AIR POLLUTION:

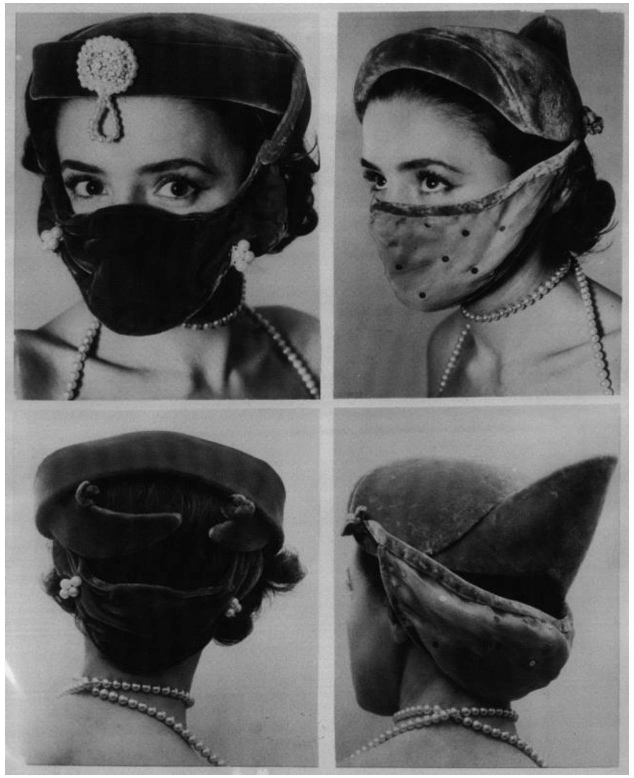
STATISTICAL AND COMPUTATIONAL METHODS

DR SHANE O`SULLIVAN

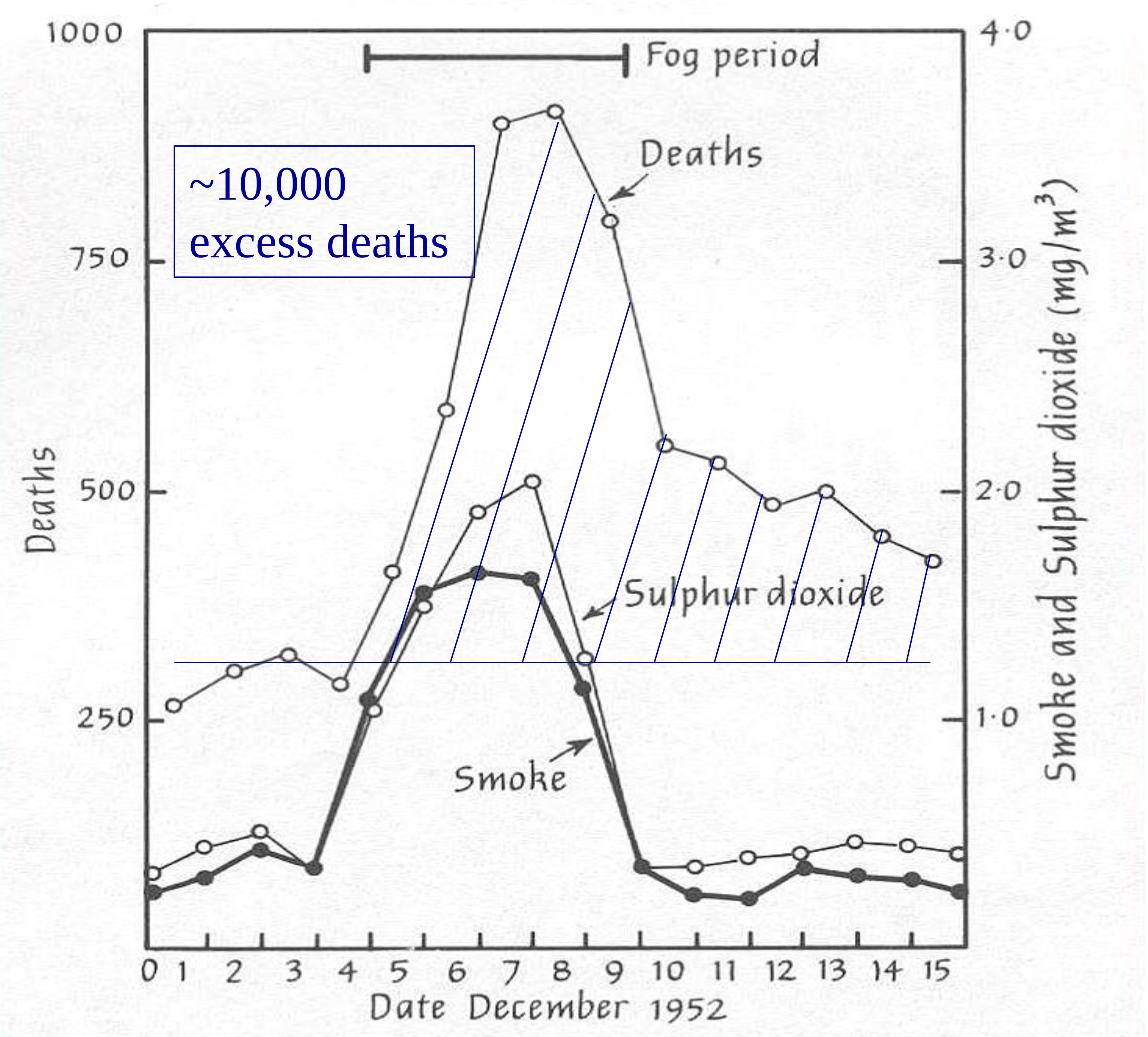
Daytime in London, 1952

Particulate levels – 3,000 µg/m^3

Designer Smog Masks - London 1950's



Daily mortality:

