

Lista de Exercícios No. 1

Cálculo Vetorial e Aplicações (MAP 215)
Cálculo Diferencial e Integral III (MAT 205)

1. Calcule o gradiente dos seguintes campos escalares:

(a) $\varphi(x, y, z) = x^2 \sin(xy) + y^2$

(b) $\varphi(x, y, z) = \cos(xe^y)$

(c) $\varphi(x, y, z) = x^5 y^2 z^3$

(d) $\varphi(x, y, z) = x^2 - y^2 - z^2$

(e) $\varphi(x, y, z) = \ln(x^2 - y^2 - z^2)$

(f) $\varphi(x, y, z) = x^{z^2}$

(g) $\varphi(x, y, z) = \tan(x^4 \sin(yz))$

2. Calcule a divergência e o rotacional dos seguintes campos vetoriais:

(a) $E(x, y, z) = (y, z, x)$

(b) $E(x, y, z) = (y^3, xz, z^9)$

(c) $E(x, y, z) = (\sin(yz), \tan(y^2), \cos(xz))$

(d) $E(x, y, z) = (\ln(x^2 y), xz, e^{xy^2})$

(e) $E(x, y, z) = (x^{z^2}, \ln(xz), y)$

(f) $E(x, y, z) = (1/x, 1/y, 1/z)$

(g) $E(x, y, z) = (1/\sin(yz), \cos(xyz), e^{xy})$

(h) $E(x, y, z) = ((x^2 + y^2)/z^2, y^{z^4}, z/(x^2 + y^3))$

3. Calcule a divergência dos campos vetoriais obtidos no item 1.

4. Calcule os gradientes dos campos escalares obtidos no item 2 como divergências.

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Resultados

1. (a) $(2x \sin(xy) + x^2y \cos(xy), x^3 \cos(xy) + 2y, 0)$
(b) $(-e^y \sin(xe^y), -xe^y \sin(xe^y), 0)$
(c) $(5x^4y^2z^3, 2x^5yz^3, 3x^5y^2z^2)$
(d) $2(x, -y, -z)$
(e) $2(x, -y, -z)/(x^2 - y^2 - z^2)$
(f) $(z^2x^{z^2-1}, 0, 2zx^{z^2} \ln(x))$
(g) $(4x^3 \sin(yz), x^4z \cos(yz), x^4y \cos(yz))/\cos^2(x^4 \sin(yz))$
2. • Divergências:
 - (a) 0
 - (b) $9z^8$
 - (c) $2y/\cos^2(y^2) - x \sin(xz)$
 - (d) $2/x$
 - (e) $z^2x^{z^2-1}$
 - (f) $-1/x^2 - 1/y^2 - 1/z^2$
 - (g) $-xz \sin(xyz)$
 - (h) $2x/z^2 + z^4y^{z^4-1} + 1/(x^2 + y^3)$

• Rotacionais:

- (a) $(-1, -1, -1)$
 (b) $(-x, 0, -3y^2)$
 (c) $(0, y \cos(yz) + z \sin(xz), -z \cos(yz))$
 (d) $(2xye^{xy^2} - x, -y^2e^{xy^2}, z - 1/y)$
 (e) $(1 - 1/z, 2zxz^2 \ln(x), 1/x)$
 (f) $(0, 0, 0)$
 (g) $\left(xe^{xy} + xy \sin(xyz), -y \frac{\cos(yz)}{\sin^2(yz)} - ye^{xy}, -yz \sin(xyz) + z \frac{\cos(yz)}{\sin^2(yz)}\right)$
 (h) $\left(-\frac{3y^2z}{(x^2 + y^3)^2} - 4z^3y^{z^4} \ln(y), -\frac{2(x^2 + y^2)}{z^3} + \frac{2xz}{(x^2 + y^3)^2}, -\frac{2y}{z^2}\right)$
3. (a) $2 \sin(xy) + 4xy \cos(xy) - x^2y^2 \sin(xy) - x^4 \sin(xy) + 2$
 (b) $-e^{2y} \cos(xe^y) - xe^y \sin(xe^y) - x^2e^{2y} \cos(xe^y)$
 (c) $20x^3y^2z^3 + 2x^5z^3 + 6x^5y^2z$
 (d) -2
 (e) $-\frac{2}{x^2 - y^2 - z^2} - \frac{4(x^2 + y^2 + z^2)}{(x^2 - y^2 - z^2)^2}$
 (f) $z^2(z^2 - 1)x^{z^2-2} + 2x^{z^2} \ln(x) + 4z^2x^{z^2} \ln(x)^2$
 (g) $(12x^2 - x^4z^2 - x^4y^2) \frac{\sin(yz)}{\cos^2(x^4 \sin(yz))} + (32x^6 \sin^2(yz) + 2x^8(y^2 + z^2) \cos^2(yz)) \frac{\sin(x^4 \sin(yz))}{\cos^3(x^4 \sin(yz))}$
4. (a) $(0, 0, 0)$
 (b) $(0, 0, 72z^7)$
 (c) $(-\sin(xz) - xz \cos(xz), 2/\cos^2(y^2) + 8y^2 \sin(y^2)/\cos^3(y^2), -x^2 \cos(xz))$
 (d) $(-2/x^2, 0, 0)$
 (e) $(z^2(z^2 - 1)x^{z^2-2}, 0, 2zx^{z^2-1} + 2z^3x^{z^2-1} \ln(x))$
 (f) $(2/x^3, 2/y^3, 2/z^3)$
 (g) $(-z \sin(xyz) - xyz^2 \cos(xyz), -x^2z^2 \cos(xyz), -x \sin(xyz) - x^2yz \cos(xyz))$
 (h) $\left(\frac{2}{z^2} - \frac{2x}{(x^2 + y^3)^2}, z^4(z^4 - 1)y^{z^4-2} - \frac{3y^2}{(x^2 + y^3)^2}, -\frac{4x}{z^3} + 4z^3y^{z^4-1} + 4z^7y^{z^4-1} \ln(y)\right)$