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Edge clique graphs and some classes of chordal graphs ☆

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Abstract

The *edge clique graph* of a graph G is one having as vertices the edges of G, two vertices being adjacent if the corresponding edges of G belong to a common clique. We describe characterizations relative to edge clique graphs and some classes of chordal graphs, such as starlike, starlike-threshold, split and threshold graphs. In particular, a known necessary condition for a graph to be an edge clique graph is that the sizes of all maximal cliques and intersections of maximal cliques ought to be triangular numbers. We show that this condition is also sufficient for starlike-threshold graphs. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Edge clique graphs were introduced and first studied by Albertson and Collins in 1984 [1]. Afterwards, a few papers have been written about the subject, as those by Raychaudhuri [13,14], Chartrand et al. [3], and the article [2]. Some of the results concerning this class of graphs have been described by McKee [10] and by Prisner in a survey on line graphs [12] and in the book about graph operators [11]. It should be noted that these graphs were also implicitly used by Kou, Stockmeyer and Wong in 1978 [8].

As for characterizations of edge clique graphs for special classes of graphs, some partial results in this direction are as follows. If a graph is chordal, so is its edge clique graph [1,13,14]. A similar property applies, if the graph is strongly chordal [13], an

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interval graph [3] or a line graph [3,14]. If $\omega(G) \leqslant 3$ and G is planar, so is its edge clique graph [1]. Also, if $\chi(G) \leqslant 3$ and G is planar, then the edge clique graph of G is perfect [1]. Finally, a sufficient condition for a graph G, so that its edge clique graph remains in the same class as G, has been presented in [14].

Given the above partial results, it is reasonable to look for characterizations of edge clique graphs of subclasses of chordal graphs. In this paper, we describe characterizations relative to four such subclasses, namely starlike graphs, starlike-threshold graphs, split graphs and threshold graphs. In addition, we solve the inverse problem for these classes. The inverse problem for a given class $\mathscr{C}$ consists of characterizing which are the graphs whose edge clique graphs belong to $\mathscr{C}$. Finally, we also consider the problem of characterizing which graphs of a given class are edge clique graphs. We solve this problem for both starlike-threshold and split graphs.

Starlike graphs were introduced by Gustedt [7], split graphs by Földes and Hammer [5], while threshold graphs were introduced by Chvátal and Hammer [4]. Starlike-threshold graphs form a class between starlike and threshold graphs, and they arise naturally when studying edge clique graphs of threshold graphs. Split graphs and threshold graphs have been intensively studied. For example, see the books by Golumbic [6] and Mahadev and Peled [9].

G denotes an undirected graph, with vertex set $V(G)$ and edge set $E(G)$. For $v \in V(G)$, denote by $N_G(v)$ the set of neighbours of v in G, while $N_G[v] = \{v\} \cup N_G(v)$. When convenient, drop the index G of the notation. If $u, v \in V(G)$ are neighbours, denote by uv the edge whose ends are u and v. A vertex adjacent to no other vertex is called *isolated*. For $S \subseteq V(G)$, say that S is a *clique* when S induces a complete subgraph in G. In particular, if $N[v]$ is a clique, then v is a *simplicial* vertex. A *maximal clique* is one not properly contained in any other. In case that S induces a subgraph with no edges in G, then S is an *independent set*. On the other hand if $|S| = \binom{n}{2}$, for some $n = 0, 1, \ldots$, then S is a *triangular subset* and $|S|$ a *triangular number*.

Let G be a graph. The *edge clique graph*, $K_e(G)$, of G is the one whose vertices are the edges of G, two vertices being adjacent in $K_e(G)$, when the corresponding edges of G belong to the same clique. A necessary condition for a graph H to be the edge clique graph of some graph G is that all its maximal cliques and intersections of maximal cliques ought to be triangular. That is,

Proposition 1 (Albertson and Collins [1]). *There exists a one-to-one correspondence between maximal cliques (intersections of maximal cliques) of G and of H, whenever $H = K_e(G)$. Moreover, if C is a maximal clique (intersection of maximal cliques) of G, then the corresponding clique of H is formed by the vertices which correspond to the edges of G with both ends in C.*

An example of a graph (Fig. 1) with triangular maximal cliques and intersections of maximal cliques that is not an edge clique graph has been described in [3].

