

# MAC0323 Algoritmos e Estruturas de Dados II

Edição 2020 – 2

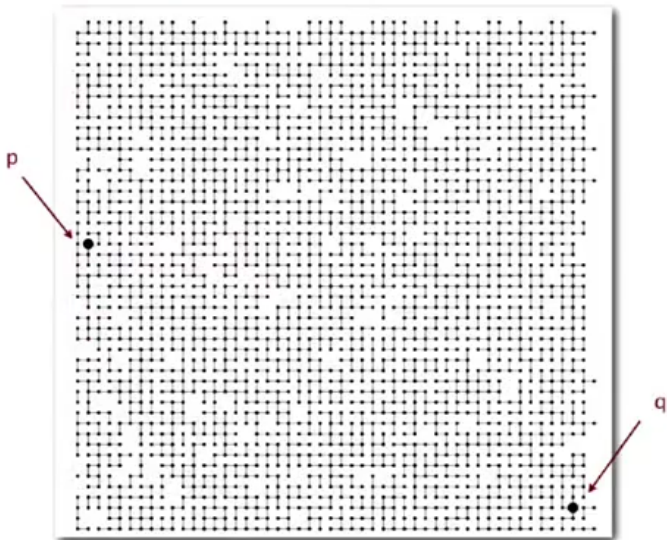


Fonte: [ash.atozviews.com](http://ash.atozviews.com)

Compacto dos melhores momentos

AULA 3

**Problema:**  $p$  e  $q$  estão ligados?

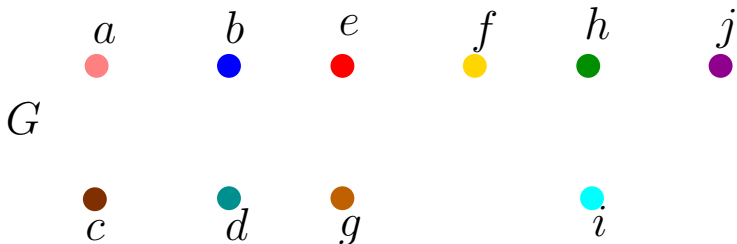


Fonte: [algs4](#)

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

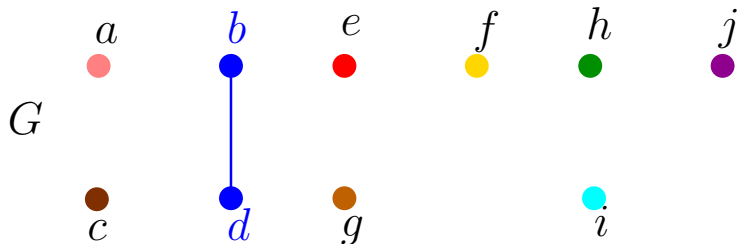
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$\{a\}$   $\{b\}$   $\{c\}$   $\{d\}$   $\{e\}$   $\{f\}$   $\{g\}$   $\{h\}$   $\{i\}$   $\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

$(b, d)$

$\{a\}$

$\{b, d\}$

$\{c\}$

$\{e\}$

$\{f\}$

$\{g\}$

$\{h\}$

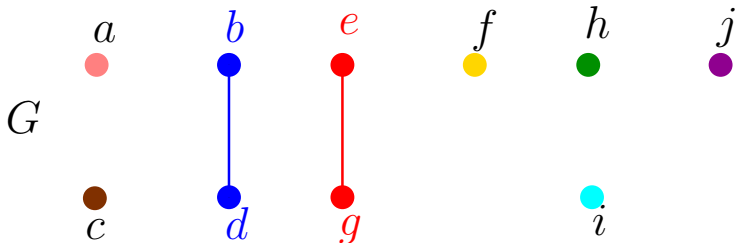
$\{i\}$

$\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

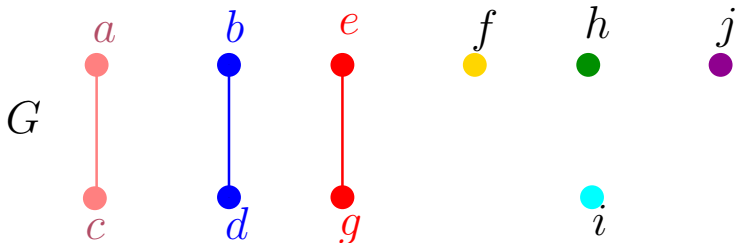
---

$(e, g)$      $\{a\}$      $\{b, d\}$      $\{c\}$      $\{e, g\}$      $\{f\}$      $\{h\}$      $\{i\}$      $\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

