and translator of C.G. Jung's works to Portuguese.

Abstract. This article considers distinct ways of understanding the world, referred to in psychology as *functions of consciousness* or as *cognitive modes*, having as the scope of interest epistemology and natural sciences. Inspired by C.G. Jung's *simile of the spectrum*, we consider three basic cognitive modes associated to: (R) embodied instinct, experience, and action; (G) reality perception and learning; and (B) concept abstraction, rational thinking, and language. RGB stand for the primary colors: red, green, and blue. Accordingly, a conceptual map between cognitive modes and primary and secondary colors is built based on the physics and physiology of color perception and epistemological characteristics of the aforementioned cognitive modes, leading to logical relations structured as an *hexagon of opposition*. Finally, this model of cognitive modes is applied to the analysis and interpretation of some important episodes in the historical development of physics and technology.

Keywords. Epistemology; Cognitive modes; Compositional and oppositional structures; Jung's simile of the spectrum; History and philosophy of physics.

Colors symbolize qualities, which can be interpreted in various ways. Psychologically this points to orienting functions of consciousness, of which at least one is unconscious and therefore not available for conscious use. C.G. Jung (CW, IX, pr.582, abridged).

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1. Introduction

Epistemology or knowledge theory is the branch of philosophy concerned with studying how we learn about our environment and then verify and justify the acquired knowledge. In this article, I restrict my interest in epistemology to the scope of natural sciences. My interests also take into consideration the human subject, observer, or agent of learning, and how he or she uses and integrates distinct ways of understanding the world – ways often refereed in psychology as *functions of consciousness* or as *cognitive modes*, see Wilde (2011). With this goal in mind, I follow in the footsteps of Swiss psychologist Carl Gustav Jung (1875-1961), who used conceptual models where *colors symbolize qualities* constituting a color-coded system that *points to orienting functions of consciousness*, as stated in the opening quotation. The best known of these systems concerns Jung's categorization of *psychological types* – that is *not* a system used in this article. Instead, I develop in the sequel an alternative system of color-coded cognitive modes based on Jung's celebrated *simile of the spectrum*.

The systems of color-coded cognitive modes used by Jung are in no way arbitrary. First, these colors and modes relate to associations Jung frequently found in patient's dreams or historically recorded imagery, which also relate to the etymology of color terms and the evolution and organizational patterns of these terms found in human languages. Second, these color codings have significant connections to the physics of color formation. Third, these color codings have significant connections to the physiology of color perception. These physical and physiological connections are frequently overlooked in the psychology literature. Nevertheless, the aforementioned connections are specially interesting for the epistemological applications I have in mind, for they correspond to, respectively, external vs. internal or objective vs. subjective aspects of color processing in particular or knowledge representation in general.

Section 2 reviews basic notions of modern color theory. Section 3 relates color theory and logical structures. Section 4 develops a model inspired by Jung's simile of the spectrum in which color-coded cognitive modes and their logical structure are interpreted in the context of epistemology and philosophy of science. Sections 5, 6, and 7 examine some examples of how these cognitive modes can be interpreted in the development of scientific disciplines. Section 8 presents some directions for further research and final remarks.

2. Modern Color Theory

This section presents an abridged and selective chronology of modern color theory, focusing on relevant concepts needed for this paper. Modern color theory starts with the publication of Isaac Newton's *Opticks* (1704), where he showed how (a) a ray of white sunlight can be decomposed by a prism into a *spectrum* of color hues, forming a linear continuum ranging from red to violet, as commonly seen

FIGURE 1. Newton's (1704) *Opticks* (updated) color wheel (circular perception structure) vs. linear structure of light spectrum; Helmholtz (1867) RGB/LMS cone receptors response curves.

in a rainbow, see Figure 1r.[1] Moreover, Newton showed that (b) different color sensations can be generated by mixing light of specific spectral hues; for example, a sensation of violet can be generated by mixing red and blue. Furthermore, (c) color sensations like magenta or purple are not produced by light from any single locus in the linear spectrum; instead, they can only be produced by various mixtures of red and blue. Hence, Newton suggested that (d) human perception of colors is better represented by a *color wheel*, where the red and violet ends of the linear spectrum are joined to form a circle. Figure 1l depicts an updated version of Newton's color wheel, see MacEvoy (2005). In this article, the color wheel's violet-magenta-purple region joining the extremities of the linear spectrum is called the *paradoxical region*. At the same time, the magenta-purple hues span the more restricted *non-spectral* region. In his famous *simile of the spectrum*, C.G.Jung (CW, VIII, pr.414-416, pp.3167-3169) compared this representation to an Ouroboros – a serpent biting its own tail at the paradoxical region of the color wheel, see Figure 1c and Atmanspacher (1996).

In 1801, using a simple thermometer, William Herschel was able to detect infrared radiation, invisible to the human eye, located beyond the red end of the spectrum, see Simon (1966). In the same year, using photochemical reactions, Johann Wilhelm Ritter detected ultraviolet radiation beyond the violet end of the visible spectrum. Hence, considering the color wheel representation, it is an understandable *façon de parler* to speak of hues at the paradoxical region as *neither ultra-violet nor infra-red but an undivided blend of both*, see Sabini (2000, p.23).

Meanwhile, Thomas Young (1802) postulated that the human perception of color is based on three types of light receptors at the eye's retina. Hermann von Helmholtz (1867) and James Clerk Maxwell (1860) were able to verify Young's intuition in a series of experiments designed to elucidate peculiarities of human perception of color. These three receptors are nowadays denominated L-M-S *cones*, that are sensitive to radiation roughly located, respectively, at the red, green, and blue regions of the spectrum, see Figure 1r. Maxwell's (1860) triangle uses

[1]Positional figure locators: c=center, t=top, b=bottom, l=left, r=right.

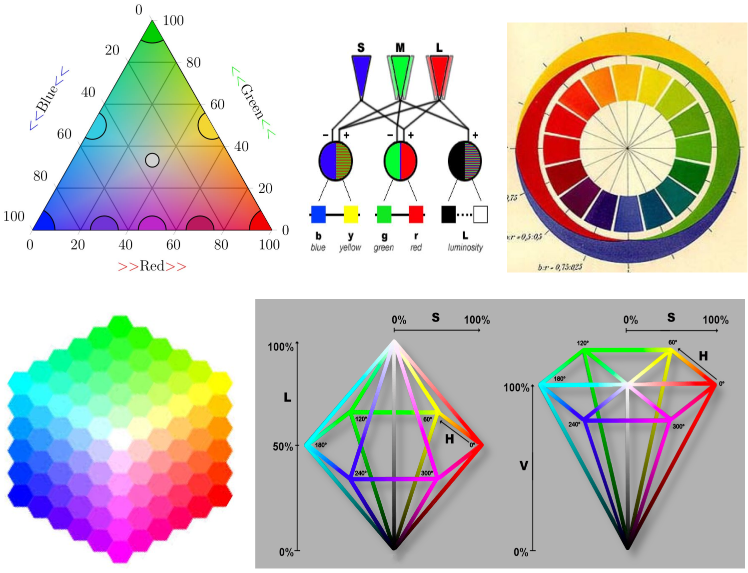

FIGURE 2. Top: Maxwell (1860) RGB compositional diagram including approximate position of violet, purple, primary and secondary colors; Neural network re-encoding trichromatic (RGB) inputs into oppositional (RGBY) outputs; Hering (1878) circle of four antagonistic archetypal colors (RGBY). Bottom: Hexagonal tiling and color schemata, from Runge (1810) and Wundt (1892) to HSL/HSV encoding by Smith, Joblove and Greenberg (1978).

a convenient system of coordinates to specify color hues by their red, green, and blue (RGB) components. In this system of coordinates, known in mathematics as (de Finetti's) compositional diagram, each coordinate is in the $[0, 1]$ interval, and all coordinates must add up to 1; see Figure 2tl, Longair (2008), Finetti (1957, S.77) and Stern (2017).