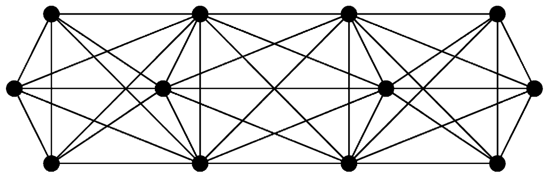

Fig. 1.

A graph G is *starlike* when there exists a partition $C, D_1, \ldots, D_s$ of its vertices, such that C is a maximal clique and, for all $u \in D_i$, $v \in D_j$, $i \neq j$ implies $uv \notin E(G)$, while if $i = j$, then $N[u] = N[v]$. In this case, $C, D_1, \ldots, D_s$ is called a *starlike partition* of G. It follows that each D_i is a clique contained in exactly one maximal clique C_i, and $D_i = C_i \backslash C$. Through this paper, $C, C_1, \ldots, C_s$ denote the maximal cliques of a starlike graph and $C, D_1, \ldots, D_s$ the corresponding starlike partition of the graph.

A *split graph* is one whose vertices can be partitioned into a clique and an independent set. It follows that if G is a split graph, then it is a starlike graph, with a starlike partition $C, D_1, \ldots, D_s$, such that $|D_i| = 1$, $1 \leqslant i \leqslant s$.

A *starlike-threshold graph* G is a starlike graph admitting an ordering of its maximal cliques $C, C_1, \ldots C_s$, such that $C \cap C_i \subseteq C \cap C_{i+1}$. By letting $D_i = C_i \setminus C$, the sequence $C, D_1, \ldots, D_s$ is a special starlike partition of G, called *starlike-threshold partition*. In addition, if $|D_i| = 1$, G is a *threshold graph*.

Let G be a graph. An *edge component* of G is a (not necessarily induced) maximal subgraph G_1 of G, in which for any pair of edges $e, f \in E(G_1)$, there exists a sequence of edges $e_1, \ldots, e_k \in E(G_1)$, such that $e_1 = e$, $e_k = f$ and e_i, e_{i+1} belong to a common clique of G. If G_1 consists of a single vertex, then it is called *trivial*. The set of non trivial edge components of G form a partition of its edges. Say that G is *edge connected* when it has at most one non trivial edge component. The following is straightforward.

Proposition 2. *Let G be a graph. Then $K_{\mathrm{e}}(G)$ is connected if and only if G is edge connected.*

Moreover, there is a one-to-one correspondence between the edge components of G and the connected components of $K_{\mathrm{e}}(G)$.

In the sequel, we look at those edge clique graphs which are a collection of disjoint complete graphs.

A *generalized block graph* is a graph whose edge components are complete subgraphs.

Proposition 3. *Let $H = K_{\mathrm{e}}(G)$. Then G is a generalized block graph if and only if H is formed by a collection of vertex disjoint triangular cliques.*

The class of starlike graphs is considered in Section 2, while starlike-threshold graphs is the subject of Section 3. For the latter class, we show that the necessary condition stating that the maximal cliques and intersections of maximal cliques of H ought to be triangular is also sufficient. Section 4 considers split and threshold graphs. Further, we formulate a characterization of the split graphs which are edge clique graphs.

Finally, the theorem below describes a characterization of threshold graphs, which will be used in the study of their edge clique graphs.

Theorem 1 (Chvátal and Hammer [4]). *A graph G is a threshold graph if and only if G is both a starlike-threshold and a split graph.*

2. Star-like graphs

In this section, we study the characterization problems related to edge clique graphs and starlike graphs. The following class of graphs will be of interest.

Let G be a graph, and $G_1,\dots,G_t$ its edge components, $t \geqslant 1$. Then G is a *generalized starlike graph* when one of the components, say G_1, is starlike, while $G_2,\dots,G_t$ are complete subgraphs.

