Morfologia matemática para processamento de imagens

Gonzalez - Capítulo 9

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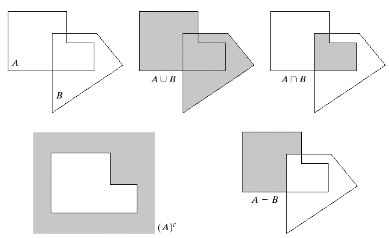
Morfologia Matemática

- utilizada para extrair componentes que podem ser utilizadas na representação e descrição da forma de regiões da imagem, como:
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Z^2 e Z^3

- conjuntos, em morfologia matemática, representam objetos em uma imagem:
  - imagem binária (0 = branco, 1 = preto) :
    - o elemento do conjunto é a coordenada (x,y) do pixel que pertence ao objeto ⇨ Z^2
  - imagens em níveis de cinza:
    - o elemento do conjunto é formado pela coordenada (x,y) e a intensidade do pixel que pertence ao objeto ⇨ Z^3

Teoria dos conjuntos



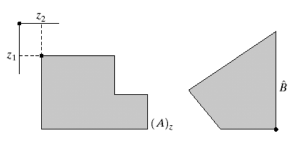
a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflexão e Translação

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$



a b

FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

Operações lógicas

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Exemplo

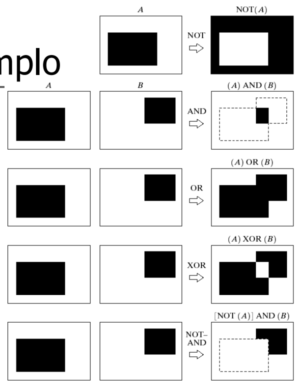
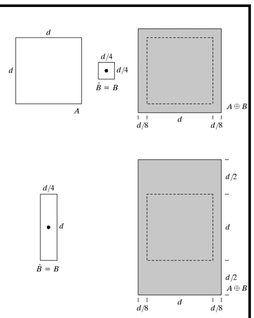


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Dilatação



a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

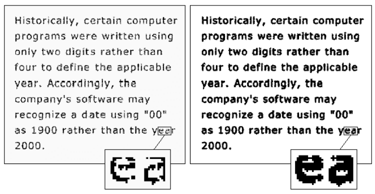
B = elemento estrutural

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

Dilatação

- Há outras definições possíveis, mas note a semelhança dessa definição com a operação de convolução com uma máscara
- essa operação é bastante utilizada na prática para preencher buracos, sendo mais eficiente que um filtro passa baixas.
  - justifique.

Dilatação: tapando buracos

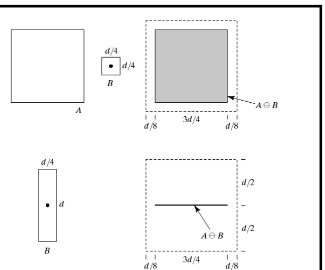


a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

Erosão



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

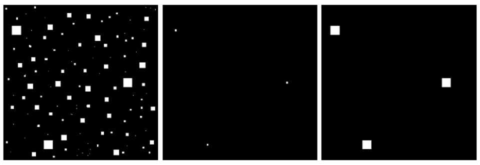
$$A \ominus B = \{z | (B)_z \subseteq A\}$$

Dualidade

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \ominus B)^c = \{z \,|\, (B)_z \subseteq A\}^c$$
$$(A \ominus B)^c = \{z \,|\, (B)_z \cap A^c = \phi\}^c$$
$$(A \ominus B)^c = \{z \,|\, (B)_z \cap A^c \neq \phi\}$$
$$= (A^c \oplus \hat{B})$$

Erosão: eliminação de detalhes irrelevantes

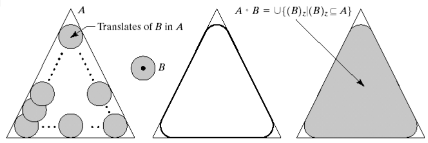


a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

elemento estrutural B = 13x13 pixels

Abertura



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

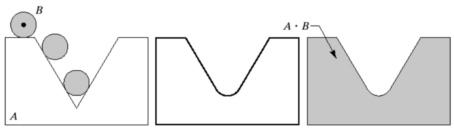
$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

Abertura

- utilizada para suavizar contornos de um objeto
- interpretação: considere que o elemento estruturante seja uma bola, então a abertura de A por B, (A ◦B), é dado pela intersecção de todas as translações de B que estejam dentro de A.