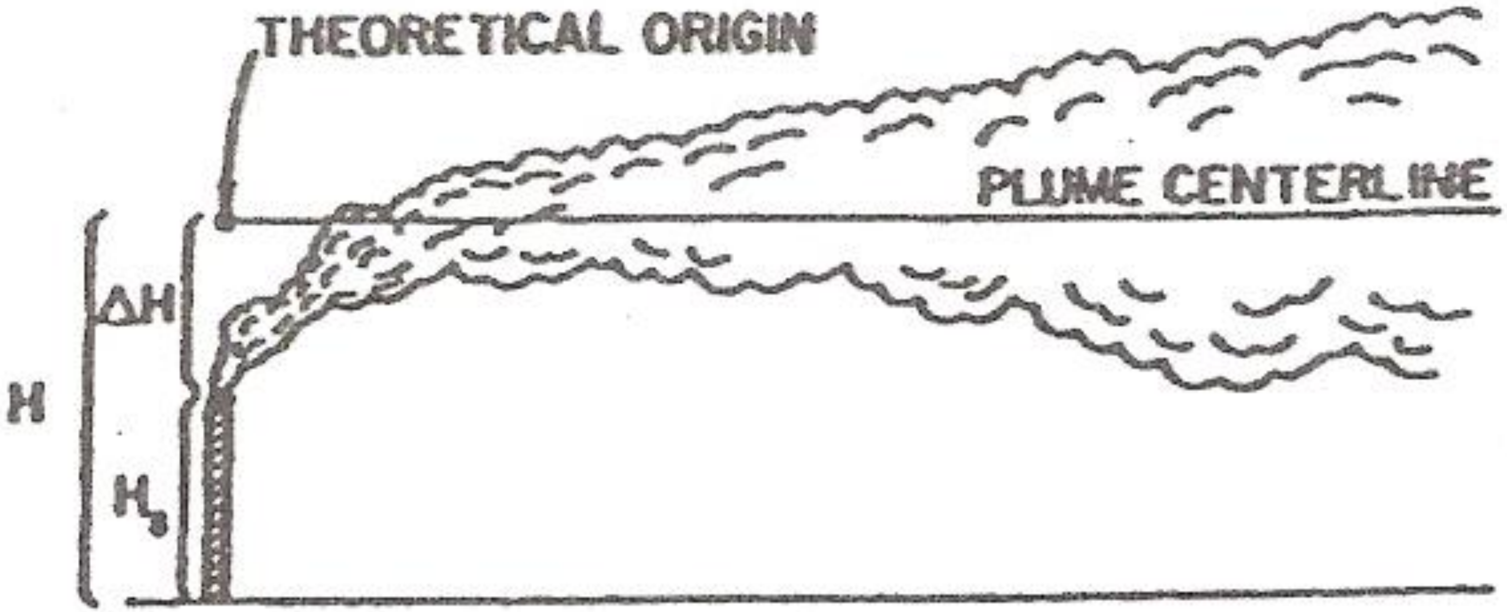


Plume Dispersion

Objectives:

Assess environmental impact of an emission source in terms of legislated standards

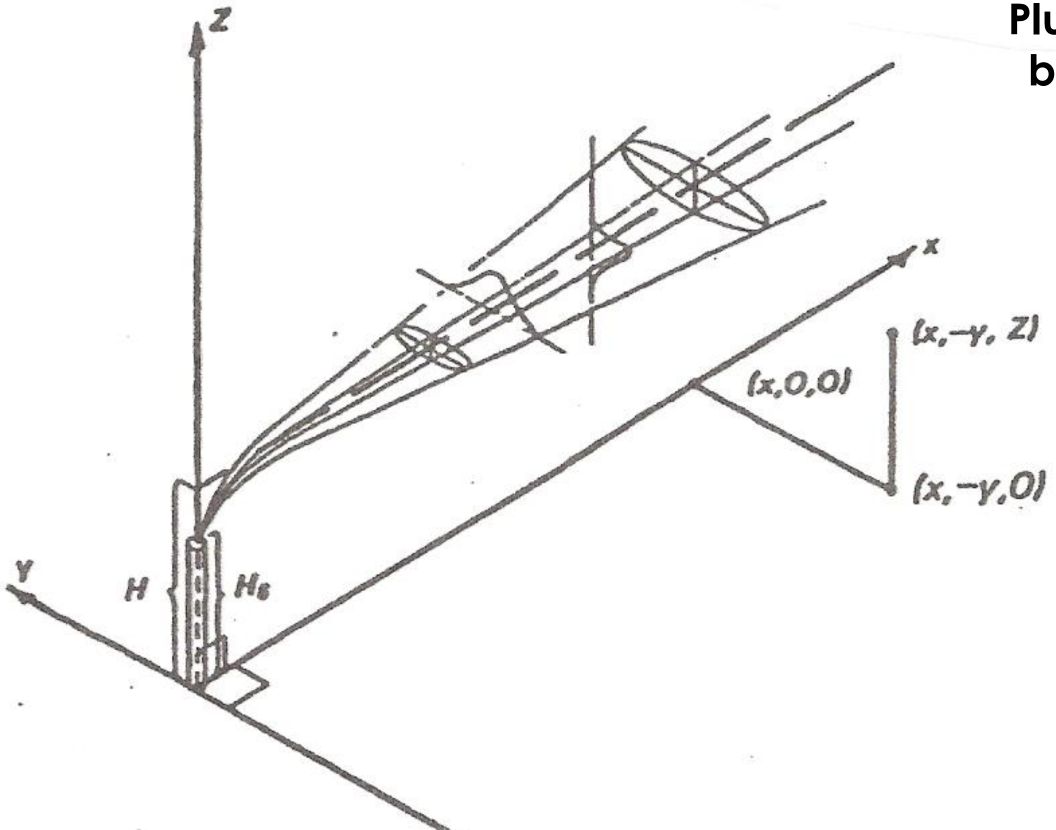
Using statistics we can estimate the impact of an emission source from an industrial plant:



