Exercício 3.1, página 83

$D = (8, 2, 2, 11, 4, 7)$, um plano AAS s, $n = 2$ e $N = 6$

a) Encontre a distrib de $\bar{y}$ e mostre que $E(\bar{y}) = 4$.

i) $\mu = \frac{1}{N} \sum_{i=1}^{N} Y_i = \frac{1}{6}(8 + 2 + 2 + 11 + 7 + 4) = \frac{34}{6} = \boxed{5,6667}$

μ é a média populacional.

ii) seja as amostras:

s	(12)	(23)	(14)	(15)	(26)	(23)	(24)	(25)	(26)	(34)	(35)	(36)	(45)	(46)	(56)
$\bar{y}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{19}{2}$	$\frac{12}{2}$	$\frac{15}{2}$	$\frac{4}{2}$	$\frac{13}{2}$	$\frac{6}{2}$	$\frac{9}{2}$	$\frac{13}{2}$	$\frac{6}{2}$	$\frac{9}{2}$	$\frac{15}{2}$	$\frac{18}{2}$	$\frac{11}{2}$

temos que $\bar{y}$ tem distribuição:

$\bar{y}$	$4/2$	$6/2$	$9/2$	$10/2$	$11/2$	$12/2$	$13/2$	$15/2$	$18/2$	$19/2$	
	$1/15$	$2/15$	$2/15$	$2/15$	$1/15$	$1/15$	$2/15$	$2/15$	$1/15$	$1/15$	$15/15$

iii) Vamos verificar $E(\bar{y})$,

$$(*) E(\bar{y}) = \frac{1}{30}\left(4 + 6.2 + 9.2 + 10.2 + 11.1 + 12 + 13.2 + 15.2 + 18 + 19\right)$$

$E(\bar{y}) = \frac{1}{30} \cdot 170 = \boxed{5,6667}$, assim temos que $\boxed{E(\bar{y}) = \mu}$

(*) sabendo que

$$E(\bar{y}) = \frac{4}{2}\left(\frac{1}{15}\right) + \frac{6}{2}\left(\frac{2}{15}\right) + \frac{9}{2}\left(\frac{2}{15}\right) + \frac{10}{2}\left(\frac{2}{15}\right) + \frac{11}{2}\left(\frac{1}{15}\right) + \frac{12}{2}\left(\frac{1}{15}\right) + \frac{13}{2}\left(\frac{2}{15}\right) + \frac{15}{2}\left(\frac{2}{15}\right) + \frac{18}{2}\left(\frac{1}{15}\right)$$

$+ \frac{19}{2}\left(\frac{1}{15}\right) =$; colocamos $\left(\frac{1}{30}\right)$ em evidência.

b) Mostre que $Var(\bar{y}) = (1-f)\, S^2/n$, sendo $f = n/N$ ②

i) Vamos calcular S^2, sendo esta a variância amostral populacional então:

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \mu)^2 = \; ; \text{ com } \mu = 34/6 ;$$

$$= S^2 = \frac{1}{5}\left\{\left(8-\frac{34}{6}\right)^2 + 2\cdot\left(2-\frac{34}{6}\right)^2 + \left(11-\frac{34}{6}\right)^2 + \left(4-\frac{34}{6}\right)^2 + \left(7-\frac{34}{6}\right)^2\right\} =$$

$$= \frac{1}{5}\left\{\left(\frac{14}{6}\right)^2 + 2\cdot\left(-\frac{22}{6}\right)^2 + \left(+\frac{32}{6}\right)^2 + \left(-\frac{10}{6}\right)^2 + \left(\frac{8}{6}\right)^2\right\} =$$

$$= \frac{1}{5}\left\{\frac{196}{36} + 2\cdot\frac{484}{36} + \frac{1024}{36} + \frac{100}{36} + \frac{64}{36}\right\} =$$

$$= \frac{1}{5}\left\{\frac{2.352}{36}\right\} = \frac{2.352}{180} ;$$

ii) Aplicamos o resultado à $(1-f)\, S^2/n$; sendo $\begin{cases} f = 2/\cancel{6} = \boxed{1/3} \\ e\ n = 2 \end{cases}$

