

Solução Prova II - MAE 116 - Biologia

1. Eventos:

H (M): Homem (Mulher)

F (NF): Fumante (Não Fumante)

i) $P(F) = P(M \cap F) + P(H \cap F) = 0.6 * 0.6 + 0.4 * 0.35 = 0.24 + 0.26 = 0.5$

ii) $P(NF/M) = 0.4$

iii)

$$P(M/NF) = \frac{P(M \cap NF)}{P(NF)} = \frac{P(M)P(NF/M)}{P(NF)} = \frac{0.26}{0.5} = 0.52.$$

2. Temos as equações

$$P(X \leq 25) = 0.82 \quad \rightarrow \quad \frac{25 - \mu}{\sigma} = 0.915.$$

$$P(X \geq 20) = 0.70 \quad \rightarrow \quad \frac{20 - \mu}{\sigma} = -0.524.$$

Resolvendo as equações acima, temos

$$\mu = 21.82, \quad \sigma = 3.47,$$

de modo que

$$P[X \geq 22] = P[Z \geq \frac{22 - 21.82}{3.47}] = 1 - A(0.0514) = 0.48.$$

3.

Eventos:

A^1 : azul retirada da urna I

A^2 : bola azul retirada da urna II

B^2 : bola branca retirada da urna II

i)

$$P(A^1 \cap A^2) = \frac{2}{42} = 0.0476.$$

ii)

$$P(A^2/A^1) = \frac{1}{6} = 0.167$$

iii)

$$P(A^1/B^2) = \frac{P(A^1 \cap B^2)}{P(B^2)} = \frac{6}{8+9+6} = \frac{6}{23} = 0.261.$$

4.

i) X : 3 # de portadores entre os $n=3$ indivíduos selecionados

X : 0, 1, 2, 3

$X \sim \text{Binomial}(3, 0.25)$

$$P(X = 0) = 1 \cdot 1 \cdot (.75)^3 = \frac{27}{64}$$

$$P(X = 1) = 3 \cdot (0.25) \cdot (.75)^2 = \frac{27}{64}$$

$$P(X = 2) = 3 \cdot (0.25)^2 \cdot (.75)^1 = \frac{9}{64}$$

$$P(X = 3) = 1 \cdot (0.25)^3 \cdot (.75)^0 = \frac{1}{64}$$

$$E[X] = 0 + \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = 0.75 = \frac{3}{4} = np.$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = \frac{27}{64} + \frac{36}{64} + \frac{9}{64} - \left(\frac{3}{4}\right)^2 = \frac{9}{16} = np(1-p) = 0.56$$

ii) $X \sim \text{Binomial}(n, p)$, $n=64$, $p=1/4$.

$$P[20 < X < 25] = P[X = 21] + P[X = 22] + P[X = 23] + P[X = 24] = P[21 \leq X \leq 24]$$

$$\begin{aligned}
&= P\left[\frac{21-16}{\sqrt{12}} \leq Z \leq \frac{24-16}{\sqrt{12}}\right] \\
&= P\left[\frac{5}{3.5} \leq Z \leq \frac{8}{3.5}\right] = P[1.43 \leq Z \leq 2.29] \\
&= A(2.29) - A(1.43) = 0.057.
\end{aligned}$$