Z-axis starts from ground level

Plume dispersion occurs in the y- and z-directions

Plume is convected by the wind in the x-direction



Turbulent dispersion:

y-direction:

$$C \propto \exp\left\{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right\}$$

z-direction:

$$C \propto \exp\left\{-\frac{1}{2}\left(\frac{z}{\sigma_z}\right)^2\right\}$$

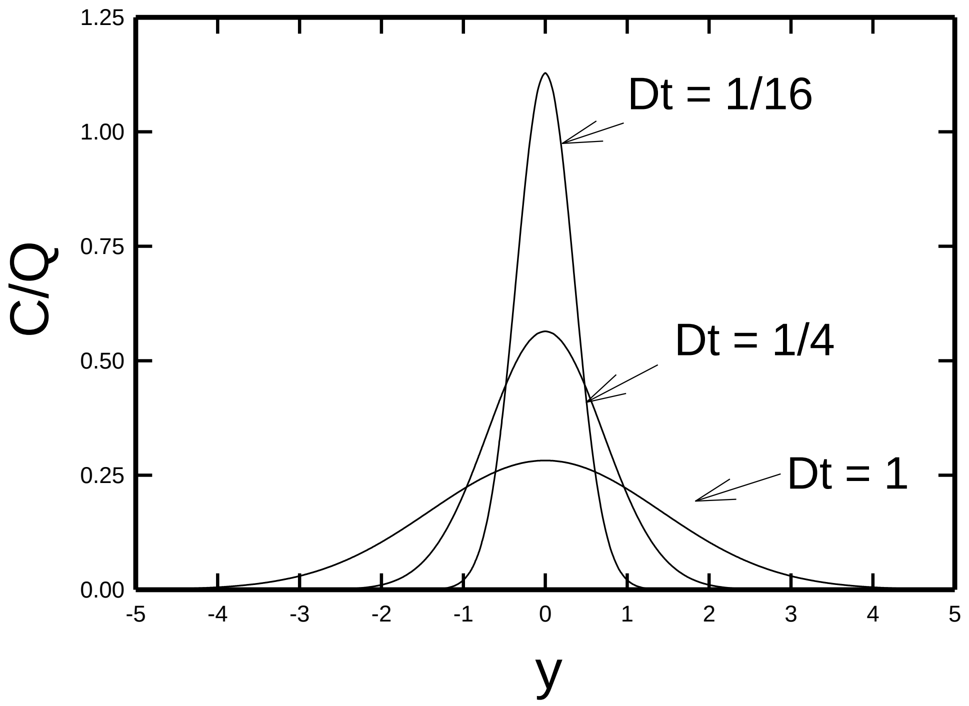
Where σ_y and σ_z are dispersion parameters that must be estimated

Particle diffusion:

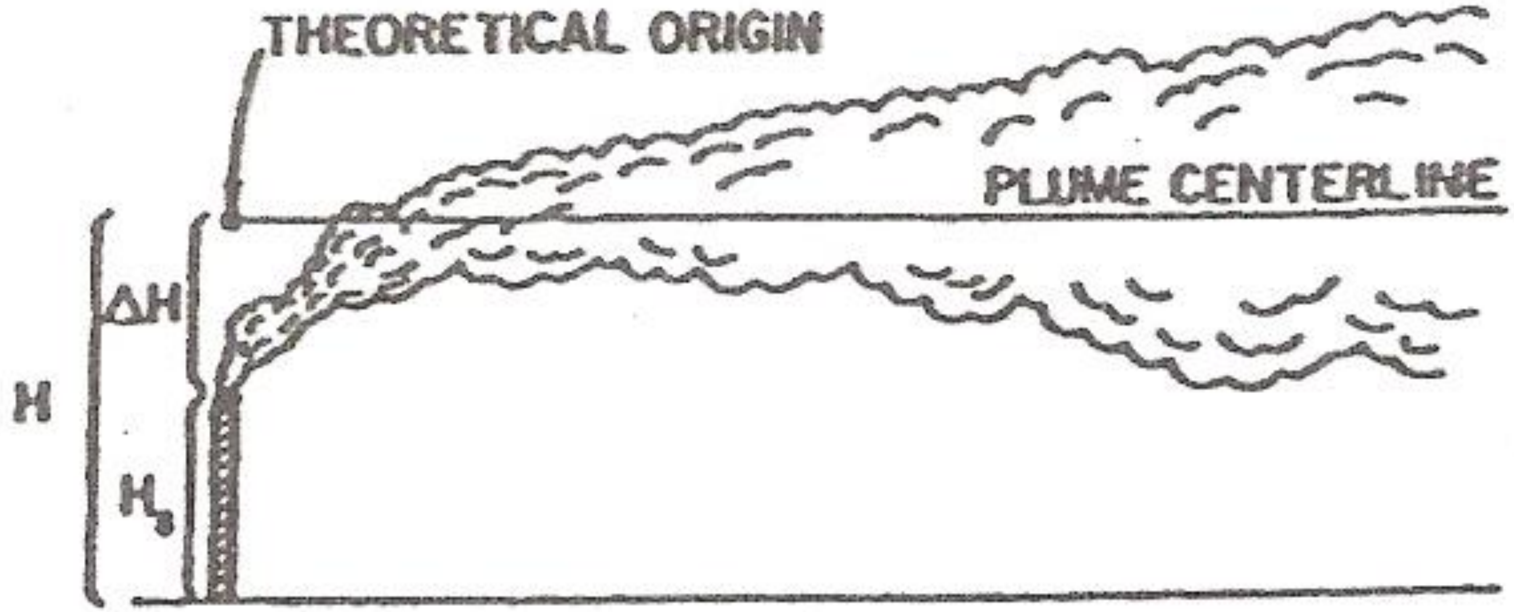
$$C = \frac{Q}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{y^2}{\sigma^2}\right\}$$

