Sol. Prova - CAP. 8-9

1.

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6\},$$
$$\mathcal{U}_1 = \{1\} \quad \mathcal{U}_1 = \{2, 3, 4\}, \quad \mathcal{U}_1 = \{5, 6\}.$$

i. $a = 2$, AASc no 1. e 2. estágios

a.

$$P[s = (1, 3)] = P[(\mathcal{U}_1, \mathcal{U}_2)]P[(1, 3)|(\mathcal{U}_1, \mathcal{U}_2)] = (1/9).(1).(1/3) = 1/27.$$

b.

$$P[s = (3, 3)] = P[(\mathcal{U}_2, \mathcal{U}_2)]P[(3, 3)|(\mathcal{U}_1, \mathcal{U}_2)] = (1/9).(1/3).(1/3) = 1/81.$$

ii. $a = 2$ com PPT e AA no segundo estgio

a.

$$P[s = (1, 3)] = P[(\mathcal{U}_1, \mathcal{U}_2)]P[(1, 3)|(\mathcal{U}_1, \mathcal{U}_2)] = (1/6).(3/6).1.(1/3) = 1/36.$$

b.

$$P[s = (3, 3)] = P[(\mathcal{U}_2, \mathcal{U}_2)]P[(3, 3)|(\mathcal{U}_1, \mathcal{U}_2)] = (3/6).(3/6).(1/3).(1/3) = 1/36.$$

2.

i.

a.

$$\mu_\alpha = \frac{B_\alpha}{\bar{B}}.\hat{\mu}_\alpha.$$

$$V_{2c} = \frac{1}{A}\sum_{\alpha=1}^{A}(\frac{B_\alpha}{\bar{B}}.\mu_\alpha - \mu)^2 + \frac{1}{A}\sum_{\alpha=1}^{A}\frac{B_\alpha}{\bar{B}}.\sigma_\alpha^2.$$

b. $Z_\alpha = \frac{B_\alpha}{N}$,

$$\hat{\tau}_{PPZ} = \frac{B_\alpha}{Z_\alpha}.\hat{\mu}_\alpha = N\hat{\mu}_\alpha$$

$$\hat{\mu}_{PPZ} = \frac{\hat{\tau}_{PPZ}}{N} = \hat{\mu}_\alpha.$$

$$V_{PPZ} = Var[\hat{\mu}_{PPZ}] = \frac{1}{N^2}\sum_{\alpha=1}^{A} Z_\alpha(\frac{\tau_\alpha}{Z_\alpha} - \tau)^2 + \sum_{\alpha=1}^{A}\frac{B_\alpha}{Z_\alpha}\frac{\sigma_\alpha^2}{b_\alpha}.$$

ii)

Cong. 1: $\mu_1 = 12$, $\tau_1 = 12$, $\sigma_1^2 = 0$, $B_1 = 1$.

Cong. 2: $\mu_2 = 10$, $\tau_2 = 30$, $\sigma_2^2 = 26/3$, $B_2 = 3$.

Cong. 1: $\mu_3 = 9$, $\tau_3 = 18$, $\sigma_3^2 = 1$, $B_3 = 2$.

$$V_{2c} = Var[\hat{\mu}_{2c}] = \frac{1}{3}(\frac{1}{2}12-10)^2+(\frac{3}{2}10-10)^2+(\frac{3}{2}9-10)^2]+\frac{1}{3}[(\frac{1}{2})^2.0+(\frac{3}{6})^2\frac{26/3}{b_\alpha}+(\frac{3}{6})^2\frac{26/3}{b_\alpha}$$

$$= 14 + \frac{13}{2b_2} + \frac{1}{b_3}.$$

$$V_{PPZ} = Var[\hat{\mu}_{PPZ}] = \frac{1}{6^2}[\frac{1}{6}(\frac{12}{1/6}-60)^2+\frac{3}{6}(\frac{30}{1/2}-60)^2+\frac{2}{6}(\frac{18}{1/3}-60)^2+[0+\frac{9}{1/2}+\frac{26/3}{b_2}+\frac{4}{1/3}\frac{1}{b_3}]$$

$$= 1 + \frac{156}{36b_2} + \frac{1}{3b_3}.$$

3. $\mathcal{U} = \{1,2,3,4,5,6\}$.

$$\mathcal{U}_1 = \{1\}, \quad \mathcal{U}_2 = \{2,3,4\}, \quad \mathcal{U}_3 = \{5,6\}.$$

TNA $\rightarrow (2,1) \rightarrow s() = (\mathcal{U}_2, \mathcal{U}_1)$.

$d_s(Y) = \{(7,9,14),(12)\}$, com $B_2 = 3$, $B_1 = 1$.

$$\tau_2 = 30, \quad \tau_1 = 12.$$

Assim,

$$\hat{\tau}_c = A\hat{\bar{\tau}} = 3\frac{(30+12)}{2} = 63,$$

com

$$\hat{V}_c = A^2 \frac{1}{a(a-1)} \sum_{\alpha \in s} (\tau_\alpha - \hat{\tau}_c)^2$$

$$= \frac{3^2}{2.(1)}[(30-21)^2 + (12-21)^2] = \frac{9.(162)}{2} = 729.$$

ii. Faixas $1/1 - 4/5 - 6$.

TNA $\rightarrow (3,5) \rightarrow s(\mathcal{U}) = (2,3)$

$\mathcal{U}_2 \rightarrow s = (2,3) \rightarrow \hat{\mu}_2 = (7+9)/2 = 8.$

$\mathcal{U}_3 \rightarrow s = (1,1) \rightarrow \hat{\mu}_2 = (8+8)/2 = 8.$

$$\hat{\mu}_{cPPZ} = \frac{1}{6.(2)} \sum_{\alpha \in s} \frac{\hat{\tau}_\alpha}{Z_\alpha} = \frac{1}{12}[\frac{3.(8)}{3/6} + \frac{2.(8)}{2/6}] = \frac{72}{6} = 12.$$

$$\hat{V}_{PPZ}(\hat{\mu}_{PPZ}) = \frac{\hat{V}_{PPZ}(\hat{\tau}_{PPZ})}{N} = \frac{1}{N^2 a(a-1)} \sum_{\alpha \in s} (\frac{B_\alpha \hat{\mu}_\alpha}{Z_\alpha} - \hat{\tau}_{PPZ})^2$$

$$\frac{1}{6^2.2}[(\frac{3.(8)}{3/6} - 72) + (\frac{2.(8)}{2/6} - 72)^2] = 16.$$