$(a, c)$

$\{a, c\}$

$\{b, d\}$

$\{e, g\}$

$\{f\}$

$\{h\}$

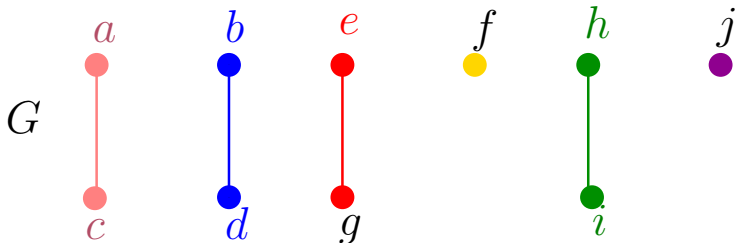
$\{i\}$

$\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

$(h, i)$

$\{a, c\}$

$\{b, d\}$

$\{e, g\}$

$\{f\}$

$\{h, i\}$

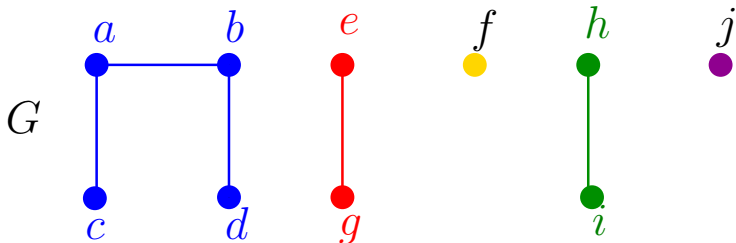
$\{j\}$



# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: **grafo dinâmico**



aresta

componentes

$(a, b)$

$\{a, b, c, d\}$

$\{e, g\}$

$\{f\}$

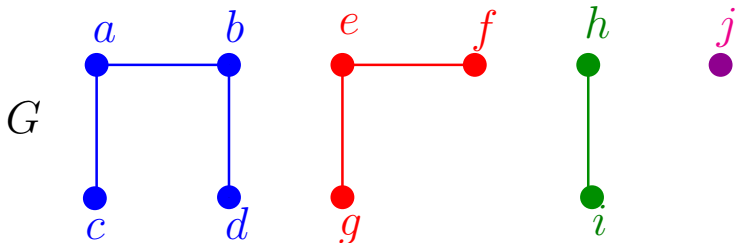
$\{h, i\}$

$\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: grafo dinâmico



aresta

componentes

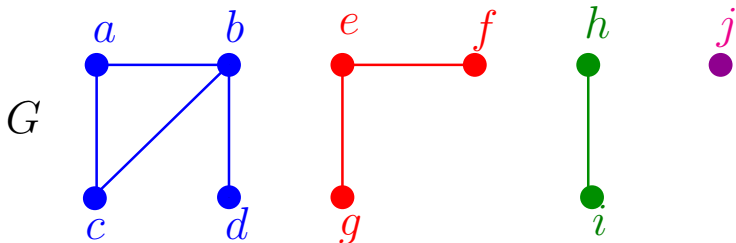
---

$(e, f)$      $\{a, b, c, d\}$      $\{e, f, g\}$      $\{h, i\}$      $\{j\}$

# Coleção disjunta dinâmica

Conjuntos são **modificados ao longo do tempo**

Exemplo: **grafo dinâmico**



aresta

componentes

$(b, c)$

$\{a, b, c, d\}$

$\{e, f, g\}$

$\{h, i\}$

$\{j\}$

# Union-find

## 1.5 Case Study: Union-Find

# Interface uf.h

---

Arquivo uf.h

---

<code>void</code>	<code>ufInit(int n)</code>	inicializa <code>n</code> sites com nomes inteiros <code>0, \dots, n-1</code>
<code>void</code>	<code>ufUnion(int p, int q)</code>	acrescenta ligação entre <code>p</code> e <code>q</code>
<code>int</code>	<code>ufFind(int p)</code>	retorna id do componente de <code>p</code>
<code>bool</code>	<code>ufConnected(int p, int q)</code>	true se <code>p</code> e <code>q</code> estão no mesmo componente
<code>int</code>	<code>ufCount()</code>	número de componentes
<code>void</code>	<code>ufFree()</code>	destroi a ED

---

# QuickFindUF

```
ufInit(10)
```

```
uf
```

```
+-----+
| id (static)  n: 10 (static)  count: 10 (static) |
| |
| |           +---+---+---+---+---+---+---+---+---+
| +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|           +---+---+---+---+---+---+---+---+---+
|           0   1   2   3   4   5   6   7   8   9
|
| Métodos:
|   count(), connected(), find(), union(), free()
|
+-----+
```

## Implementação: QuickFindUF.c

```
void ufInit(int n) {
    int i;
    id = mallocSafe(n * sizeof(int));
    for (i = 0; i < n; i++) id[i] = i;
    nUF = count = n;
}

/* retorna o id do componente de p */
int ufFind(int p) {
    return id[p];
}

/* p e q estão no mesmo componente? */
bool ufConnected(int p, int q) {
    return ufFind(p) == ufFind(q);
}
```

## Implementação: QuickFindUF.c

```
void ufUnion(int p, int q) {
    int i, p1 = ufFind(p), q1 = ufFind(q);
    if (p1 == q1) return;
    for (i = 0; i < nUF; i++)
        if (id[i] == p1) id[i] = q1;
    count--;
}

int ufCount() {
    return count;
}

void ufFree() {
    free(id);
    id = NULL;
    count = 0;
}
```



## Consumo de tempo

`ufInit`( $n$ )             $\Theta(n)$

`ufUnion`( $p$ ,  $q$ )         $O(n)$

`ufFind`( $p$ )             $\Theta(1)$

Uma sequência de  $m$  operações  
pode consumir tempo  $\Theta(m^2)$  no pior caso.