where $$\sigma^2 = 2\,D\,t$$

D **is the particle diffusivity**



Transverse turbulent dispersion

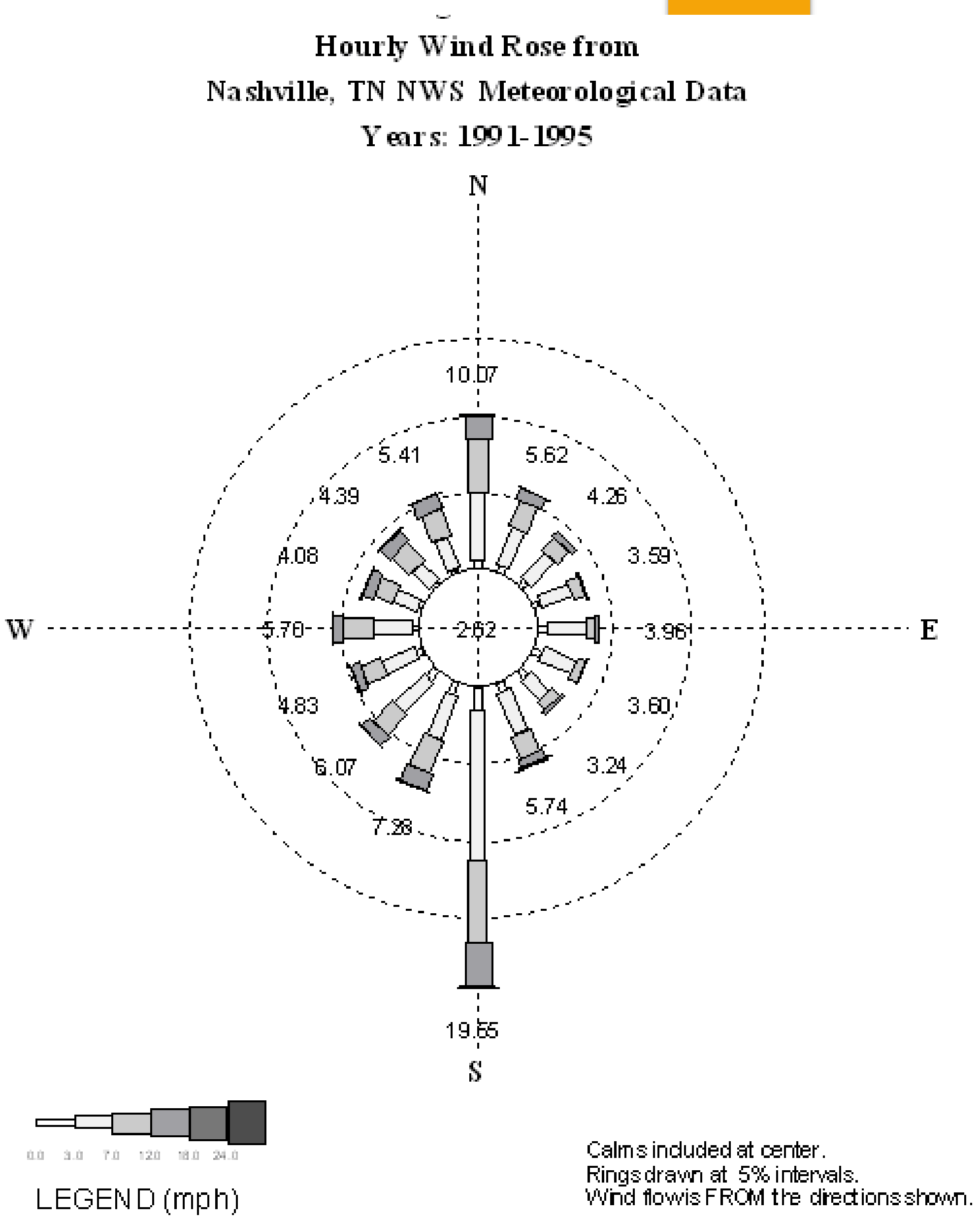


σ_y and σ_z must be estimated

Ground-level concentrations will depend on …

Wind …

<u>Wind rose</u> shows average direction and magnitude of the wind vector

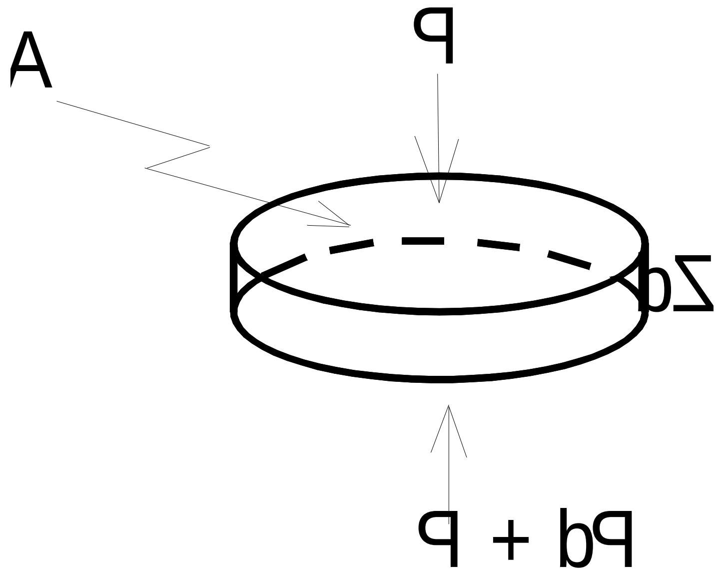


Atmospheric stability …

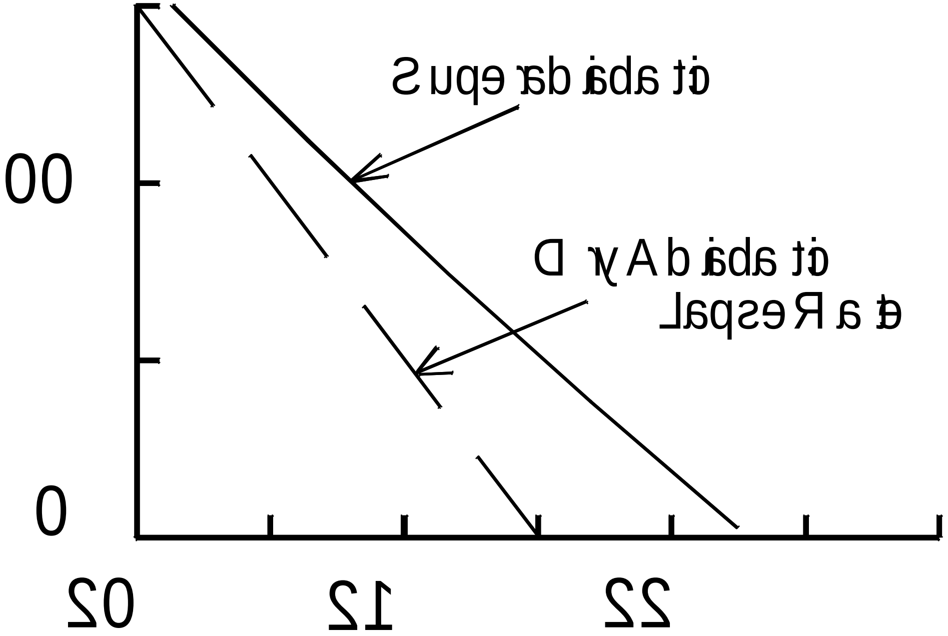
Dry adiabatic lapse rate (stable, neutral atmosphere)