Theorem 2. *Let $H = K_e(G)$. Then G is a generalized starlike graph if and only if H is a starlike graph.*

Proof. Let G be a generalized starlike graph and $G_1,\dots,G_t$ its edge components, $t \geqslant 1$, where G_1 is a starlike graph and $G_2,\dots,G_t$ are all complete. Without loss of generality assume that G has no isolated vertices. Let $C, D_1,\dots,D_s$ be a starlike partition of G_1, and $C, C_1,\dots,C_s$ its corresponding maximal cliques. Let $H_1 = K_e(G_1)$. By Proposition 1, H_1 has $s+1$ maximal cliques $C', C_1',\dots,C_s'$, and the vertices of C_i' are the edges of G_1 having their both ends in C_i, $1 \leqslant i \leqslant s$. Similarly for C'. Define $D_i' = C_i' \backslash C'$. We show that $C', D_1',\dots,D_s'$ is a starlike partition of H_1.

Observe that the vertices of D_i' are the edges of G_1 having at least one of its ends in D_i.

In the sequel, we show that $C', D_1',\dots,D_s'$ is a partition of $V(H_1)$. Examine the intersections of these subsets. Clearly, $C' \cap D_i' = \emptyset$, by definition. Suppose that $D_i' \cap D_j' \neq \emptyset$, $i \neq j$, and let $v \in V(H_1)$ be a vertex of this intersection. By the above observation, v is an edge of G_1 having at least one end in D_i, and at least one end in D_j. The existence of such an edge contradicts the fact that $C, D_1,\dots,D_s$ is a starlike partition of G_1. Consequently, $C', D_1',\dots,D_s'$ is indeed a partition of $V(H_1)$.

It remains to show that the partition is starlike. Let $u \in D_i'$ and $v \in D_j'$. Then u is an edge of G_1 with at least one end in D_i, while v has at least one end in D_j. If $i \neq j$, because G_1 is starlike, there is no clique of G_1 containing both edges u and v. Therefore the pair uv is not an edge of H_1. While if $i = j$, consider $z \in N_{H_1}[u]$. If $z \in C_i'$, then $z \in N_{H_1}[v]$. When $z \notin C_i'$, since $uz \in E(H_1)$, there exists C_k', such that $u, z \in C_k'$. Clearly,

$i \neq k$. Therefore $u \in C'_i \cap C'_k$. Since $u \notin C'$, it follows that $u \in D'_i \cap D'_k$, contradicting the fact that $C', D'_1, \ldots, D'_s$ is a partition. Therefore the situation $z \notin C'_i$ does not occur, meaning that $N_{H_1}[u] \subseteq N_{H_1}[v]$. Similarly, $N_{H_1}[v] \subseteq N_{H_1}[u]$. Consequently, $N_{H_1}[u] = N_{H_1}[v]$ and, by definition, H_1 is a starlike graph with partition $C', D'_1, \ldots, D'_s$.

Finally, consider the remaining edge components of G. Clearly, the graph defined by $G_2, \ldots, G_t$ is a generalized block graph. By Proposition 3, its edge clique graph is formed by a collection of vertex disjoint cliques. Consequently, H is a starlike graph with partition $C', D'_1, \ldots, D'_s, D'_{s+1}, \ldots, D'_{s+t-1}$.

Conversely, let $H = K_e(G)$ be a starlike graph. The aim is to prove that G is a generalized starlike graph. Let $H_1, \ldots, H_t$ be the connected components of H. By Proposition 2, G is formed by the edge components $G_1, \ldots, G_t$, where $H_i = K_e(G_i)$, with the possible addition of isolated vertices. Since H is starlike, at most one of its connected components, say H_1, is not complete. Examine H_1 and G_1. Let $C', D'_1, \ldots, D'_s$ be a starlike partition of H_1, and $C', C'_1, \ldots, C'_s$ its corresponding maximal cliques. By Proposition 1, G_1 has exactly $s+1$ maximal cliques $C, C_1, \ldots, C_s$, corresponding to $C', C'_1, \ldots, C'_s$, respectively. Define $D_i = C_i \setminus C$, $1 \leqslant i \leqslant s$. The following facts will be useful.

Fact 1. *$u, v \in C_i \cap C_j$, $u \neq v$ and $i \neq j \Rightarrow u, v \in C$.*

Since $u \neq v$ and $u, v \in C_i \cap C_j$, it follows that the edge uv of G is a vertex of H_1 belonging to C'_i and C'_j. Because H_1 is starlike and $i \neq j$, $C'_i \cap C'_j \subseteq C'$. Hence uv is a vertex of C', implying that $u, v \in C$.