Consumo de tempo amortizado  
de cada operação é  $O(m)$ .

Hmm. Em `ufUnion()`, seria razoável alterarmos  
o **menor número possível de posições** do vetor `id`.  
Para isso precisamos saber qual conjunto  
tem o menor número de itens...





Fonte: [Pinterest](#)

# AULA 4

# QuickUnionUF

A **ideia** é trocar o indicador `id[]` do componente por um indicador do `pai[]` do elemento.

Por sua vez, se `p` é um elemento,

`pai[pai[p]]` é o **avô** de `p`

`pai[pai[pai[p]]]` é o **bisavô** de `p`,

`pai[pai[pai[pai[p]]]` é o **tataravô**,

...

# QuickUnionUF

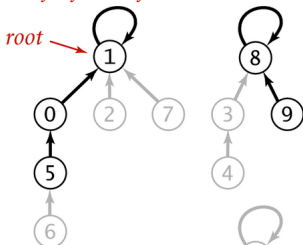
O **representante** ou **nome** de um componente será o elemento que é o pai de si mesmo.

É intuitivo representarmos a estrutura através de um conjunto de **árvores disjuntas** (= **floresta**) onde as raízes das árvores são os sítios **p** tais que

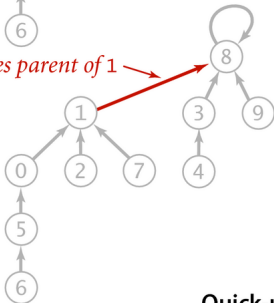
$$p == \text{pai}[p].$$

# Estrutura disjoint-set forest

*id[] is parent-link representation of a forest of trees*



*8 becomes parent of 1*



*find has to follow links to the root*

p	q	0	1	2	3	4	5	6	7	8	9
5	9	1	1	1	8	3	0	5	1	8	8

*find(5) is  
id[id[id[5]]]*

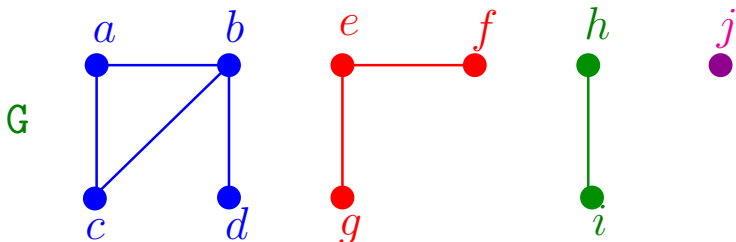
*find(9) is  
id[id[9]]*

*union changes just one link*

p	q	0	1	2	3	4	5	6	7	8	9
5	9	1	1	1	8	3	0	5	1	8	8
		1	8	1	8	3	0	5	1	8	8

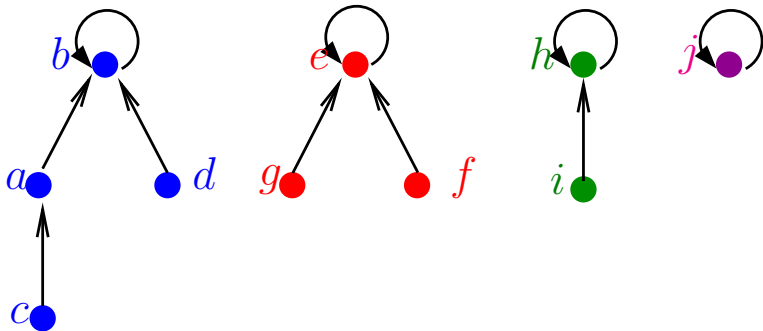
Quick-union overview

## Estrutura *disjoint-set forest*



- ▶ cada conjunto tem uma **raiz**, que é o seu representante
- ▶ cada nó **p** tem um **pai**
- ▶  $\text{pai}[p] = p$  se e só se **p** é uma raiz

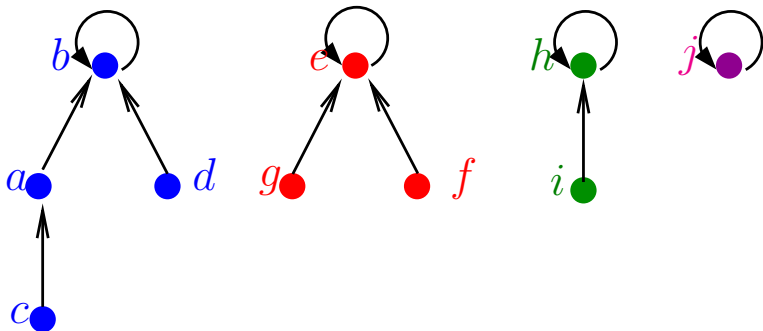
## Estrutura *disjoint-set forest*



- ▶ cada conjunto tem uma raiz
- ▶ cada nó  $p$  tem um pai
- ▶  $\text{pai}[p] = p$  se e só se  $p$  é uma raiz