The sensitivity curves of LMS/RGB receptors depicted in Figure 1r are normalized, i.e., these curves are plotted with maxima of the same hight. However, their absolute sensitivities are quite different: S/B receptors have a much smaller (neural output density) response than M/G receptors, that, in turn, have a smaller response than L/R receptors. Furthermore, these receptors have distinct and nonlinear response curves to color hue, resulting in highly nonlinear combined response curves for brightness, color saturation, or other qualitative aspects of color perception around the hue circle. Hence, additional calibration points around the hue circle are needed for sound color encoding systems. For this purpose, the hexcone (hexagonal cone) color encoding system includes calibration points located midway between the primary colors (RGB), corresponding to the secondary colors cyan, magenta, and yellow (CMY), see Figure 2b.

The primary colors constitute an additive basis, i.e., different color hues can be generated by mixing RGB light sources of different intensities. In particular, each of the secondary colors is generated by mixing two primary colors, namely, C=G+B, M=R+B, and Y=R+G. In contrast, the secondary colors constitute a subtractive basis, i.e., different color hues can be generated by sending white light through CMY filters of different intensities (like artists do by mixing paints).

The hexcone and similar color encoding systems were first envisioned by Philipp Otto Runge (1810), explored scientifically by Wilhelm Wundt (1892, 1896), and greatly developed for TV broadcasting and computer graphics in order to achieve good quality renderization of color images at high processing speed; see Joblove and Greenberg (1993), Silberstein (1942), and Smith (1978). Hexcone encoding and similar systems are now ubiquitous, underlying color information structure in the modern world. The logical structure of such hexagonal color models is further examined in the next section.

The tripolar color model, developed by Maxwell and Helmholtz, explained how distinct physical light sources and filters can be combined to obtain different colors. Meanwhile, Ewald Hering (1878) developed an alternative quadripolar color model based on four archetypal colors, or *Urfarben*, organized as antagonistic processes opposing red vs. green and yellow vs. blue, see Figure 2tr. Hering's model was able to explain some color phenomena related to perception latency, see Hering (1878, 1964) and Turner (1993). Hering's model could also explain recurring organization patterns for color words found in human languages. Interestingly, the same colors and structure are used by Jung to color-code oppositional cognitive modes in his theory of personality types; see Jung (1939; 1940, p.48), Laughlin (2015), and Wilde (2011).

At the beginning of XX century, Erwin Schrödinger (1920a,b, 1925, 2017) showed how to combine the aforementioned tripolar and quadripolar models into an integrated color theory, but the functional transform underlying this integration is still a matter of current research. For example, Chittka et al. (1992, 1996) show how neural networks responsible for post-processing of signals generated by cone receptors conform to the oppositional structure anticipated by Hering, see Figure 2tc. From a logical point of view, the simplest structure able to integrate the aforementioned tripolar and quadripolar models is the hexagon of opposition, studied in the next section. For general overviews of color theory and its historical development, see Kuehni and Schwarz (2008), and MacEvoy (2005).

The aforementioned tripolar models describe color processing at the interface between the human eye and the external environment. In contrast, quadripolar models describe processes at a corresponding interface to the internal world of an embodied human mind. My interest in epistemology demands simultaneous attention to both external phenomena and their internal representation. Following Jung's intuition, I use the framework provided by color theory for this purpose, for vision is arguably the most important human sense for perception of phenomena in the external environment. It should therefore have a comparable influence and

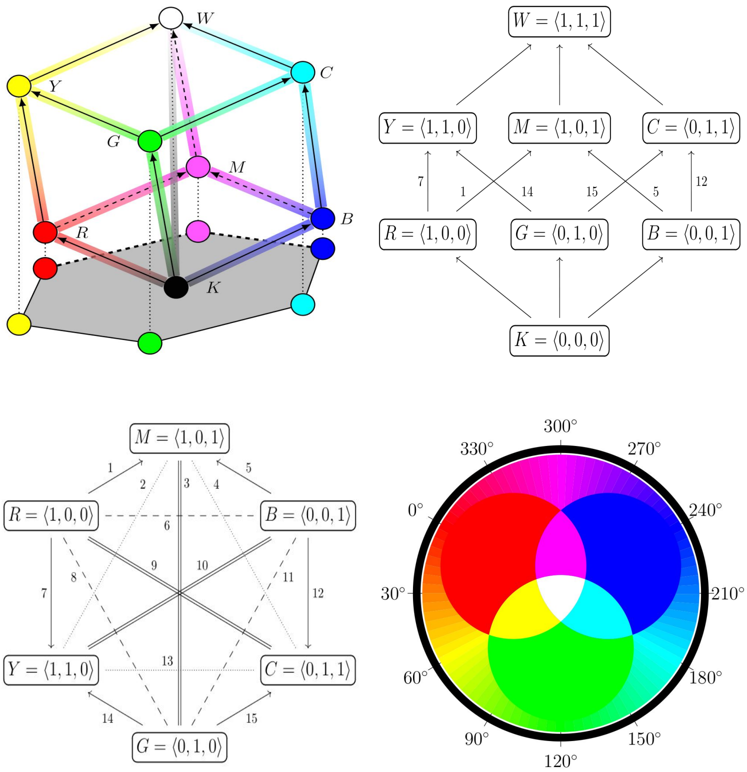

FIGURE 3. Top left: Cubic diagram of color entailment relations projected into the hexagon; Top right: Hasse diagram for (transitive) mereological relations of entailment or inferiority ($\longrightarrow$); Bottom left: Hexagon of opposition for additive (RGB) and subtractive (CMY) colors with corresponding mereological or bit-string relations of complementarity ($=\!=$), contrariety ($--$), and sub-contrariety ($\cdots$); Bottom right: Color wheel showing hue continuum in standard angular coordinate.

importance in internal human representation and psychological processing. Accordingly, Section 4 develops a model for epistemology and philosophy of science, in which primary and secondary colors are interpreted as cognitive modes.