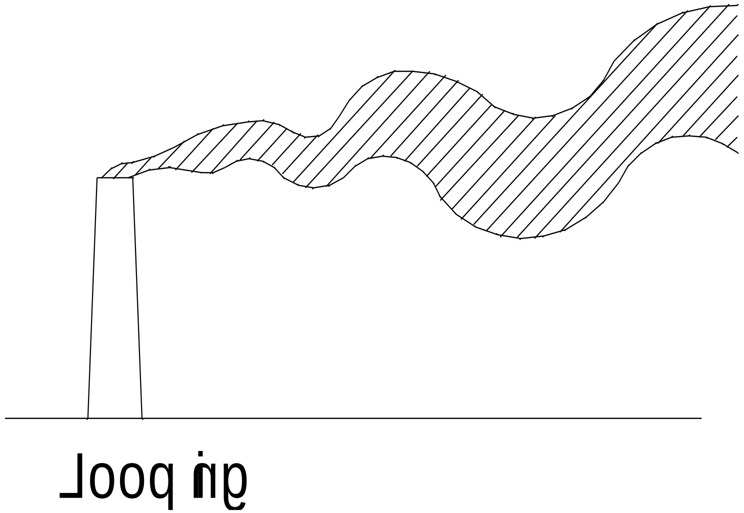
$$\frac{dT}{dZ} = -1^{\circ}C/100\,m$$

Natural balance between hydrostatic head, ρ g dA dZ, and pressure forces



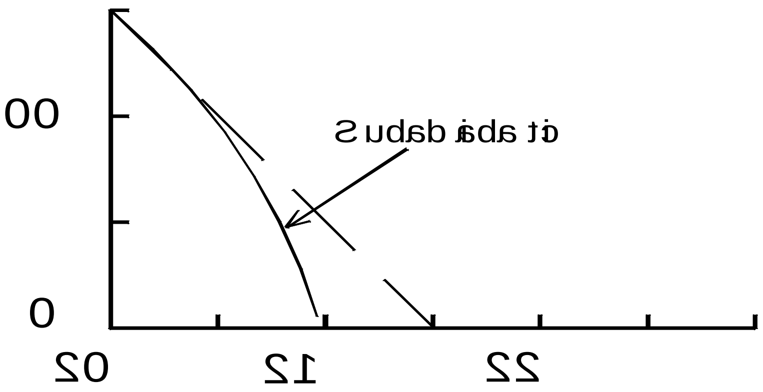
Super-adiabatic lapse rate:

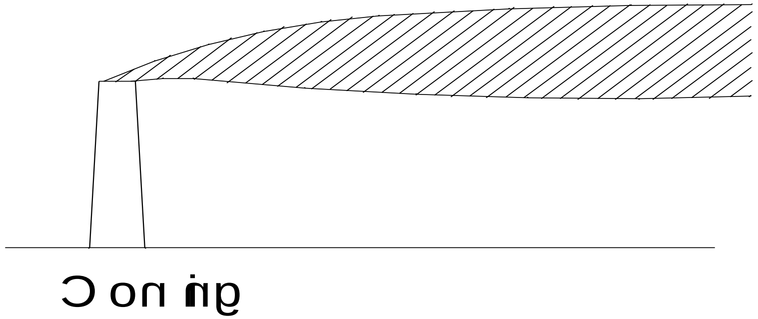




A “buoyant” atmosphere

Sub-adiabatic lapse rate:





The Gaussian plume model:

The concentration of material downwind in the x-direction varies as the inverse of the local transport velocity, i.e.,

$$C \propto \frac{1}{U}$$

A Gaussian type distribution is used in the y-direction:

$$C \propto A \exp\left\{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right\}$$

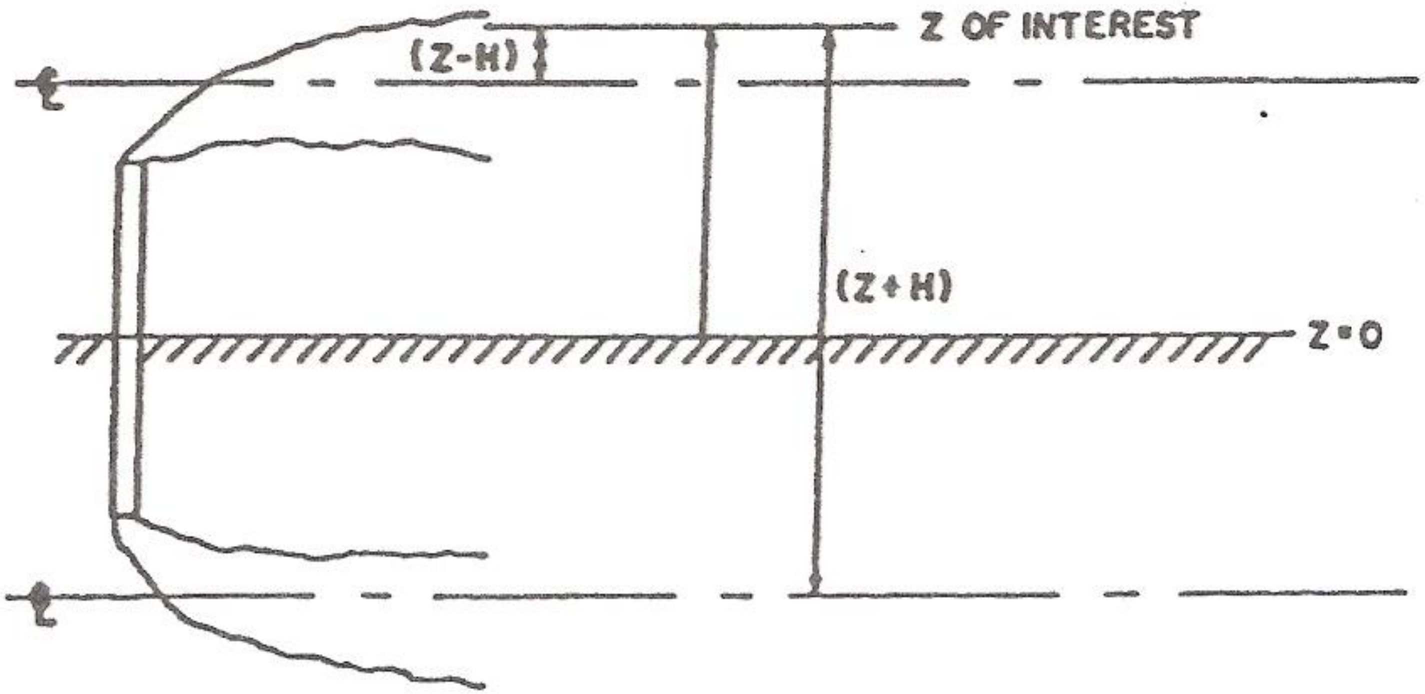
Starting to look like a Normal distribution!