Fact 2. *$D_i \cap D_j \neq \emptyset$ and $i \neq j \Rightarrow C \cap C_i \cap C_j = \emptyset$.*

Let $i \neq j$ and $u \in D_i \cap D_j$. Suppose there exists $v \in C \cap C_i \cap C_j$. Then $u \neq v$, because $u \notin C$. In addition, it follows that $u, v \in C_i \cap C_j$. Applying Fact 1, conclude that $u, v \in C$, contradicting $u \notin C$. Consequently, no such v may exist.

Fact 3. *$D_i \cap D_j \neq \emptyset$, $C \cap C_i \neq \emptyset$ and $i \neq j \Rightarrow C \cap C_j = \emptyset$.*

Let $i \neq j$, $u \in D_i \cap D_j$ and $x \in C \cap C_i$. Suppose there exists $y \in C \cap C_j$. By Fact 2, $x \notin C_j$. Then x, y and u are mutually adjacent vertices of G_1. That is, xu and yu are adjacent vertices of H_1. Note that xu is a vertex of D'_i, because $x, u \in C_i$ and $u \notin C$. Similarly, yu is in D'_j. Since $C', D'_1, \ldots, D'_s$ is a starlike partition of H_1, $D'_i \cap D'_j = \emptyset$, meaning that xu and yu do not belong to a common clique of G_1, a contradiction. Hence there can be no $y \in C \cap C_j$.

Fact 4. *$C \cap C_i \neq \emptyset$.*

Suppose $C \cap C_i = \emptyset$, for some i. Since H_1 is the edge clique graph of G_1, $C' \cap C'_i = \emptyset$. Because H_1 is starlike, the latter implies that H_1 is disconnected, a contradiction. Hence $C \cap C_i \neq \emptyset$.

The idea is to prove that $C, D_1, \ldots, D_s$ is a starlike partition of G_1. The following argument shows that $C, D_1, \ldots, D_s$ is a partition of $V(G_1)$. We recall that $C \cap D_i = \emptyset$, by definition. Suppose there exist i and j, such that $D_i \cap D_j \neq \emptyset$ and $i \neq j$. By Fact 3, either $C \cap C_i = \emptyset$ or $C \cap C_j = \emptyset$, contradicting Fact 4. Therefore, $C, D_1, \ldots, D_s$ is indeed a partition of $V(G_1)$.

In the sequel, examine a pair of vertices $u, v \in V(G_1)$, $u \neq v$, $u \in D_i$ and $v \in D_j$. Suppose $i \neq j$ and $uv \in E(G_1)$. Consequently, u and v belong to some maximal clique C_k. Since $u, v \notin C$, it follows $u, v \in D_k$. If $k = i$, then $D_j \cap D_k \neq \emptyset$. If $k \neq i$, the consequence is $D_i \cap D_k \neq \emptyset$. Any of these situations contradicts $C, D_1, \ldots, D_s$ to be a partition of $V(G_1)$. Therefore $uv \notin E(G_1)$. Examine the second alternative $i = j$, and consider $z \in N_{G_1}[u]$. If $z \in C_i$, then $z \in N_{G_1}[v]$. When $z \notin C_i$, since $uz \in E(G_1)$, there exists C_k, such that $u, z \in C_k$. Clearly, $i \neq k$. Therefore $u \in C_i \cap C_k$. Since $u \notin C$, it follows that $u \in D_i \cap D_k$, contradicting the fact that $C, D_1, \ldots, D_s$ is a partition. Therefore the situation $z \notin C_i$ does not occur, meaning that $N_{G_1}[u] \subseteq N_{G_1}[v]$. Similarly, $N_{G_1}[v] \subseteq N_{G_1}[u]$. Consequently, $N_{G_1}[u] = N_{G_1}[v]$, implying that G_1 is a starlike graph with partition $C, D_1, \ldots, D_s$.

Finally, consider the remaining connected components $H_2, \ldots, H_t$ of H. Each one is a complete graph. Therefore, by Proposition 3 the corresponding edge components $G_2, \ldots, G_t$ of G are also complete graphs. Consequently, G is a generalized starlike graph. This completes the proof of Theorem 2. $\square$

3. Starlike-threshold graphs

In this section we study the problems related to edge clique graphs and starlike-threshold graphs. The following definition is similar to that of the previous section, for starlike graphs.