3. Logic Structures and Color Theory

The superposition or compositional properties of primary and secondary colors entail a rich and intuitive algebraic structure that has been extensively explored in mathematical and philosophical studies, see Jaspers (2012, 2017) and Silva

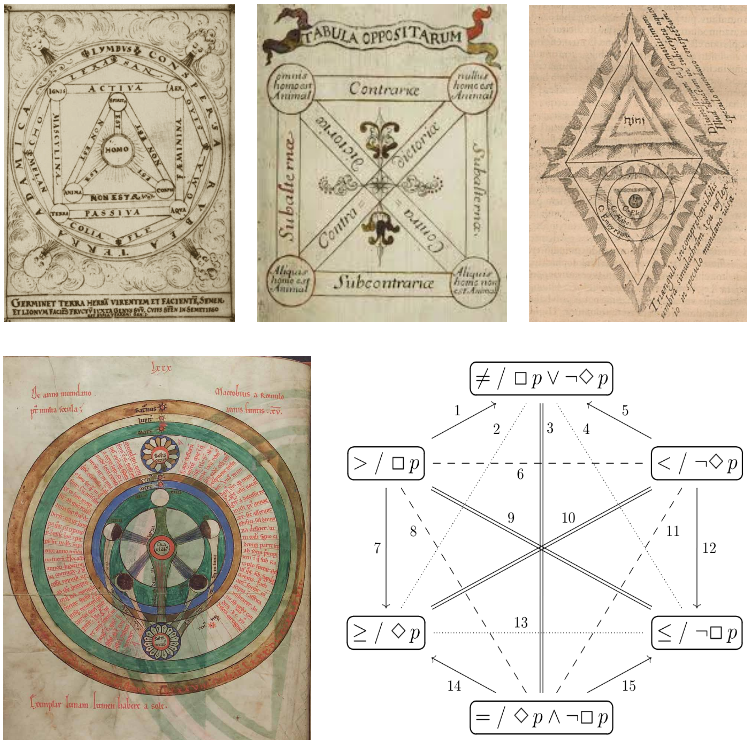

FIGURE 4. Top & bottom-left: Medieval diagrams of tri-, quadri- and hexa-polar oppositional structures; Bottom-right: Blanché (1966) hexagon of opposition for $(\Box, \Diamond, \neg)$ modal logic operators of necessity, possibility and negation, or $(<, >, =, \neq)$ (in)equality relations, including oppositional relations of contradiction ($=\!=$), contrariety ($- -$), sub-contrariety ($\cdots$) and subalternation ($\longrightarrow$).

(2017). Formally, a bit-string $\langle r, g, b\rangle$ in the 3-dimensional Boolean space $\{0, 1\}^3$ is used to represent the colors Red (R), Green (G), Blue (B), Cyan (C), Yellow (Y), Magenta (M), Black (K) and White (W), as shown in the cubic diagram at Figure 3tl. Analogously, a vector $\langle r, g, b\rangle$ in the 3-dimensional Euclidean unit cube $[0, 1]^3$ is used to represent a continuum of color hues, as (partially) depicted in Figure 3tl. Arrows in these diagrams represent color intensity gradients for the Euclidean color cube, and entailment or inferiority relations for the Boolean color cube.

The entailment relations in the Boolean color cube impose a (transitive) order structure captured by the algebraic lattice depicted in Figure 3tr; for further details, see Birkhoff and MacLane (1997), Jaspers (2012, 2017), Demey (2020), Demey and Smessaert (2014, 2016, 2018). The geometric orthogonal projection of the color cube along the K-W axis generates the color hexagon, as depicted in

Figure 3l. In addition to the entailment relations directly inherited from the color cube, the color hexagon includes other important logical relations corresponding to color-theoretic properties: *Contrariety* relations, represented in the hexagon by dashed lines ($--$), interconnect elements of the additive color basis. Meanwhile, *sub-contrariety relations*, represented in the hexagon by dotted lines ($\cdots$), interconnect elements of the subtractive color basis, see Figure 3bl. Accordingly, bit-string codes of any two contrary colors have null or K=$\langle 0,0,0\rangle$ intersection or minimum. Meanwhile, bit-string codes of any two sub-contrary colors have full or W=$\langle 1,1,1\rangle$ union or maximum. Finally, *complementarity* relations, represented in the color hexagon by parallel lines ($=$), interconnect colors with complementary bit-string codes.

Curiously (or insightfully), one can observe a synchronic evolution of the human understanding and the historical development, on the one hand, of color theories and their logical structures and, on one other hand, of inference systems formalizing human reasoning and their logical structures. Classical and medieval logic orbits around tripolar and quadripolar structures known as triangles and squares of opposition, see Figure 4tl,tc. Only in modern times, since Blanché (1953), were these structures generalized so as to integrate tripolar and quadripolar oppositional relations; see also Sesmat (1951), Gallais (1974, 1982) and Jaspers and Seuren (2016). The simplest structure of this kind is the logical hexagon of opposition, depicted in Figure 4br.

Figure 4br illustrates oppositional relations in the logical hexagon either by arithmetic equality and inequality operators ($<, >, =, \neq$), or by modal logic operators of necessity, possibility and negation ($\Box, \Diamond, \neg$). Applying to the logical hexagon the same convention used in the color hexagon: Implication or subalternation relations are represented by arrows ($\longrightarrow$); Contrariety relations are represented by dashed lines ($--$); Sub-contrariety relations are represented by dotted lines ($\cdots$); and Contradiction relations are represented by parallel lines ($=$). Contradictory statements have opposite truth-false values; Contrary statements cannot both be true, although they might both be false; and Sub-contrary statements cannot both be false, although they might both be true.

I take the existing isomorphism between the color hexagon and the logical hexagon as a sign reinforcing Jung's intuition of seeking color-coded systems for representing cognitive modes or as evidence corroborating the validity of following this path. Moreover, the same basic oppositional structure, or further generalizations thereof, is used to represent a great variety of inference systems; see Béziau (2012, 2015), Bueno-Soler and Carnielli (2016), Carnielli and Coniglio (2016), Demey (2020), Demey and Smessaert (2014, 2016, 2018), Dubois and Prade (1982, 2012), Esteves et al. (2017, 2019), Moretti (2009, 2012), and Stern et al. (2017). These extensions and generalizations engender additional homeomorphisms between logical structures found in color theory and (sub-)structures of those inference systems. Coherently, I take these homeomorphisms as additional evidence supporting the path taken by C.G.Jung. Figure 4 shows some medieval illustrations of oppositional diagrams concerned with either language and argumentation, see Demey (2020), or alchemy and gnostic philosophy, see Gieser (2005, p.184), Diotallevi (2018, p.45), Petraeus (1550, pl.6), and Pauli (1955, pl.1). Each of these diagrams presents a fragment of the full hexagon of opposition, whose interpretations in logic and color theory were analyzed in this and the preceding sections, see Figure 4br. Moreover, these diagrams were conceived as *conceptual maps*, that bridge and interconnect different fields of study by seeking, identifying, and abstracting common underlying logical structures, see Tang and Karunanithi (2018). I believe the success of these enterprises further reinforces the validity of Jung's intuitions that motivate this article.

4. Epistemic Color-Coded Cognitive Modes

Eugen Bleuler (1857-1939) was the director of Burghölzli psychiatric hospital from 1898 to 1927. Jung worked at Burghölzli from 1900 to 1909, where he developed several key ideas in analytical psychology. Bleuler (1881, 1925) was particularly interested in chromesthesia and other paradoxical phenomena related to color perception. Jung was also aware of Wilhelm Wundt's (1892) psychometric studies, including color theory and perception. Hence, we can safely assume Jung had a good understanding of the complex structure and rich interconnections implied by his simile of the spectrum. Surprisingly, some interpretations found in psychology textbooks present Jung's simile in over-simplified fashion. Sometimes, it is even compared to an allegory of Frederic Myers' (1891, pp.298-306; 1892, 333-336), whose simple linear structure completely fails to capture essential aspects of Jung's simile. The next abridged quotation presents, in a condensed form, Jung's formulation of the simile of the spectrum:

> *[We] employ once more the simile of the spectrum... The dynamism of instinct is lodged as it were in the infra-red part of the spectrum, whereas the instinctual image lies in the ultra-violet part. If we remember our color symbolism, then, as I have said, red is not such a bad match for instinct. But for spirit, as might be expected, blue would be a better match than violet. Violet is the 'mystic' color, and it certainly reflects the indubitably 'mystic' or paradoxical quality of the archetype in a most satisfactory way. Violet is a compound of blue and red, although in the spectrum it is a color in its own right. ... Because the archetype is a formative principle of instinctual power, its blue is contaminated with red: it appears to be violet... The creative fantasy of the alchemists sought to express this abstruse secret of nature by means of an other, no less concrete, symbol: the Ouroboros, or tail-eating serpent.*
>
> Jung (CW, VIII, pr.414-416, pp.3167-3169, abridged).