Então:

$$Var(\bar{y}) = (1-1/3)\cdot\frac{2\,352}{180}\cdot\frac{1}{2} =$$

$$Var(\bar{y}) = \frac{\cancel{2}}{3}\cdot\frac{2.352}{180}\cdot\frac{1}{\cancel{2}} = \frac{2.352}{540} = \boxed{4,3555.}$$

$\boxed{Var(\bar{y}) = 4.3555, \text{ pela corolário } 3.4}$

iii) Vamos calcular $Var(\bar{y})$ através da distrib. de $\bar{y}$: ③

sendo $Var(\bar{y}) = E(\bar{y}^2) - (E(\bar{y}))^2$, então:

$$E(\bar{y}^2) = \left(\frac{4}{2}\right)^2 \frac{1}{15} + \left(\frac{6}{2}\right)^2 \cdot \frac{2}{15} + \left(\frac{9}{2}\right)^2 \cdot \frac{2}{15} + \left(\frac{10}{2}\right)^2 \left(\frac{2}{15}\right) + \left(\frac{11}{2}\right)^2 \cdot \frac{1}{15} + \left(\frac{12}{2}\right)^2 \cdot \frac{1}{15} + \left(\frac{13}{2}\right)^2 \frac{2}{15} + \left(\frac{15}{2}\right)^2 \cdot \frac{2}{15} + \left(\frac{18}{2}\right)^2 \cdot \frac{1}{15} + \left(\frac{19}{2}\right) \cdot \frac{1}{15}$$

$$E(\bar{y}^2) = \frac{1}{60}\Big(16 + \underbrace{36.2}_{72} + \underbrace{81.2}_{162} + \underbrace{100.2}_{200} + 121 + 144 + \underbrace{169.2}_{338} + \underbrace{225.2}_{450} + 324 + 361\Big) =$$

$$E(\bar{y}^2) = \frac{1}{60}(2.188) = \boxed{\frac{2.188}{60}}$$

assim temos que:

$$Var(\bar{y}) = \frac{2.188}{60} - \left(\frac{170}{30}\right)^2 = \frac{2.188}{60} - \frac{28.900}{900} =$$

$$= \frac{(2.188).15 - 28.900}{900} =$$

$$= \frac{32.820 - 28.900}{900}$$

$$= \frac{3.920}{900} = \boxed{4,3555\ldots}$$

iv) Pelo resultado acima, e pelo encontrado em (ii), concluímos que: $Var(\bar{y}) = (1-f).S^2/n$, para amostra aleatória simples sem reposição: AASs.

Exercício 3.2. página $\underline{83}$

$\Rightarrow$ Mesmos dados que 3.1, mas com o plano de $\underline{AASc}$

a) Encontre a distrib $\bar{y}$ e mostre que $E(\bar{y}) = \cancel{\quad}\, \mu$

i) seja $\underline{s}$ a amostra:

s:	(11)	(12)	(13)	(14)	(15)	(16)	(22)	(23)	(24)	(25)	(26)	(33)	(34)	(35)	(36)	(44)	(45)	(46)	(55)	(56)
$\bar{y}$:	$\frac{16}{2}$	$\frac{10}{2}$	$\frac{10}{2}$	$\frac{19}{2}$	$\frac{12}{2}$	$\frac{15}{2}$	$\frac{4}{2}$	$\frac{4}{2}$	$\frac{13}{2}$	$\frac{6}{2}$	$\frac{9}{2}$	$\frac{4}{2}$	$\frac{13}{2}$	$\frac{6}{2}$	$\frac{9}{2}$	$\frac{22}{2}$	$\frac{15}{2}$	$\frac{18}{2}$	$\frac{8}{2}$	$\frac{11}{2}$
$P(\bar{y})$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$

s	(66)
$\bar{y}$:	$\frac{14}{2}$
$P(\bar{y})$:	$\frac{1}{36}$

$$\frac{2.15}{36} + \frac{1.6}{36} = \frac{36}{36} = \boxed{1}$$

ii) Vamos calcular $E(\bar{y})$ (*) $\Rightarrow$ distrib. de $\bar{y}$ p/ $\underline{AASc}$