Let G be a graph, and $G_1, \ldots, G_t$ its edge components, $t \geqslant 1$. Then G is a *generalized starlike-threshold graph* when one of the components, say G_1, is starlike-threshold, while $G_2, \ldots, G_t$ are complete subgraphs.

The proof of the following theorem is basically similar to that of Theorem 2.

Theorem 3. *Let $H = K_e(G)$. Then G is a generalized starlike-threshold graph if and only if H is a starlike-threshold graph.*

Below, we describe exactly which starlike-threshold graphs are edge clique graphs. In particular, the necessary condition for a graph to be an edge clique graph, given by Proposition 1, is shown to be sufficient for starlike-threshold graphs.

Theorem 4. *Let H be a starlike-threshold graph. Then H is an edge clique graph if and only if its maximal cliques and intersections of maximal cliques are triangular.*

Proof. We need only to prove the sufficiency. By hypothesis, H is a starlike-threshold graph in which all its maximal cliques and intersections of maximal cliques are triangular. Without loss of generality, assume that H is connected, otherwise apply the techniques already employed in Theorems 2 and 3. We show that there exists a (starlike-threshold) graph G, such that $H = K_e(G)$. Let $C', D'_1, \ldots, D'_s$ be a starlike-threshold partition of H, and $C', C'_1, \ldots, C'_s$ its corresponding maximal cliques. Since $C', C'_1, \ldots, C'_s$ and $C' \cap C'_i$ are all triangular cliques, there are positive integers c, c_i, n_i satisfying $|C'| = \binom{c}{2}$, $|C'_i| = \binom{c_i}{2}$, $|C' \cap C'_i| = \binom{n_i}{2}$, $1 \leqslant i \leqslant s$. Observe that a starlike-threshold graph is uniquely determined by the numbers s, $|C|$, $|C_i|$ and $|C \cap C_i|$, $1 \leqslant i \leqslant s$. We construct G, by describing a starlike-threshold partition $C, D_1, \ldots, D_s$ of it, with corresponding maximal cliques $C, C_1, \ldots, C_s$, where $D_i = C_i \setminus C$.

Start by defining C as a clique of cardinality c. Afterwards, define $n_0 = 0$, $C_0 = \emptyset$ and for $1 \leqslant i \leqslant s$, construct C_i as follows. Define $C \cap C_i = (C \cap C_{i-1}) \cup C_i^*$, where $C_i^* \subseteq C \setminus C_{i-1}$ and $|C_i^*| = n_i - n_{i-1}$. Further, define a clique D_i as consisting of $c_i - n_i$ new vertices. Finally, for each pair of vertices $u, v \in V(G)$, such that $u \in C \cap C_i$ and $v \in D_i$, include in G the edge uv. Consequently, C_i is a maximal clique formed by c_i vertices. The construction of G is completed.

Note that the above construction requires the existence of a clique $C_i^* \subseteq C \setminus C_{i-1}$, possibly empty, of cardinality $|C_i^*| = n_i - n_{i-1}$. However, this is true by the following simple fact. Since the partition $C', D'_1, \ldots, D'_s$ is starlike-threshold, $C' \cap C'_{i-1} \subseteq C' \cap C'_i$ and therefore $n_{i-1} \leqslant n_i$. Because $|C \cap C_{i-1}| = n_{i-1}$ and $c > n_i$ it follows that there exists $C_i^* \subseteq C$, as required. Consequently, C is a maximal clique and $C \cap C_i$ is constructed successfully. So is D_i, since $c_i > n_i$. Because n_i is a positive integer, G has no isolated vertices.