Jung's simile of the spectrum is a metaphor used to explain essential aspects of *archetypes*, a concept we further discuss in Section 7. At this point, we focus on specifics of the color symbolism used in the simile, involving the colors red, blue, and violet. Red and blue correspond to the color receptors of the human retina closer to the extremes of the visible linear spectrum, while violet lies in the paradoxical region of the color wheel where the Ouroboros bites its tail, see Figure 1l,c. In the topology of the color wheel, opposite to violet, and midway in the linear spectrum between red and blue, is the locus of color green, a color that, like other colors used in the simile, finds a consistent symbolic meaning in Jung's work, as expressed in the following abridged quotations:

> *Red, the blood color, has always signified emotion and instinct.*
> Jung (CW, VIII, pr.384, p.3143).
>
> *Blue, the color of air and sky, is most readily used for depicting spiritual contents.* Jung (CW, VIII, fn.122, p.3167).
>
> *Statistically, at least, green is correlated with the sensation function [...in...] relation to the real world.* Jung (CW, IX, fn.130, pr.582, p.3840).
>
> *Of the essence of things, of absolute being, we know nothing. But we experience various effects: from 'outside' by way of the senses, from 'inside' by way of fantasy. ... the color 'green' ... is an expression, an appearance standing for something unknown but real.* Jung (CW, VII, pr.355, p.2862).

Table 1 presents the symbolic meanings of primary and secondary colors as they are used in the hexapolar epistemological model under construction in this article. The three primary colors plus violet are reinterpreted in the context of epistemology, our targeted application field, but still preserving (I hope) much of Jung's original interpretations in the context of psychology. As far as I know, cyan never found in Jung's work a distinct symbolic meaning. This is not surprising for, outside the terminology of modern color theory, few human languages (like Russian, Italian, and Hebrew, but neither German nor English) have a distinct traditional word for this color, using instead compound expressions like light-blue or greenish-blue; for pertinent references in etymology, evolutionary linguistics, and grammar of color terms, see Berlin and Kay (1999), Elliot et al. (2015), Klein (1987), MacLaury et al. (2007), Sterman and Taubes (2012), and also Samuel Preiswerk (1871), Jung's grandfather.

Yellow (citrinus or $\xi\alpha\nu\theta o\varsigma$) was Jung's "missing" color, used to reestablish oppositional symmetry and complete the quadripolar basis he used to represent psychological types (that can then be unfolded in 2^k-polar models, for k=3,4,5); see Jung (CW, XII, pr.333; 1939; 1940, p.48) and Wilde (2011). In the same way, yellow is the color still missing in our hexapolar model, where it takes a symbolic meaning specific to the model at hand, see Table 1.

In real-life experience, it is difficult to spot pure spectral colors, for processes that naturally generate light produce either a mixture of isolated frequencies (like chemical spectra) or, after some interactions in the environment (like reflection and scattering), complex mixtures in the color space. Likewise, in our epistemological analog, it is difficult to spot examples of scientific models or theories that would be well described by an isolated primary color. Far easier is to give good examples related to secondary colors (CYM), corresponding to coordinated operations in the space spanned by (at least) two primary colors.

- **Red:** Color of blood, symbol of (e)motion and instinct; Capacity to maintain embodied life (grounded existence and autopoiesis), of well adapted reactions or purposive interactions with objects in a scope of interest.
- **Yellow:** Color of metallic gold, symbol of craft work, fine artisanry, precise manufacture, industry, and technology.
- **Green:** Color of vegetation, symbol of sensory perception and sense of reality; Ability to perceive and learn existing qualitative relations in the scope of interest; Capacity to discern, detect and evaluate independence, correlation, or other forms of statistical association between quantities of interest.
- **Cyan:** Light-blue, symbol of reliable empirical statements; Ability to build, use and communicate good descriptive or predictive models of reality.
- **Blue:** Color of the sky, symbol of thinking and the rectified spirit; Capacity to distill conceptual notions or sublimate abstract ideas; Ability to relate and interconnect such concepts and retrieve or communicate pertinent relational chains in organized conceptual networks. A lexicon used to express and communicate such concepts is called (in computer science) an *ontology*.
- **Violet:** Spectral hue in the - - purple-magenta-violet paradoxical region of the color wheel; Symbol of the cryptic (or psychoid) nature of archetypal forms, halfway between adaptive instincts and their teleological representation as conscious images or ideas; Ability to find, seek or suggest meaningful associations, symbolic connections, or causal relations.

Table 1. Color-coded epistemological cognitive modes

As should be expected, well-developed scientific theories integrate all primary and secondary colors (*cauda pavonis*), hence providing the clearest views in their application areas. Nevertheless, those theories never drop from the sky *fix-und-fertig* (already fully assembled and ready to go). Usually, they are first noticed while in a dark shade of a secondary color and, from there, progressively evolve to better illuminate their fields of study. In the following sections, we discuss some examples of this kind, discerning positive aspects of scientific models or theories in an evolutionary stage appropriately described by a secondary color, as well as corresponding negative effects due to the scarcity of the opposite primary color.

5. Yellow: Invisible Carriers in Charge

This section presents case studies of technological development that, according to our epistemological model for color-coding cognitive modes, could be characterized as yellow – the secondary color made by adding red and green. The technological devices under study had to be manufactured and employed for specific purposes where they had to perform according to strict objective criteria. However, these case studies also illustrate partial successes made by trial and error, as well as the overcoming of deficiencies in cognitive mode blue, namely, how overcoming a paralyzing deadlock required a breakthrough that, in turn, could only be achieved when key concepts could be abstracted and ensuant metaphors were developed and used to illuminate blind spots previously dark to consciousness.

The XX century spans the development of electronics – the technology of generating, amplifying, and precisely controlling electrical currents. The evolution of electronics came in two great waves, characterized by the key device used to exert this control, namely, vacuum tube triodes and semiconductor transistors. Studying electronics' history is facilitated by abundant documentation, including laboratory notebooks of pioneers, scientific articles reporting important breakthroughs, textbooks on the subject written by main protagonists, and even audio and video recordings of interviews with those personalities. Finally, there are good collections of early prototypes, production samples of these artifacts, and good literature dedicated to the history of these technologies; for general references, see Hoddeson et al. (1992), Orton (2004), Redhead (1998, 2000), Shive (1959), Shockley (1950) and Tyne (1977). For additional details relevant to this section, see Bardeen and Brattain (1949), Braun (1980), Braun and MacDonald (1982), Davydov (1938), DeForest (1906), Huff (2001), Richardson (1916), Riordan and Hoddeson (1997), Langmuir (1913, 1919), Riordan et al. (1999), Shive (1949), Shockley (1949 to 1951), and also the references listed as videos and simulations.