$$E(\bar{y}) = \frac{1}{72}\{4.4 + 6.4 + 8.1 + \overset{+9.4+}{} 10.4 + 11.2 + 12.2 + 13.4 + 14 + 15.4 + 16 + 18.2 + 19.2 + 22\}$$

$$E(\bar{y}) = \frac{1}{72}(16 + 24 + 8 + 40 + 22 + 24 + 52 + 14 + 60 + 16 + 36 + 38 + 22 + 36)$$

$$E(\bar{y}) = \frac{1}{72}(408) = \frac{408}{72} = \boxed{5,6667}$$

iii) Assim, concluímos, ~~p~~ sabendo que $\mu = 34/6$, que

$E(\bar{y}) = \mu$, p/ $\underline{AASc}$

* $E(\bar{y})$4 , dado que a distribuição de $\bar{y}$ é:

$\bar{y}$	$4/2$	$6/2$	$8/2$	$9/2$	$10/2$	$11/2$	$12/2$	$13/2$	$14/2$	$15/2$	$16/2$	$18/2$	$19/2$	$22/2$
$P(\bar{y})$	$4/36$	$4/36$	$1/36$	$4/36$	$4/36$	$2/36$	$2/36$	$4/36$	$1/36$	$4/36$	$1/36$	$2/36$	$2/36$	$1/36$

$$\frac{4\times 6}{36} + \frac{4\times 2}{36} + \frac{4}{36} = \frac{36}{36} = \boxed{1}$$

$$E(\bar{y}) = \frac{4}{2}\left(\frac{4}{36}\right) + \frac{6}{2}\left(\frac{4}{36}\right) + \frac{8}{2}\left(\frac{1}{36}\right) + \frac{9}{2}\left(\frac{4}{36}\right) + \frac{10}{2}\left(\frac{4}{36}\right) + \frac{11}{2}\left(\frac{2}{36}\right) + \frac{12}{2}\left(\frac{2}{36}\right) + \frac{13}{2}\left(\frac{4}{36}\right) +$$

$$+ \frac{15}{2}\left(\frac{4}{36}\right) + \frac{16}{2}\left(\frac{1}{36}\right) + \frac{18}{2}\left(\frac{2}{36}\right) + \frac{19}{2}\left(\frac{2}{36}\right) + \frac{22}{2}\left(\frac{1}{36}\right) =$$

⇒ Continua em (iii). pág 4.

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b) veja se $Var(\bar{y})$ é igual ao resultado (3.9), pág 65,

em que $Var(\bar{y}) = \sigma^2/n$.

i) Vamos calcular $Var(\bar{y})$, diretamente:

$Var(\bar{y}) = E(\bar{y}^2) - (E(\bar{y}))^2$, então:

$$E(\bar{y}^2) = \left(\frac{4}{2}\right)^2 \cdot \left(\frac{4}{36}\right) + \left(\frac{6}{2}\right)^2 \cdot \left(\frac{4}{36}\right) + \left(\frac{8}{2}\right)^2 \cdot \left(\frac{1}{36}\right) + \left(\frac{9}{2}\right)^2 \cdot \left(\frac{4}{36}\right) + \left(\frac{10}{2}\right)^2 \left(\frac{4}{36}\right) +$$

$$+ \left(\frac{11}{2}\right)^2 \left(\frac{2}{36}\right) + \left(\frac{12}{2}\right)^2 \left(\frac{2}{36}\right) + \left(\frac{13}{2}\right)^2 \left(\frac{4}{36}\right) + \left(\frac{15}{2}\right)^2 \left(\frac{4}{36}\right) + \left(\frac{16}{2}\right)^2 \left(\frac{1}{36}\right) +$$

$$+ \left(\frac{18}{2}\right)^2 \left(\frac{2}{36}\right) + \left(\frac{19}{2}\right)^2 \left(\frac{2}{36}\right) + \left(\frac{22}{2}\right)^2 \left(\frac{1}{36}\right) =$$

assim temos que:

$$\cancel{E(\bar{y}^2) = \frac{1}{144}\left(16.4 + 36.4 + 64 + 81.4 + 100.4 + 121.2 + 144.2 + 169.4 + 196 + 900 + 256 + 648 + 722 + 484\right)}$$

$$E(\bar{y}^2) = \frac{1}{144}\left(64 + 144 + 64 + 324 + 400 + 242 + 288 + 676 + 196 + 900 + 256 + 648 + 722 + 484\right) =$$

$$E(\bar{y}^2) = \frac{1}{144}(5.408) = \boxed{\frac{5.408}{144}}$$

ii) Sendo $E(\bar{y}) = \boxed{\frac{408}{72}}$, temos que:

$$Var(\bar{y}) = \frac{5.408}{144} - \left(\frac{408}{72}\right)^2$$

$$= \frac{5.408}{144} - \frac{166.464}{5.184} =$$

$$= \frac{(5.408).36 - 166.464}{5.184} = \frac{194.688 - 166.464}{5.184} =$$

$$= \frac{28.224}{5.184} = \boxed{5,444\ldots}$$

Assim, temos que calculado diretamente; $Var(\bar{y}) = 5.4444$

iii) seja $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(y_i - \mu)$ => { temos que, pelo resultado de (3.1), (i) temos: }

$$\sigma^2 = \frac{1}{6}\cdot\left\{\frac{2.352}{36}\right\} = \frac{2.352}{216}.$$

e pelo resultado da pág 65, temos:

$$Var(\bar{y}) = \frac{\sigma^2}{n} = \frac{2.352}{216.2} = \boxed{\frac{2.352}{432}} = \boxed{5,444...}$$

Assim temos que o resultado acima é igual ao encontrado em (ii), confirmando que p/ AASc, $\boxed{Var(\bar{y}) = \sigma^2/n}$

Exercício 3.3:

Seja $P(|T - \tau| \leq B) \cong 1-\alpha$; { ache n, para $B = 0,03$, $\alpha = 0,01$, $s^2 = 3,6$. }

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i) $P(|T - \tau| \leq B) \cong 1-\alpha$

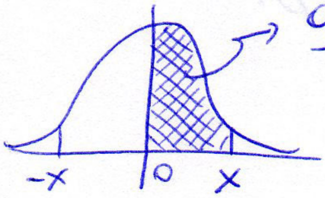
$$P\left(\frac{|T-\tau|}{N} \leq \frac{B}{N}\right) \cong 1-\alpha \Rightarrow P(|\bar{y} - \mu| \leq B/N) \cong 1-\alpha,$$

ver pag 69: (3.18) $\Rightarrow$

Assim, p/ N, e para valer (3.18) e podermos aproximar a probabilidade para uma $N(0,1)$, temos que dividir $|T-\tau|$, por N, para que sejam satisfeitas as condições de normalidade, então temos que:

ii) $P\left(\dfrac{|\bar{y}-\mu|}{\sigma/\sqrt{n}} \leq \dfrac{B}{N\sigma/\sqrt{n}}\right) \cong 1-\alpha$; $\dfrac{B}{N\sigma/\sqrt{n}} \sim N(0,1)$

iii) para $\gamma = 0,99$, temos que $z_{\gamma/2}$ vale:



$\dfrac{0,99}{2} = \boxed{0,495}$, procurando X, para a densidade de 0,495 temos que $\boxed{X = 2,58}$

iv) $\dfrac{B}{N.\sigma/\sqrt{n}} = 2,58$; logo; sabendo que $B = 0,03$, e usando s^2 no lugar de σ^2, temos que:

$$\left(\frac{B\sqrt{n}}{N.\sigma}\right)^2 = (2,58)^2 \Rightarrow nB^2 = (2.58)^2 \cdot (N^2 \cdot \sigma^2)$$

$$n = \frac{(2.58)^2 \cdot N^2 \cdot (3,6)}{B^2}$$

$$\boxed{n = \frac{23,963}{0.0009} \cdot N^2} \;//$$

$n > N$, pois nesse caso com rep. isso pode acontecer.