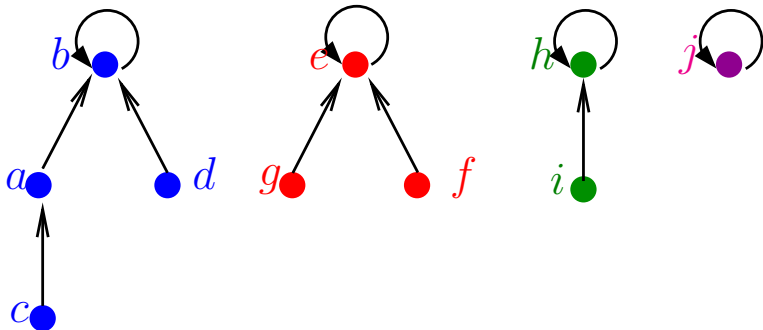
# MakeSet e FindSet



MAKESET(*p*)

1     $\text{pai}[p] \leftarrow p$

# MakeSet e FindSet



**MAKESET**(p)

1 pai[p] ← p

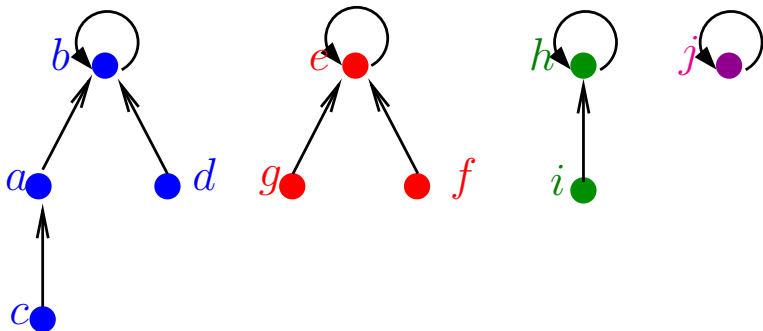
**FINDSET**(p)

1 enquanto pai[p] ≠ p faça

2 p ← pai[p]

3 devolva p

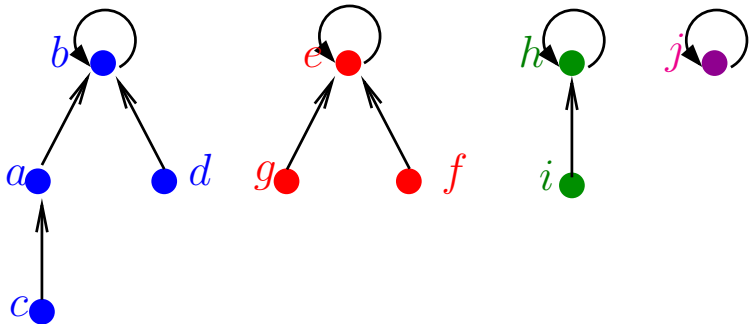
## FindSet recursivo



**FINDSET**(*p*)

- 1 se  $\text{pai}[p] = p$
- 2 então devolva *p*
- 3 senão devolva **FINDSET**( $\text{pai}[p]$ )

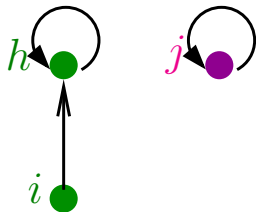
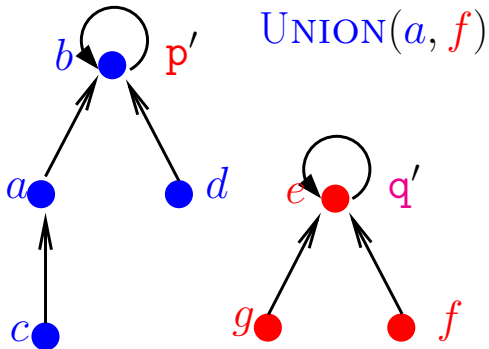
# Union



UNION( $p, q$ )

- 1  $p' \leftarrow \text{FINDSET}(p)$
- 2  $q' \leftarrow \text{FINDSET}(q)$
- 3  $\text{pai}[q'] \leftarrow p'$