Fechamento



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$A \bullet B = (A \oplus B) \ominus B$$

Dualidade $(A \bullet B)^c = (A^c \circ \hat{B})$

Propriedades

Abertura

(i) A°B é um subconjunto (subimagem) de A

(ii) Se C é um subconjunto de D, então C °B é um subconjunto de D °B

(iii) (A °B) °B = A °B

Fechamento

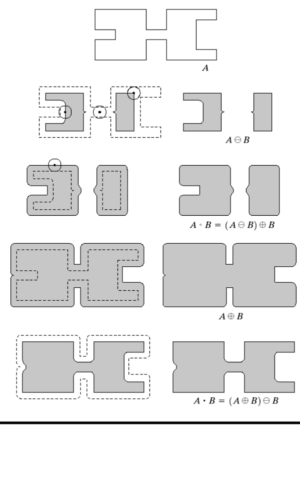
(i) A é um subconjunto (subimagem) de A•B

(ii) Se C é um subconjunto de D, então C •B é um subconjunto de D •B

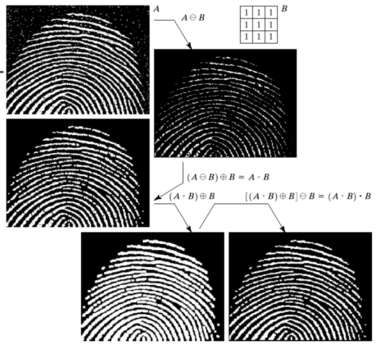
(iii) (A •B) •B = A •B

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Exemplo de abertura e fechamento. O elemento estruturante é o círculo.



a b
d c
e f

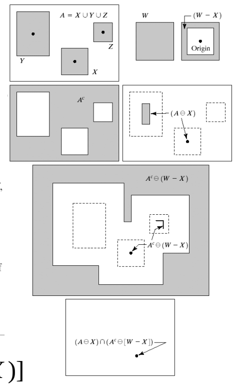
FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of *A*.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Transformação Hit-or-Miss

transformação utilizada para detecção de formas
Exemplo: conjunto A contem 3 formas, X, Y, Z
objetivo: achar X

a b
c d
e
f

FIGURE 9.12
(a) Set *A*. (b) A window, *W*, and the local background of *X* with respect to *W*, $(W - X)$.
(c) Complement of *A*. (d) Erosion of *A* by *X*.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of *X*, as desired.

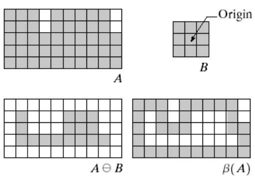


$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

Extração de contornos

a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*.
(d) Boundary, given by the set difference between *A* and its erosion.



$$\beta(A) = A - (A \ominus B)$$

Exemplo

a b

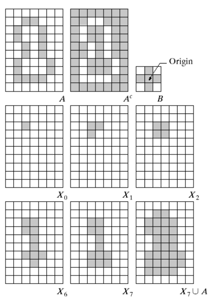
FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Preenchimento de regiões

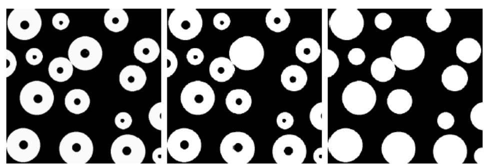
$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$

a b c
d e f
g h i

FIGURE 9.15
Region filling.
(a) Set *A*.
(b) Complement of *A*.
(c) Structuring element *B*.
(d) Initial point inside the boundary.
(e)–(h) Various steps of Eq. (9.5-2).
(i) Final result [union of (a) and (h)].