Triodes and transistors, also called valves or amplifiers, use a small input, the emitter (or cathode) to base (or grid) electric signal, to regulate a much larger output, the emitter to collector (or anode) electric current. Figure 5t,cr depicts modern diagrammatic representations and shows photographs of the earliest prototypes of these devices. In both cases (triodes and transistors), pioneering inventors had a poor understanding of the fundamental science involved: They were severely misguided by inappropriate concepts and metaphors that generated intellectual blind spots that, in turn, temporarily halted further development. In both cases, electrically charged particles flow through these devices, but the nature and behavior of these particles were a source of confusion and misunderstanding.

In the case of vacuum tubes, early researchers thought that charged particles flowing through the vacuum tube resulted from the chemical decomposition of gas molecules into positively charged cations and negatively charged anions. Figure 5crt shows an Audion, whose patent explicitly required some residual gas left in the tube for ionization, resulting in a working but very noisy and unreliable amplifier. Later on, the development of theoretical and experimental means and methods of physics and chemistry demonstrated that electrons traveling through vacuum were the carriers in charge of the relevant transport processes, a hypothesis formerly perceived as incoherent, for there is an apparent contradiction in having a current of something in empty space. The apparent paradox was solved by realizing that the electrons in question were sub-atomic particles orders of magnitude smaller than chemical molecules of ordinary matter, see Anderson (1964). Triodes and their variants built using high-vacuum tubes were reliable, had good signal-to-noise characteristics, and became the backbone of subsequent developments in electronic technology.

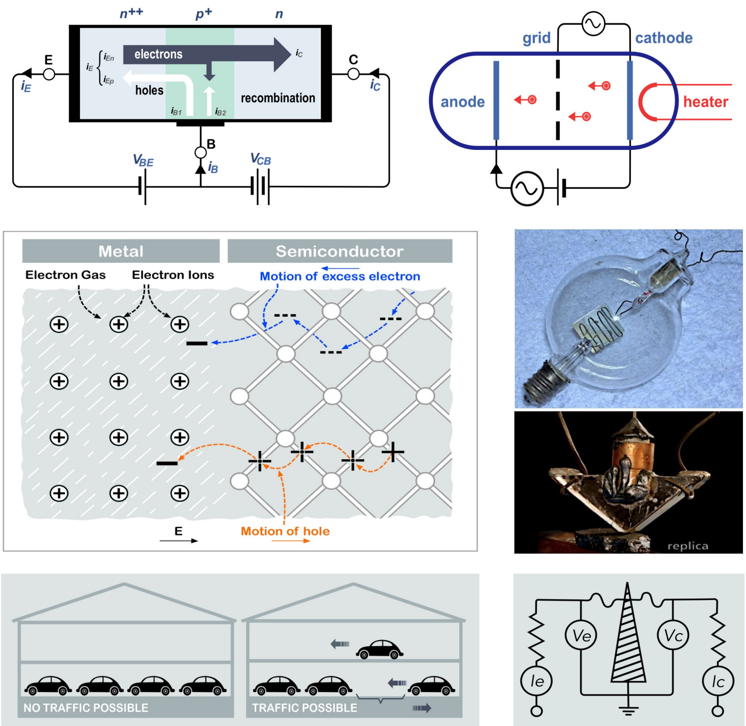

FIGURE 5. Top: Diagrammatic representation of Transistor and Triode vacuum tube. Right: Early devices relying on misleading metaphors: Audion, on gas ionization; Contact point transistor, on surface effects. Left: Shockley (1950) parking garage metaphor for the flow of holes (+) and free electrons (−) in a crystal lattice.

In the case of semiconductors, researchers had to follow a path in the opposite direction. They had to realize that not only electrons, but also (at least initially) mysterious positively charged (quasi-)particles called *holes*, had to be invoked in order to understand and control the relevant electrical flows. The concept of electron-holes, or just holes, was made explicit for the first time by Werner Heisenberg (1931). Emerging from quantum mechanics mathematical formalisms for solid-state physics, this easy-to-visualize metaphor often offers the best way to answer Heisenberg's signature question, see Hoddeson (1992, p.113,120): *How can we make that physically insightful (anschaulisch) or intuitive?*

Figure 5cl, resembling Shockley (1950, p.57), depicts free electrons (−) and positively charged holes (+) flowing as missing electrons in covalent bonds in a (doped) Silicon crystal lattice; see also Heinz (2020) and Mathew (2017). Figure 5bl, resembling Shockley (1950, p.8,9), depicts his famous *two-story parking garage* for the flow of holes (+) and electrons (−): Electric flow is impossible in the perfect crystal lattice of pure 4-valent Silicon or Germanium, but possible if the crystal is "doped" with scattered impurities of either a 5-valent element (like Phosphorus or Arsenic, introducing a free electron in the crystal) or a 3-valent element (like Boron, introducing a missing electron or hole in the lattice of covalent bonds).

Pioneering researchers trying to build a semiconductor triode were fully aware of the existence in crystalline structures of excess electrons and holes – that could be conceived as negatively and positively charged particles. Moreover, they knew that, depending on the type of semiconductor, the number of particles of one kind far exceeded the other, whence called majority and minority carriers. Furthermore, they implicitly hold a majority-only premise, namely, they tried to build semiconductor devices relying only on majority carriers, with minority carriers having a superficial or no role to play, see Hilsch and Pohl (1938) for such a device. Appreciating the importance of minority carriers was the conceptual blind spot to overcome in order to achieve a viable solid-state triode.

Figure 5crb depicts a contact point transistor, invented by John Bardeen and Walter Brattain at Bell-Labs in 1949. Like the Audion, this pioneering device worked, but just barely. Its invention was a fruit of much trial-and-error experiments guided by fuzzy ideas about the role played by minority carriers – supposed to be trapped at a semiconductor's surface or confined to its interfaces. Retrospectively, Brattain stated he had *an intuitive feel for what you could do in semiconductors, not a theoretical understanding*, see Braun and Macdonald (1982, p.40) and Shockley (1976). Figure 5br, depicts John Shive's (1949) double-surface triode, used to demonstrate the importance of in-depth (non-superficial) currents of minority carriers, the conceptual breakthrough needed for William Shockley (1950) to invent the Junction Transistor. Figure 5tl gives a diagrammatic representation of a junction transistor, where majority carriers (electrons) are responsible for the main current through the device. Nevertheless, this flow of majority carriers is controlled by a secondary current of minority carriers (holes) injected at the base (or grid). The interaction of holes from this much smaller secondary current with electrons flowing in the semiconductor constitutes the key mechanism used to efficiently and reliably control the main flow; see Heinz (2020) and Mathew (2017).