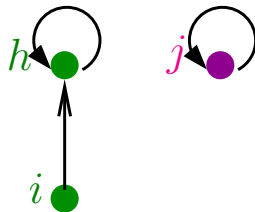
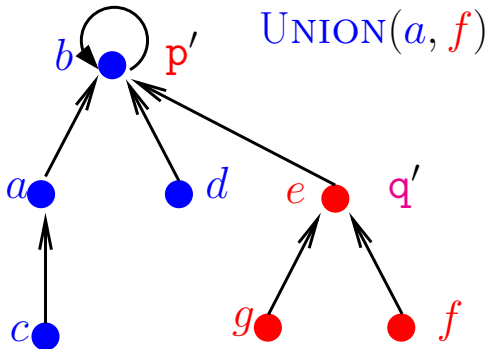
# Union



$\text{UNION}(p, q)$

- 1  $p' \leftarrow \text{FINDSET}(p)$
- 2  $q' \leftarrow \text{FINDSET}(q)$
- 3  $\text{pai}[q'] \leftarrow p'$

# Union



$\text{UNION}(p, q)$

- 1  $p' \leftarrow \text{FINDSET}(p)$
- 2  $q' \leftarrow \text{FINDSET}(q)$
- 3  $\text{pai}[q'] \leftarrow p'$

# MakeSet, Union e FindSet

MAKESET( $p$ )

1  $\text{pai}[p] \leftarrow p$

UNION( $p, q$ )

1  $p' \leftarrow \text{FINDSET}(p)$

2  $q' \leftarrow \text{FINDSET}(q)$

3  $\text{pai}[q'] \leftarrow p'$

FINDSET( $p$ )

1 **se**  $\text{pai}[p] = p$

2     **então devolva**  $p$

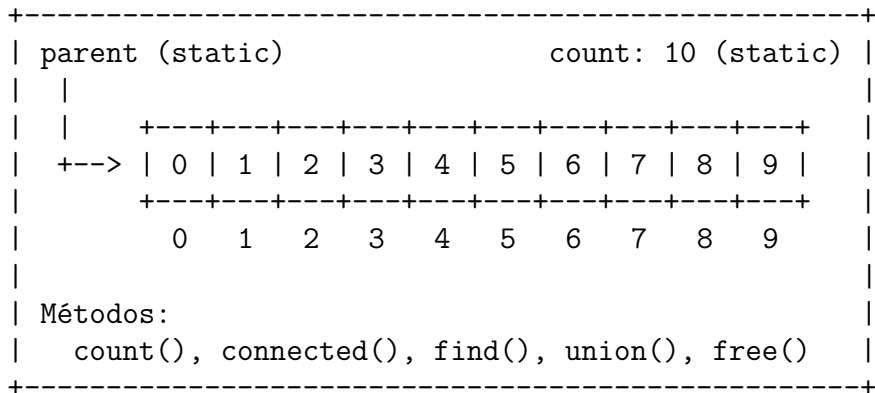
3     **senão devolva**  $\text{FINDSET}(\text{pai}[p])$

# QuickUnionUF

`ufInit(10)`



`uf`





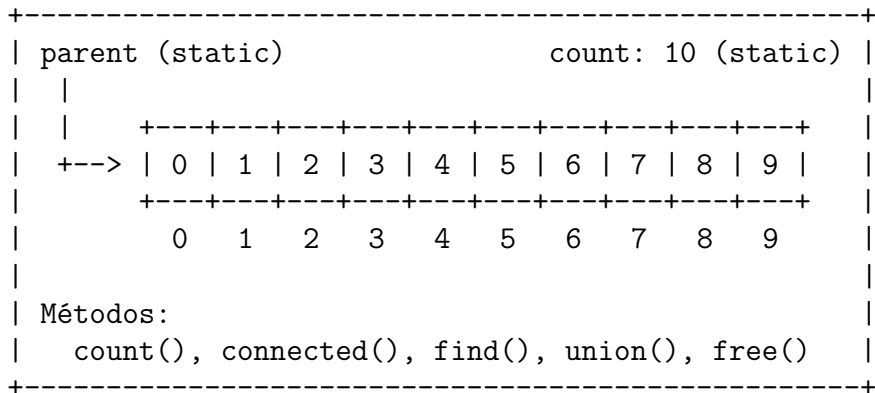
# QuickUnionUF

ufFind(3) retorna 3

ufFind(0) retorna 0

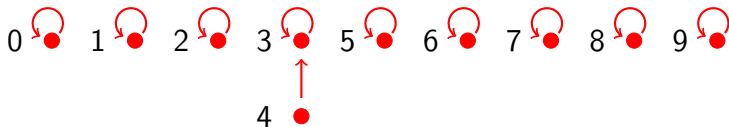


uf



# QuickUnionUF

ufUnion(4, 3)