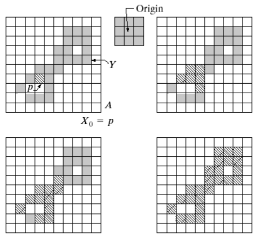
Exemplo



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extração de componentes conexos



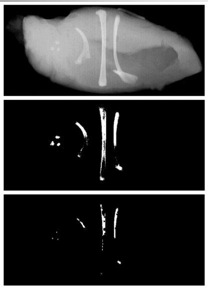
a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Exemplo

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Feixo convexo - Convex Hull

- Um conjunto A é dito convexo se um segmento de reta qualquer entre 2 pontos de A está completamente contido em A.
- O feixo convexo H de um conjunto S arbitrário é o menor conjunto convexo contendo S.
- O conjunto H-S é denominado deficiência convexa de S

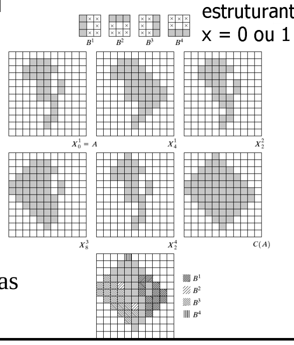
$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A, \ \text{i} = 1,2,3,4$$

Convex hull

elementos estruturantes
x = 0 ou 1

a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



$$C(A) = \bigcup_{i=1}^{4} D^i,$$

união das convergências

Algoritmo limitado ao bounding box

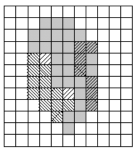


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

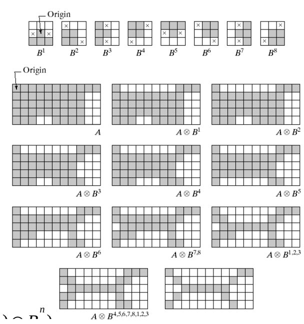
Thinning

$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

$\{B\} = \{B^1, B^2, ..., B^n\}$,

elem. estruturantes

$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$



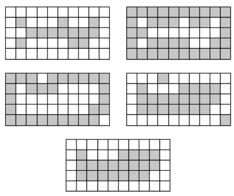
a
b c d
e f g
h i j
k l

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

Thickening

$$A \circledcirc B = A \cup (A \circledast B)$$

Como em thinning, thickening pode ser feito com um conjunto de elem. estruturantes, mas na prática, é feito através do thinning do conjunto complementar



a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons S(A)

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

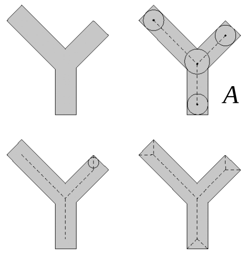
k erosões

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max\{k \mid (A - kB) \neq \Phi\}$$

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.



$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

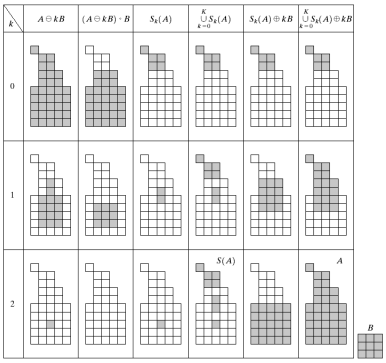


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

H = elemento estruturante3x3

Pruning

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$

a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

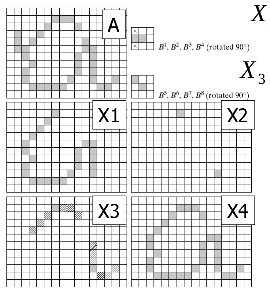


TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; i = 1, 2, 3, 4;$ $k = 1, 2, 3, \ldots; X_0^i = A$; and $D^i = X_{conv}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

5 elementos estruturantes básicos

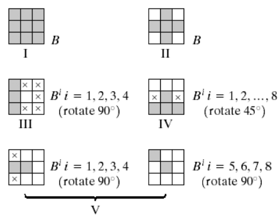


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.