Vacuum as a transport medium is a difficult thing to "see", and so is a flow of empty holes! Nevertheless, in the aforementioned case studies, overcoming associated blind spots was the pivotal step to progress, see next quotations. Not surprisingly, in 1906, 1928 and 1932, Joseph Thomson, Owen Richardson, and Irving Langmuir were awarded a Nobel Prize for elucidating the nature and laws of thermionic emission, the theoretical foundation of vacuum tube technology. Contemporary textbooks in solid-state physics are fully immersed in the quantum mechanics framework; see and compare Kittel (1953, 1976). In contrast, John Bardeen, Walter Brattain, and William Shockley shared a Nobel Prize (1956) for inventing the transistor using simplified (semi-classical) models for the dynamics of flow and interaction (drift, diffusion, and recombination) of majority and minority carriers in semiconductors. Essentially, they "only had to see" interacting flows of electrons and holes; see Ning (1997), Roosbroeck (1950), Ryder and Kircher

(1949), Shive (1949), Shockley et al. (1949, 1950, 1951), and all references listed as videos and simulations. The next quotations reveal this mindset:

> *The explanation of these effects involved both the majority and the minority carriers. The fact that minority carriers might play an important role in the understanding of semiconductor phenomena was more or less overlooked by other investigators. As we shall see later, this was another blind spot. ... In the course of these experiments it became evident that the minority carrier, even in small concentrations, played a very important role. ... It is of course not surprising that this blind spot persisted for so long. The minority carriers were, after all, present in too small concentrations in most semiconductors to matter very much.* Pearson and Brattain (1955, p.1797,1801,1802).

> *The hole, or deficit produced by removing an electron from the valence-bond structure of a crystal, is the chief reason for existence of this book.*
> Shockley (1950, Preface, 1st line).

At this point, it is worth remembering Heinz von Foerster's (2003) Principle of *The blind spot: One does not see what one does not see.* As explained in Stern (2014), if we lack an appropriate conceptual framework to represent a specific "pattern of reality", our "mind's eye" will not be able to discern this pattern, even when the conditions for its occurrence are directly available in our environment. This notion is also in tune with the etymological origin of the word theory, from Ancient Greek: $\Theta\epsilon\omega\rho\iota\alpha = \Theta\epsilon\alpha\nu + o\rho\alpha\omega$, *theoria* = *thean* (a view) +*horao* (I see). Retrospectively, once we are able to see what was hidden in a former blind spot, it may be hard to believe that someone (possibly ourselves) could not see "that" what had always been there! Even so, incorporating and integrating new theories, adopting new ways of seeing the world, and accepting its consequences, may not be easy. We often cling to old blind spots, resist change, and hold on to old ideas and/or to the old habits, modus operandi, or ways of being that grew with them. Furthermore, these inertial effects can be various, complex, multi-layered, mutually reinforcing, and, therefore, can be easily misunderstood – sometimes even misinterpreted as intentional efforts aiming to suppress innovation and progress, see Fingermann (2014), Stern (2014) and Winston (1986, 1998).

6. Cyan Science: As in Heaven Not on Earth

In Ptolemy's astronomy, a planet moves around its epicycle, a small circle whose center moves around a larger one, the planet's deferent, see Blasius (2014). All motions in heaven are explained by a composition of circular motions of this sort. Ptolemy's model can be displayed by planetaria – gear driven mechanical simulators; see Figure 6l, Freeth (2009), and Price (1974). Ptolemy's astronomy provides a *kinematic* description of planetary motions, i.e., it presents a model of orbital trajectories without regard to their *causes*, that is, without answering the question of *why* these trajectories are the way they do. Moreover, the heavenly world is conceived as an ideal reality inaccessible and alien to human beings – confined

Figure 6. Blasius (2014) mechanism based on Hipparchus of Nicaea (190-120 BC) or Claudius Ptolemaeus (100-170 AD) deferent plus epicycles astronomical models; Isaac Newton (1687) diagram of cannonball sub-orbital and orbital trajectories.

to the imperfect sub-lunar world. Hence, the astronomer is an observer completely detached from the reality he or she observes.

Newtonian Mechanics presents a *dynamic* model that derives the trajectory followed by a material body from the physical forces acting upon it. Hence, these forces are conceived as the *causes* producing and determining a given trajectory exactly the way it is. Moreover, under appropriate circumstances, these forces can be precisely measured and manipulated so that the trajectories of the bodies they impel can be controlled according to our will and power. Figure 6r shows a diagram from Newton's magnum opus, *Philosophiae Naturalis Principia Mathematica*, illustrating the smooth transition from suborbital to orbital trajectories of a cannonball. This diagram is reproduced in a Hungarian postage label (HU 3199AZf, 1977, highlighted sub-orbital trajectories), near the lift-off of a Soyuz rocket impelling an artificial satellite to orbit. It is perfectly feasible to build mechanical simulators of such forces and consequent orbits. However, these models are useful to illustrate the dynamics of Newtonian systems, not as analog computers used for orbit calculations, a task better suited to the mathematics of differential equations, see Chapman (1969), Mirenberg (1968, 2021) and Turner (1915).

Ptolemy's astronomy is cyan science. It is blue for its well established concepts and metaphors and its expression in the formal language of Greek geometry. It is green for its descriptions and predictions in excellent agreement with empirical data – up to the observational precision attainable at that time (and for many centuries later). However, it lacks the color red, for it does not admit any possible interaction between the observer and the (kind of) objects he or she observes. In contrast, Newtonian physics provides a much clearer light. It is blue for its new but well-established concepts, like positional coordinates, velocity, acceleration, and force, and for its expression in the formal language of differential and integral calculus, see Newton (1704). It is green for its accurate agreement with the best empirical data available, surpassing in this respect Ptolemaic astronomy. Moreover, it is also red because the same universal laws govern heaven and earth, where humans are no longer dis-empowered voyeurs of the sky, but partakers in a universe in which they eventually become spacecraft builders, astronauts, or cosmonauts.

7. Purple-Violet: Suggestive Instincts-Insights

> *[There] are essential phenomena of life which express themselves psychically, just as there are other inherited characteristics which express themselves physiologically. ... Among these inherited psychic factors there [are] universal dispositions of the mind, and they are to be understood as analogous to Plato's forms (eidola), in accordance with which the mind organizes its contents. One could also describe these forms as categories analogous to the logical categories which are always and everywhere present as the basic postulates of reason. Only, in the case of our "forms", we are not dealing with categories of reason but with categories of the imagination. ...following St. Augustine, I call them "archetypes".* Jung (CW, IX, pr.845, pp.5401-5402, abridged).

> *The archetypal representations (images and ideas) mediated to us by the unconscious should not be confused with the archetype as such... It seems to me probable that the real nature of the archetype is not capable of being made conscious, that it is transcendent, on which account I call it psychoid.*
> Jung (CW, VIII, pr.417, pp.3169, abridged).

Pythagoras' theorem, one of the best-known results of Euclidean geometry, establishes an invariant relationship between the lengths of the edges in a right triangle, namely, the sum of the squares of the lengths of the catheti is equal to the square of the length of the hypotenuse; for illustrative images, see Figure 7. For an intuitive understanding and beautiful visual proofs of Pythagoras theorem, see Nelsen (1993, 2020, 2015); for its history, see Ratner (2009). Felix Klein (1872, 1948) *Erlangen program* to the study of geometry is based on the following question: What kind of *transformations* can be applied to geometric figures that preserve their essential characteristics? For example: How can the position of each vertex of a triangle be moved so that the size (Pythagoras theorem suggests the quadratic norm) of its edges and its angles remain invariant? In the case of Euclidean geometry, the answer to the last question is: By the composition of a translation (linear displacement along a given direction) and a rotation (angular displacement around a given direction). A standard mathematical representation of this class of movements is given by the algebra of Complex numbers in the Euclidean (two dimensional) plane, and by the algebra of Quaternion numbers in the Euclidean (three dimensional) space. For a readable introduction to Klein's approach to geometry, see Greenberg (1993); for extensions of this program to physics, see Wigner (1949) and Stern (2011a, 2017, 2020b). Complex numbers are covered by high-school or college books; Bruno de Finetti's (1957) is my favorite. For an intuitive introduction to Quaternions, see Hanson (2006), or Conway and Smith (2003) for more abstract views.

