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Erratum

Erratum to “Dynamical R-matrices for the Calogero models” [Nucl. Phys. B 621 (2002) 523] ☆

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In the second part of the paper, which discusses the construction of Lax pairs and dynamical R-matrices for the Calogero models associated with symmetric pairs $(\mathfrak{g}_0, \theta)$ ($\mathfrak{g}_0$ being a real semisimple Lie algebra and θ its Cartan involution), the central theorem (Theorem 2) is only valid under an additional hypothesis. This additional restriction comes from the fact that Eqs. (A.28) and (A.29) stated in Appendix A of the paper are not always correct: in general, one can only guarantee the existence of a Cartan–Weyl basis satisfying

$$\sigma E_\alpha = -\epsilon_\alpha E_{\sigma\alpha}, \qquad \tau E_\alpha = -E_{-\alpha}, \qquad \theta E_\alpha = \epsilon_\alpha E_{\theta\alpha} \tag{A.28$'$}$$

and

$$N_{\theta\alpha,\theta\beta} = \frac{\epsilon_{\alpha+\beta}}{\epsilon_\alpha \epsilon_\beta} N_{\alpha,\beta}, \tag{A.29$'$}$$

where the coefficients ϵ_α are sign factors ($\epsilon_\alpha = \pm 1$), subject to the following invariance properties:

$$\epsilon_{\sigma\alpha} = \epsilon_\alpha, \qquad \epsilon_{-\alpha} = \epsilon_\alpha, \qquad \epsilon_{\theta\alpha} = \epsilon_\alpha. \tag{A.30$'$}$$

These sign factors cannot always be eliminated: they depend on the symmetric pair under consideration.

Indeed, the proof of Eqs. (A.28) and (A.29) given in Appendix A of the paper contains an error, which occurs in the 8th formula on p. 568: this equation should read

$$\begin{aligned} f_{\alpha+\beta} \widetilde{N}_{\alpha,\beta} \widetilde{E}_{\theta\alpha+\theta\beta} &= \widetilde{N}_{\alpha,\beta} \theta \widetilde{E}_{\alpha+\beta} = \theta\big([\widetilde{E}_\alpha, \widetilde{E}_\beta]\big) = [\theta \widetilde{E}_\alpha, \theta \widetilde{E}_\beta] \\ &= f_\alpha f_\beta [\widetilde{E}_{\theta\alpha}, \widetilde{E}_{\theta\beta}] = f_\alpha f_\beta \widetilde{N}_{\theta\alpha,\theta\beta} \widetilde{E}_{\theta\alpha+\theta\beta}, \end{aligned}$$

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where we have changed notation, writing $\widetilde{N}_{\alpha,\beta}$ rather than $N_{\alpha,\beta}$ for the structure constants in the original Cartan–Weyl basis. Therefore, the coefficients f_α continue to satisfy the conditions stated in the 9th formula on p. 567, but the 10th formula on p. 567 must be replaced by

$$f_{\alpha+\beta}\widetilde{N}_{\alpha,\beta} = f_\alpha f_\beta \widetilde{N}_{\theta\alpha,\theta\beta},$$

which we may write more briefly as

$$f_{\alpha+\beta} = \pm f_\alpha f_\beta,$$

since it can be guaranteed "a priori" that the structure constants are θ-invariant up to signs, i.e.,

$$\widetilde{N}^2_{\theta\alpha,\theta\beta} = \widetilde{N}^2_{\alpha,\beta}.$$

This relation is valid in any Cartan–Weyl basis, i.e., any basis satisfying the conditions (A.3)–(A.6) and (A.9) of the paper, since in any such basis, the value of $N^2_{\alpha,\beta}$ is completely determined by the shape of the α-string through β and since θ, being an automorphism, transforms the α-string through β into the $\theta\alpha$-string through $\theta\beta$.

The argument in the remainder of the proof may now be adapted to prove Eqs. (A.28′)–(A.30′) as stated above. Obviously, if the sign factors ϵ_α are all equal to 1, then the structure constants are θ-invariant. Conversely, one can use the original argument to show that if the structure constants are θ-invariant, then the Cartan–Weyl basis may be chosen so that the sign factors ϵ_α are all equal to 1.

Unfortunately, it turns out that the proof of the central theorem of this part of the paper (Theorem 2 on p. 552) breaks down when the structure constants are only θ-invariant up to signs, rather than θ-invariant. In other words, the condition $\epsilon_\alpha \equiv 1$ has to be considered as an additional selection criterion, further restricting the choice of symmetric pairs for which the method for constructing a Lax pair and a dynamical R-matrix presented in the paper works.

Finally, a more detailed analysis reveals that the question whether it is possible to eliminate these sign factors by a judiciously chosen change of signs in the choice of the root generators E_α depends on the behavior of the real roots in Δ, that is, the roots α in $\tilde{\Delta}$ for which $\theta\alpha = -\alpha$. For example, it can be shown that if $\epsilon_\alpha = 1$ for all real roots, then it is always possible to find a transformation of the form $E_\alpha \to \pm E_\alpha$ such that in the new basis, $\epsilon_\alpha = 1$ for all roots. On the other hand, it is easy to see that if there are two real roots α and β such that $\alpha + \beta$ is also a root, then it is impossible to find such a transformation, simply because in this case, $N_{\theta\alpha,\theta\beta} = N_{-\alpha,-\beta} = -N_{\alpha,\beta}$ and thus $\epsilon_{\alpha+\beta}\epsilon_\alpha\epsilon_\beta = -1$. This situation prevails for the AI-series, whereas the first case prevails for the AII-series (where there are no real roots) and for the AIII-series, corresponding to the complex Grassmannians $\mathrm{SU}(p,q)/\mathrm{S}(\mathrm{U}(p) \times \mathrm{U}(q))$ (where, in the standard basis, $\epsilon_\alpha = 1$ for all roots). This is the example discussed at length in the paper, for which all results stated in the paper remain correct.