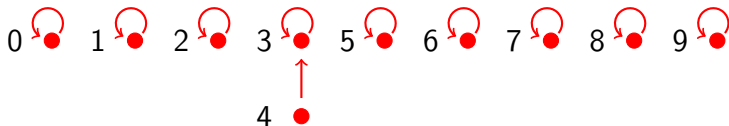
uf

```
+-----+
| parent (static)                count: 9 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 8 | 9 |
|      +---+---+---+---+---+---+---+---+---+ |
|          0   1   2   3   4   5   6   7   8   9
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# QuickUnionUF

ufFind(3) retorna 3

ufFind(4) retorna 3

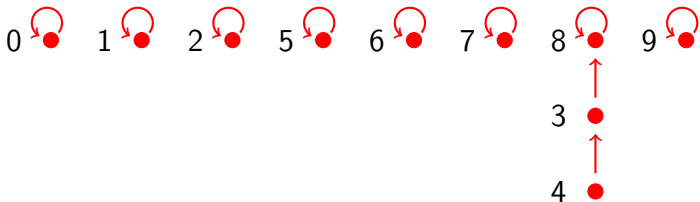


uf

```
+-----+
| parent (static)                count: 9 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 3 | 3 | 5 | 6 | 7 | 8 | 9 |
|      +---+---+---+---+---+---+---+---+---+ |
|          0   1   2   3   4   5   6   7   8   9
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# QuickUnionUF

ufUnion(3, 8)

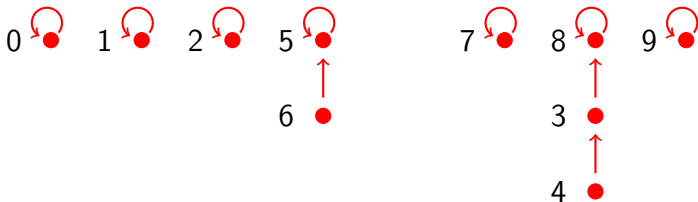


uf

-----										
parent (static)	count: 8 (static)									
0	1	2	8	3	5	6	7	8	9	
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										

# QuickUnionUF

ufUnion(6, 5)

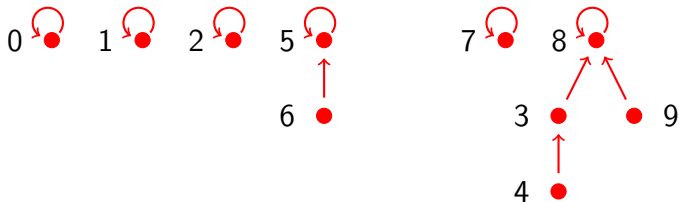


uf

uf											
parent (static)											count: 7 (static)
parent	0	1	2	8	3	5	5	7	8	9	
parent	0	1	2	3	4	5	6	7	8	9	
Métodos:											
count(), connected(), find(), union(), free()											

# QuickUnionUF

ufUnion(9, 4)

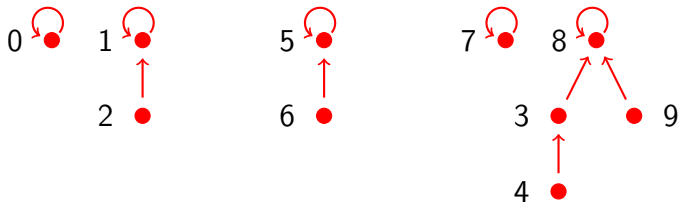


uf

-----										
parent (static)	count: 6 (static)									
0	1	2	8	3	5	5	7	8	8	
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										

# QuickUnionUF

ufUnion(2, 1)

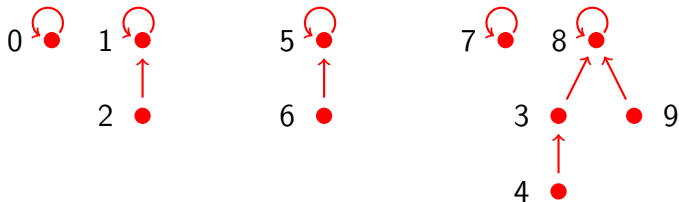


uf

-----										
parent (static)	count: 5 (static)									
0	1	1	8	3	5	5	7	8	8	
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										

# QuickUnionUF

ufUnion(8, 9)



uf

-----										
parent (static)	count: 5 (static)									
0	1	1	8	3	5	5	7	8	8	
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										



# QuickUnionUF

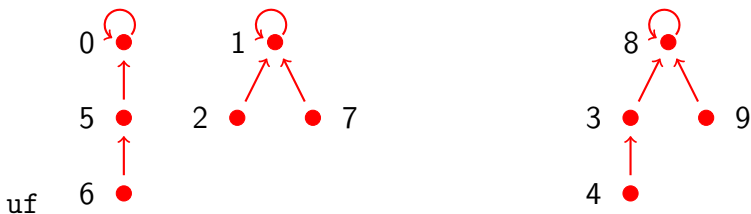
ufUnion(5, 0)



-----										
parent (static)						count: 4 (static)				
0	1	1	8	3	0	5	7	8	8	
-----										
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										