If we choose $A = \dfrac{1}{\sqrt{2\pi}\,\sigma_y}$

The distribution will provide an integrated concentration of unity across the transverse cross-section

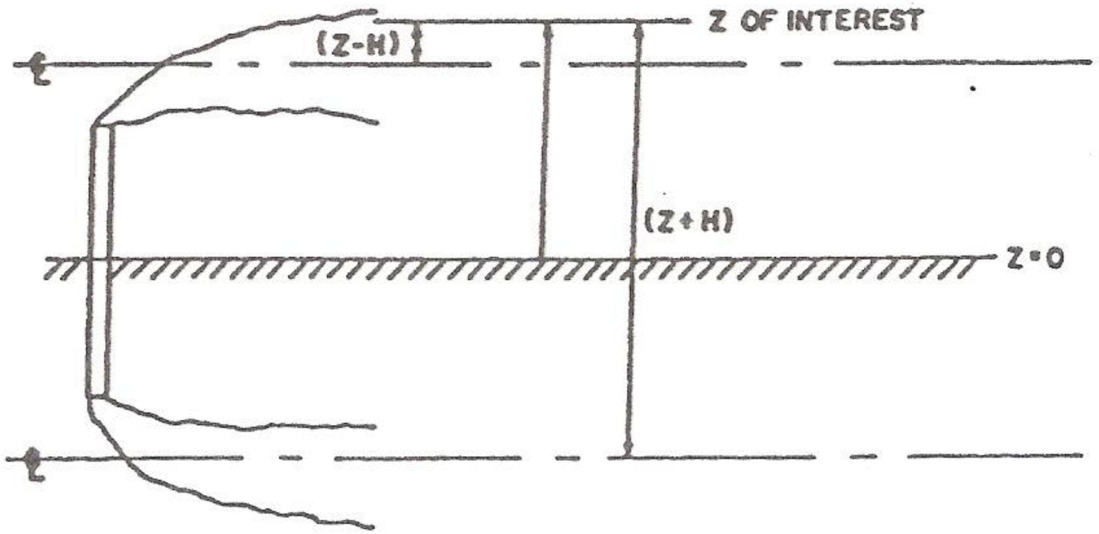
$$\frac{1}{\sqrt{2\pi}\,\sigma_y}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right\}dy = 1$$

Z- direction requires special treatment …



When the "edge" of the plume reaches the ground …
we assume perfect "reflection"!

Plume model almost complete



$$C \propto \frac{1}{\sqrt{2\pi}}\left[\exp\left(-\frac{1}{2}\left\{\frac{z-H}{\sigma_z}\right\}^2\right) + \exp\left(-\frac{1}{2}\left\{\frac{z+H}{\sigma_z}\right\}^2\right)\right]$$

(z - H) term accounts for the above ground contribution

(z + H) term accounts for the imaginary source below ground

Final form of the Gaussian plume model:

$$C(x, y, z, H) = \frac{Q}{2\pi\sigma_y\sigma_z U}\exp\left\{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right\}$$

$$\times\left[\exp\left(-\frac{1}{2}\left\{\frac{z-H}{\sigma_z}\right\}^2\right) + \exp\left(-\frac{1}{2}\left\{\frac{z+H}{\sigma_z}\right\}^2\right)\right]$$

- Product of the y- and z-direction distributions
- Q is the emission rate in mass per unit time

Variation of C with x is contained in the behaviour of σ_y and σ_z with downstream position, x, from the emission source.

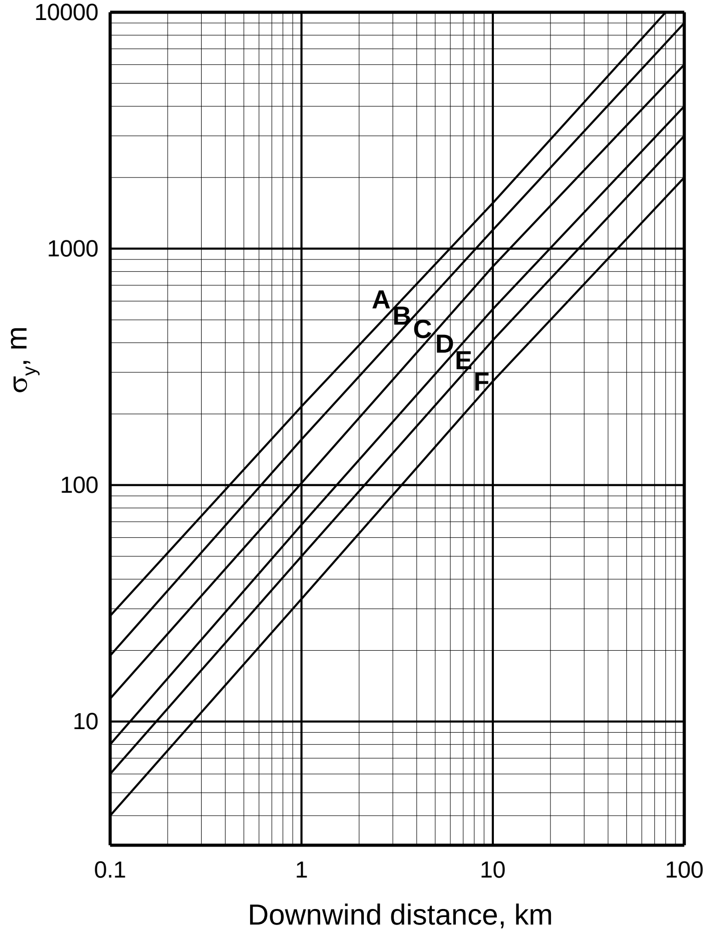
$$C(x, y, z, H) = \frac{Q}{2\pi\sigma_y\sigma_z U}\exp\left\{-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right\}$$