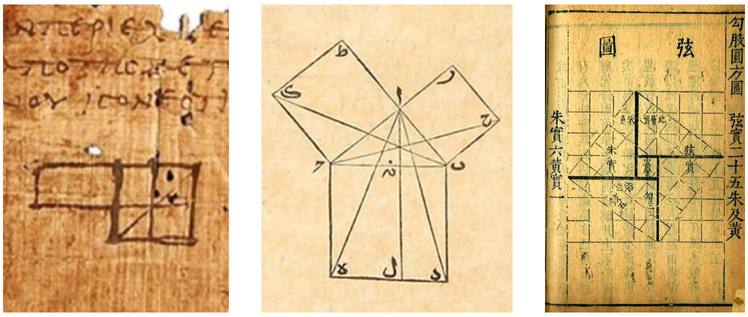

FIGURE 7. Euclid of Alexandria (300 BC) and Zhoubi Suanjing (100 BC) diagrammatic demonstrations of Pythagorean theorem.

Complex and Quaternion arithmetic are standard tools of Computer Graphics and Robotics because they efficiently encode the possibilities and constraints that govern the movement of physical objects in two and three-dimensional space. Nevertheless, mechanical robots (robota= slave worker) are machines conceived to emulate the movements human workers are capable of. Meanwhile, computer graphics can emulate human visual perception of physical objects as they are moved or illuminated under changing conditions. Hence, humans must have internal means and methods, like neural networks, that biologically encode equivalent algebraic structures. Every time we do manual labor, be it a plumber or a surgeon, we coordinate our visual perceptions and fine motor skills by using phylogenetically inherited capabilities that are ontogenetically trained and developed during our lives. Using Jung's terminology, in this context far removed from his original field of psychology, we could say that Complex and Quaternion algebras are good descriptions of archetypal forms of movement the human body is capable of.

Abraham Kaplan's *Law of the Instrument* states: *If your only tool is a hammer, then every problem looks like a nail.* Humans are finite beings that have quite limited resources. If we have a tool that works well in a context, it is only natural to try and test it everywhere we can. In the case at hand, Complex and Quaternion algebras are archetypal forms of movement that seem to be well-adapted to our environment, i.e., it seems they efficiently encode essential geometric properties of the space we live in. Moreover, these archetypes are great contributors to human intuition, for we use them all the time in our daily activities. As predicted by Kaplan's law of the instrument, we naturally try to use the same archetypal forms to study different phenomena and, behold, sometimes it works miraculously well!

James Clerk Maxwell (1831-1879) equations of electromagnetism can be written as a quaternion differential equation – although vector calculus is an equivalent and nowadays more popular formalism, see Crowe (1967), Edmonds (1998), Purcell (1963). As an applied tool, the same equations are at the core of electronic engineering. As fundamental physics, Maxwell equations can be verified by extremely

FIGURE 8. Blindfolded Fortuna (lady luck), Pizan (1414), and Justice, Dammartin (1700); Francis Galton (1889) Quincunx, demonstrating convergence to Normal (Gaussian) distribution.

precise empirical experiments. The extraordinarily precise agreement between simple and compact mathematical formulation of physical theories and empirical tests motivated Wigner's (1960) famous comments on the *Unreasonable effectiveness of mathematics in the natural sciences*, see Stern (2011a,c) and references therein.

The suggestive power of archetypal insights has, however, a double-edged nature: It may either inspire and drive a work of genius, or else engender persistent and misleading mirages. Abraham Kaplan's aphorism, a.k.a. the *Law of the Hammer* when applied with a pejorative meaning, can explain a conceptual opposite of a blind spot, namely, some persistent forms of wishful thinking and self-illusion. The term *apophania* (from $\alpha\pi o$ = away $+\phi\alpha\iota\nu\omega$ = bring to light, show, reveal) was coined by Klaus Conrad to describe frequent misidentifications of patterns or meanings at the onset of schizophrenia; see Conrad (1958), Escamilla (2016), and Mishara (2010). The closely related *Gambler's fallacy* or *pareidolia* (from $\pi\alpha\rho\alpha$ = beside, instead $+\epsilon\iota\delta\omega\lambda o\nu$ = form, shape) refers to perceptions of inexistent patterns in random data. Pareidolia explains some misleading beliefs or pathological behaviors of gamblers; see Stern (2008a) and references therein. *Statistical retrospective fishing expeditions* and other variations of the gambler's fallacy are the root cause of many misconceived experimental designs or mistaken statistical analyses. Such spurious arguments are often found in pseudo-science, academic deception, or professional malpractice. Jung himself warned about the double-edged power of archetypal insights, a source of inspiration for genius and fools alike:

> *The golden apples drop from the same tree, whether they be gathered by an imbecile locksmith's apprentice or by a Schopenhauer.*
>
> Jung (CW, VII, pr.229, p.2789).

Notwithstanding Jung's harsh warning, I must say that even the most brilliant scientists I know – those who have had the grace of their Eureka or Schopenhauer moments, also had plenty more of dumb locksmith's apprentice moments – trying to use the wrong key to open a door, or even struggling to properly use a good working key. In Stern (2011b, 2021), we carefully dissect some paradigmatic cases of pseudo-scientific studies concerning parapsychology, extra-sensory perception, and the medical (ab)use of phosphoethanolamine and hydroxychloroquine. The strong insights and suggestive power offered by intuitive (violet) archetypal ideas – that is, archetypal forms that correspond to firmly embodied (red) instincts that are also represented in well-established (blue) conceptual ontologies – may shed some light on psychological aspects of these bizarre cases.

Double-blind and randomized statistical trials are the gold standard used to test and accept or reject statistical hypotheses. Figure 8l depicts a medieval personification of Luck ($T\upsilon\chi\eta$, Tyche), blindfolded and spinning the wheel of fortune. Figure 8c depicts Justice ($\Delta\iota\kappa\eta$, Dike) holding her classical instruments, sword and scales, and also blindfolded – representing impartial judgment, an iconograpic innovation of that time. Figure 8r shows Francis Galton (1822-1911) Quincuncx machine, used to demonstrate the asymptotic convergence of means of random variables to the Normal or Gaussian distribution, a core result of Mathematical Statistics, see Kunert et al. (2001) and Gelman et al. (2003). These three images provide some intuition for the key ideas supporting double-blind and randomized statistical trials; for technical details see references in the next paragraphs. All the case studies analyzed in Stern (2011b, 2021) involve blunt denials of (green) statistical theory and practice, either by contesting the validity of standard mathematical reasoning, or by disputing the ethics of conducting double-blind and randomized experimental trials, or by recourse to unfounded conspiracy theories, etc. Hence, *Caveat emptor*: Any pragmatic or rhetorical attempt to avoid submitting an empirical model to test at this crucible – in which predictive models are validated or falsified – should be taken as a warning flag for pseudo-science; for related discussions, see Coulter (1991), Kurz (2005) and Pigliucci and Boudry (2013). Nevertheless, there are many more important aspects of pseudo-science, some of them, I suspect, relating to the suggestive power of archetypal ideas.