# QuickUnionUF

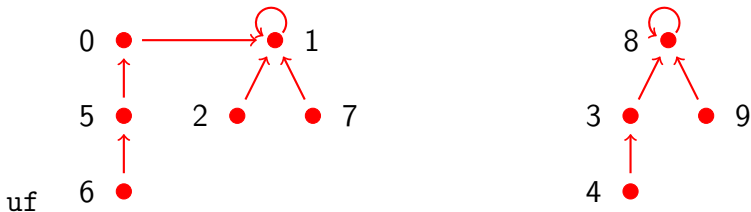
ufUnion(7, 2)



-----									
parent (static)	count: 3 (static)								
0	1	1	8	3	0	5	1	8	8
0	1	2	3	4	5	6	7	8	9
Métodos:									
count(), connected(), find(), union(), free()									
-----									

# QuickUnionUF

ufUnion(6, 1)



-----										
parent (static)	count: 2 (static)									
1	1	1	1	8	3	0	1	1	8	8
0	1	2	3	4	5	6	7	8	9	
Métodos:										
count(), connected(), find(), union(), free()										
-----										

## Arquivo QuickFindUF.c: esqueleto

```
#include "uf.h"

static int *pai;
static int count; /* no. componentes */

void ufInit(int n) {...}
void ufUnion(int p, int q) {...}
int ufFind(int p) {...}
bool ufConnected(int p, int q) {...}
int ufCount() {...}
void ufFree() {...}
```

## Implementação: QuickUnionUF.c

```
void ufInit(int n) {
    int i;
    pai = mallocSafe(n * sizeof(int));
    for (i = 0; i < n; i++) pai[i] = i;
    count = n;
}

/* retorna o id do componente de p */
int ufFind(int p) {
    while (pai[p] != p) p = pai[p];
    return p;
}
```

## Implementação: QuickUnionUF.c

```
bool ufConnected(int p, int q) {  
    return ufFind(p) == ufFind(q);  
}  
  
void ufUnion(int p, int q) {  
    int p1 = ufFind(p), q1 = ufFind(q);  
    if (p1 != q1) return;  
    pai[q1] = p1;  
    count--;  
}
```

## Implementação: QuickUnionUF.c

```
bool ufConnected(int p, int q) {
    return ufFind(p) == ufFind(q);
}

void ufUnion(int p, int q) {
    int p1 = ufFind(p), q1 = ufFind(q);
    if (p1 != q1) return;
    pai[q1] = p1;
    count--;
}

int ufCount() {
    return count;
}

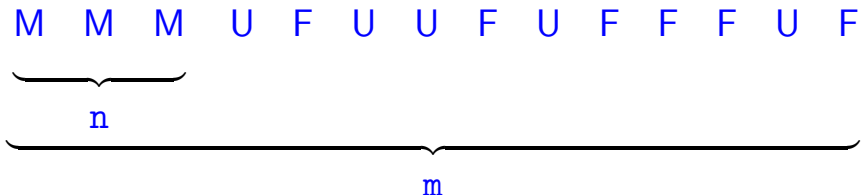
void ufFree() {
    free(pai);
    pai = NULL;
    count = 0;
}
```

## Consumo de tempo

ufInit( $n$ )             $\Theta(n)$

ufFind( $p$ )             $O(n)$

ufUnion( $p, q$ )         $O(n)$



Custo total da sequência:

$$n \Theta(1) + m O(n) + n O(n) = O(mn)$$



# Experimentos

```
% time client < tinyUF.txt  
2 components  
0.003seg
```

```
% time client < mediumUF.txt  
3 components  
0.005seg
```

```
% time client < largeUF.txt  
:-(
```

# WeightedQuickUnionUF

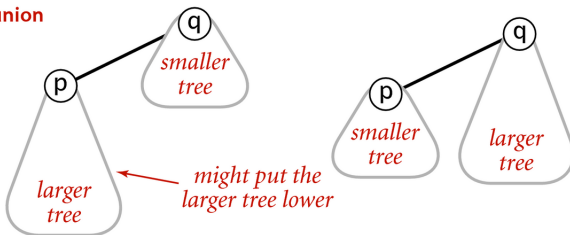
Ideia:

ligar a raiz da árvore com menos elementos  
na raiz da árvore com mais elementos.

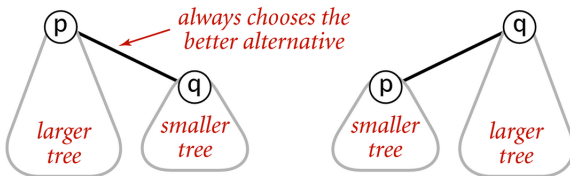
Isso seria a política natural para  
tornarmos o quick-find mais eficiente.