$$\times\left[\exp\left(-\frac{1}{2}\left\{\frac{z-H}{\sigma_z}\right\}^2\right)+\exp\left(-\frac{1}{2}\left\{\frac{z+H}{\sigma_z}\right\}^2\right)\right]$$

Stability classes A - F

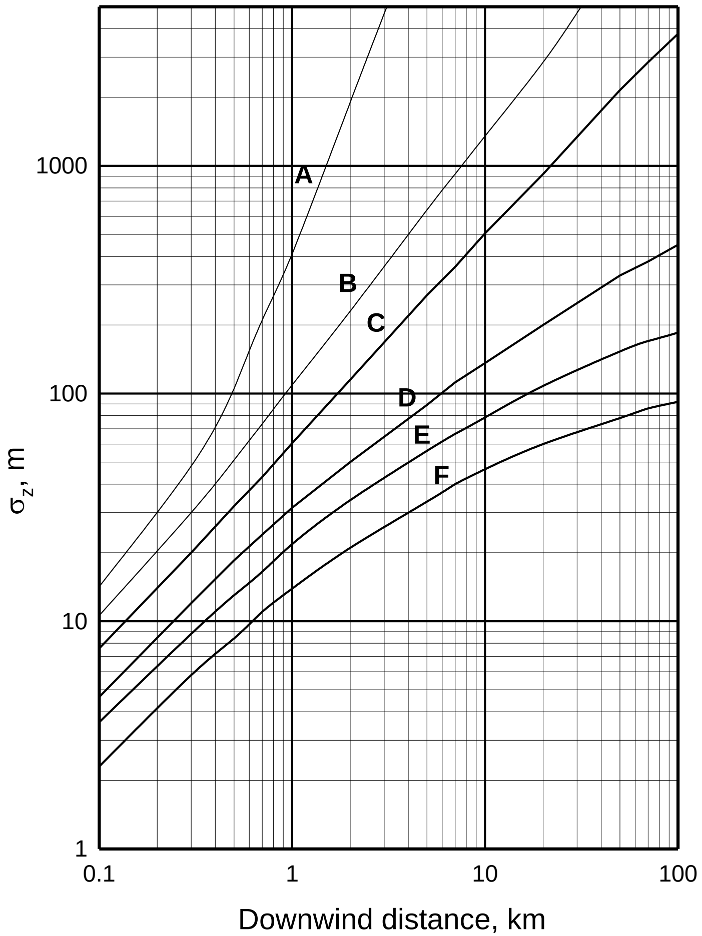
Surface wind speed (at 10 m) in m sec^{-1}	Day			Night	
	Incoming solar radiation			Thinly overcast or $\geq 4/8$ low cloud	$\leq 3/8$ Cloud
	Strong	Moderate	Slight		
< 2	A	A-B	B		
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
> 6	C	D	D	D	D

Mathematical models for σ_y



$$\sigma_y = a\,x^p$$

Mathematical models for σ_z



$$\sigma_z = b\,x^q$$

- for class C, and
- piecewise (in x) for other different classes

Detailed dispersion parameter estimates for model forms: σ_y and σ_z

Dispersion Class	σ_y		σ_z		Range of Application for σ_z, x, meters
	a	P	b	q	
A	0.55202	0.85927	0.07791	1.1148	100 - 300
			0.01036	1.4787	300 - 500
			0.000218	2.1057	500 - 3000
B	0.38267	0.87024	0.1186	0.9711	100 - 500
			0.05419	1.1028	500 - 20000
C	0.22759	0.88812	0.11486	0.9074	100 - 20000
D	0.16209	0.87944	0.09261	0.8529	100 - 500
			0.25616	0.6927	500 - 3000
			0.52179	0.6020	3000 - 20000
E	0.11826	0.88167	0.08182	0.8178	100 - 500
			0.22387	0.6575	500 - 3000
			0.74895	0.5052	3000 - 20000
F	0.07948	0.88144	0.05498	0.8101	100 - 500
			0.1549	0.6485	500 - 3000
			0.92323	0.4234	3000 - 20000

But …

The accuracy of these parameters depends on the weather conditions and downstream distance from the emission source. In general, errors in the estimate of σ_z on the order of 3 can occur for longer distances under unstable and stable cases. Errors in σ_z are expected to be correct within a factor of 2 for the cases:

(i) all stability classes for distances out to a few 100 meters.

(ii) neutral to moderately unstable conditions out to a few km.

(iii) Unstable conditions in the lower 1000 m of the atmosphere with a marked inversion for distances out to 10 km or more.

The errors in σ_y are generally less than those for σ_z. The estimates of σ for both directions are based on a 10-minute time average, the dispersion tends to be greater for longer times due to meandering of the plume in the cross-stream directions.