Clearly, G is a starlike graph, because $C, D_1, \ldots, D_s$ is a starlike partition of it. In addition, $C \cap C_{i-1} \subseteq C \cap C_i$. Then G is a starlike-threshold graph. It remains to show that $H = K_e(G)$. Let H'' be the edge clique graph of G. By Theorem 3, H'' is a starlike-threshold graph, with corresponding partition $C'', D''_1, \ldots, D''_s$ and maximal cliques $C'', C''_1, \ldots, C''_s$, where $D''_i = C''_i \setminus C''$, and $C'' \cap C''_{i-1} \subseteq C'' \cap C''_i$. Since $|C| = c$, $|C''| = \binom{c}{2}$. Examine each maximal clique C''_i of H''. From the definitions of $C \cap C_i$ and C_i, we derive $|C'' \cap C''_i| = \binom{n_i}{2}$ and $|C''_i| = \binom{c_i}{2}$. Comparing H and H'' we conclude that they coincide. Consequently, H is an edge clique graph. $\square$

Theorem 4 cannot be extended to starlike graphs. For example, the graph of Fig. 1 is starlike, satisfies the conditions of Theorem 4, but is not an edge clique graph.

4. Split and threshold graphs

In this section, we examine edge clique graphs of split and threshold graphs. In addition, we describe the split graphs which are edge clique graphs and we solve the inverse problem for this class. The following definitions are useful.

Let G be a graph, and $G_1, \ldots, G_t$ its edge components, $t \geqslant 1$. Then G is a *generalized split graph* when one of the components, say G_1, is split, while each of $G_2, \ldots, G_t$ consists of a single edge.

Let G be a starlike graph, with partition $C, D_1, \ldots, D_s$ and maximal cliques $C, C_1, \ldots, C_s$, where $D_i = C_i \setminus C$. Say that G is *singular* when $|C \cap C_i| = \binom{|D_i|}{2}$, for all $1 \leqslant i \leqslant s$.

Theorem 5. *Let $H = K_e(G)$. Then G is a generalized split graph if and only if H is a singular starlike graph.*

The proof of the above theorem is again similar to that of Theorem 2.

In the sequel, we examine which split graphs are edge clique graphs. We conclude that these graphs are very restricted.

Theorem 6. *Let H be a split graph. Then H is an edge clique graph if and only if H consists of a triangular clique, together with zero or more additional isolated vertices.*

Proof. Let H be a split graph and an edge clique graph. Then the vertices of H can be split into a clique C and an independent set I. Because H is an edge clique graph, C must be triangular. Suppose that the theorem is false. Then $|I| \geqslant 1$ and I is not a collection of isolated vertices. Consequently, there exists $v \in I$, such that $N_H(v) \neq \emptyset$. Clearly, $N_H[v]$ is a maximal clique of H, while $N_H(v)$ is an intersection of maximal cliques. These conditions imply that $N_H(v)$ and $N_H[v]$ must be both triangular. Since they differ by one, there are no more than two possibilities: $|N_H(v)| = 0$ or 1. The first alternative does not occur because $N_H(v) \neq \emptyset$. In the second case, $|N_H(v)| = 1$, v has a unique neighbour $w \in C$. This implies that the vertices v, w form a maximal clique of size two, meaning that it is not triangular. Hence, H is not an edge clique graph, a contradiction. Consequently, H is a triangular clique together with zero or more isolated vertices.

The converse is straightforward. □

The problem of characterizing which are the graphs whose edge clique graphs are split graphs also has a simple solution. Namely, the class is formed by the graphs with at most one edge component being a complete graph and the others consisting of a single edge or vertex.

Finally, we consider edge clique graphs and threshold graphs. The following definition is similar to the case of split graphs.

A graph with edge components $G_1, \dots, G_t$ is a *generalized threshold graph* when one of the components, say G_1, is a threshold graph and each of $G_2, \dots, G_t$ consists of a single edge.

From Theorem 1, it follows that edge clique graphs of threshold graphs can be described by combining the corresponding results for starlike-threshold and split graphs. Therefore, the theorem below is a consequence of Theorems 1, 3 and 5.

Theorem 7. *Let $H = K_e(G)$. Then G is a generalized threshold graph if and only if H is a singular starlike-threshold graph.*

5. Conclusions

We have described characterizations relative to edge clique graphs and four subclasses of chordal graphs. We have solved the inverse problem for these classes and described which starlike-threshold, split and threshold graphs are edge clique graphs. It remains to characterize which starlike graphs are edge clique graphs. We have also proved that the condition of having triangular maximal cliques and intersection of maximal cliques is sufficient for a starlike-threshold graph to be an edge clique graph. It would be interesting to identify other classes of graphs with this property.

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