Notwithstanding the former caveat, there are valid methodological and ethical concerns regarding clinical (and similar) trials that should be addressed using state of the art means and methods. For example, contemporary clinical trials should: Dynamically optimize (minimize) sample sizes, see Fossaluza et al. (2015), Lauretto et al. (2012, 2017); Assure cryptographically secure, traceable and auditable randomization procedures, see Marcondes et al. (2019), Saa and Stern (2019), Stern et al. (2020); Provide information and conclusions that are, on the one hand, logically coherent and, on the other hand, understandable and consistently interpretable, see Borges and Stern (2007), Pereira et al. (1999, 2008, 2020), Stern et al. (2018, 2020, 2021); Protect participants against discernible sub-optimal treatments or practices; etc. Moreover, in my opinion, clinical trials should inform participating patients and agents of the general framework (including goals and ethics) of clinical trials, and how they differ (and so they must) from standard medical practice, in a way that is far more comprehensive than often done.

8. Final Remarks

As is the case in color theory, both tripolar and quadripolar structures coexist in Jung's theory of psychic functions, polarities that, we argued, can only be reconciled using an hexagonal logical structure. For example, as already mentioned, Jung categorization of psychological types has a basic quadripolar oppositional structure. Nevertheless, Jung (CW, X, pr.555-557, pp.4591-4592) suggest that *all man's psychic functions have an instinctual foundation* and that, in turn, *the world of unconscious instincts* has a tripolar structure corresponding to:

(R) Self-assertion – associated with Nietzsche's *Wille zur Macht* or will to power, with the Adlerian standpoint in psychology, and Augustinian *Superbia*;

(G) Imitation impulse – a reality principle associated with *the Learning capacity, a quality almost exclusive to man, based on the instinct for imitation found in animals. It is in the nature of this instinct to disturb other instinctive activities and eventually to modify them*;

(B) Sex drive – associated with *preservation of the species*, Freudian libido, and Augustinian *Concupiscentia*.

Considering our color-coding of cognitive modes, it seems natural to associate power and learning with the colors red and green. Finally, the association of sex to the color blue can be motivated by the following analogy: From a biological point of view, the most archaic forms of sex are, in essence, exchange of genetic information (horizontal gene transfer mechanisms are much older than genetic recombination in sexual reproduction); see Dawkins (1976), Michod and Levin (1988), Inhasz and Stern (2010), and Spielrein (1912). Moreover, genetic information is organized around basic units of meaningful information that are encoded in DNA as genes. Analogously, conceptual thinking is organized around basic units of meaningful information that are encoded in language as words. Each in its respective domain, genes and words constitute a basic linguistic vocabulary or a basic repertoire of abstractions used for dealing with life and communication, i.e., they constitute basic *ontologies* for their respective domains, see Stern (2014, 2017, 2020b). The possible similarities or parallelisms between psychology, evolution biology, and epistemology suggested by the analogies or correspondences considered so far motivates[2] a few lines of future research:

Psychology has a great expertise in developing qualitative and quantitative *instruments* to sketch psychological profiles of human subjects, see Wilde (2011). In future research, we would like to explore the possibility of developing similar

[2]Perhaps these analogies can also shed some light on the dual character of $E\rho\omega\varsigma$, namely, on the one hand, the young Eros (desire) – the playful god of love and, on the other hand, Eros the elder – equated in Orphic tradition to $\Phi\alpha\nu\eta\varsigma$ (Phanes, from $\phi\alpha\iota\nu\omega$= bring to light, show, reveal), a primordial god generator of life and the first to bring light to human consciousness.

instruments to sketch epistemological profiles of scientific theories (as they stand in a given instance). We consider some tools of Bayesian statistics, like survey techniques for elicitation, aggregation, and statistical analysis of expert opinion, to be specially promising for the task at hand.

Recent studies in neuroscience suggest that the neural networks in charge of fine motor skills (used for specialized brain processes that are approximately described by Complex and Quaternion algebras) are reused for other tasks, a phenomenon related to what is known in computer science as *code reuse*. For example, Hesslow (2002) suggests that the same code developed for motor-visual perception, control, and coordination of human fine motor skills, is reused for simulating and anticipating actions and intentions of other individuals. Furthermore, Rizzolatti and Arbib (1998) advance a *linguistic hypothesis*, suggesting that the same code is reused for language processing. As a consequence, a *pre-linguistic grammar* closely related to the aforementioned algebras should lie underneath the basic structure of human language. Furthermore, Ramachandran (2007) suggests that the same code is reused, once again, to support abstract concepts related to consciousness and self-awareness.

Complex and Quaternion numbers are members of the small but important family of *normed division algebras*, which also include Real numbers and Octonions. These algebras represent translations and rotations in 1, 2, 3, and higher dimensional spaces. In modern science, we "keep finding" those algebras everywhere we look; see Casanova (1976), Dixon (1994), Hemkumar and Cavallo (1994), Josipovic (2019), Lounesto (2001), and Stern (2008a). Hence, the questions:

Do we keep finding these structures in the universe because they really are out there? Or is that what we keep seeing because these structures are a priory encoded in the equipment we have to perceive and interact with the world? Or is it the case that these are the structures (or a priory categories) that we have because those were the ones selected along our phylogenetic evolutionary path as the best fit or the better adapted to the world as it is? For further considerations on the interplay between archetype theory and evolution biology and psychology, see Hogenson (2019), Samuels (1998), and Stevens (1998, 2002). Finally: To what extent must the explanations we find most intuitive, see Shive and Weber (1982), be related to our inventory of inborn mental structures or archetypal categories? Perhaps these investigations may help us to better understand a celebrated statement by Johanes Kepler[3], as quoted and translated by Wolfgang Pauli (1955, p.163-164):

> *The traces of geometry are expressed in the world so that geometry is, so to speak, a kind of archetype of the world.*

[3] *Geometriae vestigia in mundo expressa, sic ut geometria sit quidam quasi mundi archetypus.* Johanes Kepler, De Stella Nova (1606).

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Figures:[4] Images I generated using LaTeX and TikZ or pb-diagram: fig.1rb; fig.2tl; fig.3tl,tr,bl; 3br (thanks Henrik Midtiby); fig.4br. Images prepared for this paper by graphic artist Alex Freitas: fig.2bl,bc,br; fig.5tl,tr,cl,bl,br. Images used with the kind permission of the author or responsible for copyright: Bruce MacEvoy (2005): fig.1l; fig.2tc,tr; Lorenz Demey (2020): fig.4tc; Edward Godshalk (2009): fig.5crt; William Hammack (2010): fig.5crb; Christopher Blasius (2014): fig.6l; Imre Marozsán for Magyar Posta: fig.6r. Images from books in public domain, more than 70 years past after dates of publication and author's death: fig.1c, from Cleopatra (1000, 1887, p.128); fig.1rt, from Helmholtz (1867, p.291); fig.4tl, from Petraeus (1550, pl.6); fig.4tr, from Fludd (1617, p.21); fig.4bl, from Audomaro (1121, p.128); fig.6r, from Newton (1728, p.195); fig.7lc, from Euclid (300BC); fig.7r, from Zhoubi Suanjing (100BC); fig.8l, from Pizan (1414, p.129); fig.8c, from Dammartin (1700, p.122); fig.8r, from Galton (1889, p.63).

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[4]Positional locators: c=center, t=top, b=bottom, l=left, r=right.

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