# WeightedQuickUnionUF

quick-union

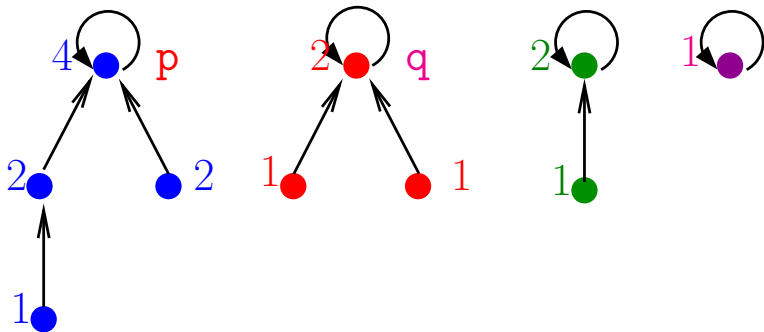


weighted



Weighted quick-union

## union by rank



$\text{rank}[p] = \text{posto do nó } p$

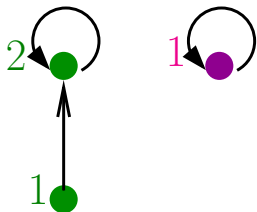
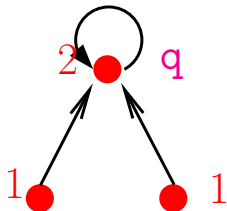
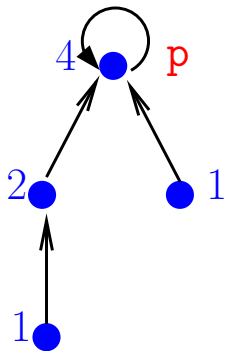
**MAKESET** ( $p$ )

1  $\text{pai}[p] \leftarrow p$

2  $\text{rank}[p] \leftarrow 0$

## union by rank

UNION (p, q)



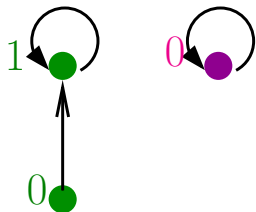
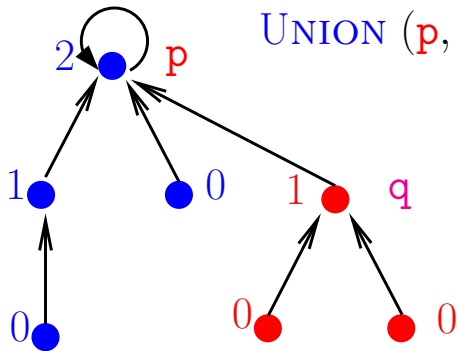
$\text{rank}[p] = \text{posto do nó } p$

MAKESET (p)

1  $\text{pai}[p] \leftarrow p$

2  $\text{rank}[p] \leftarrow 0$

## union by rank



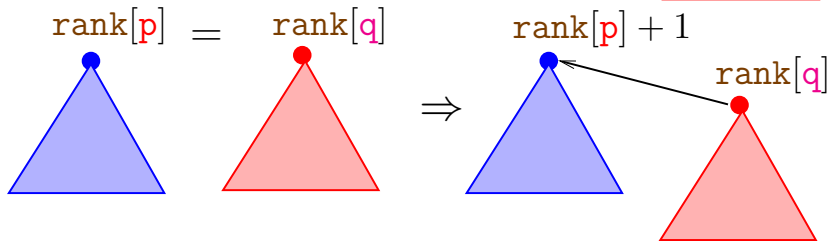
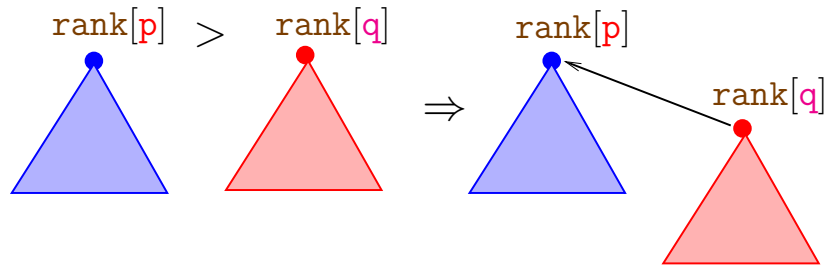
$\text{rank}[p] = \text{posto do nó } p$

MAKESET (p)

1  $\text{pai}[p] \leftarrow p$

2  $\text{rank}[p] \leftarrow 0$

## union by rank



## union by rank

UNION (p, q)  $\triangleright$  com "union by rank"

1  $p' \leftarrow \text{FINDSET}(p)$

2  $q' \leftarrow \text{FINDSET}(q)$   $\triangleright$  supõe que  $p' \neq q'$

3 se  $\text{rank}[p'] > \text{rank}[q']$

4     então  $\text{pai}[q'] \leftarrow p'$

5     senão  $\text{pai}[p'] \leftarrow q'$

6             se  $\text{rank}[p'] = \text{rank}[q']$

7                 então  $\text{rank}[q'] \leftarrow \text{rank}[q'] + 1$



## Variante: *union by size*

`size[p]`: número de nós na árvore enraizada em `p`

**UNION** (`p`, `q`)  $\triangleright$  com “union by size”

```
1  p'  $\leftarrow$  FINDSET (p)
2  q'  $\leftarrow$  FINDSET (q)  $\triangleright$  supõe que p'  $\neq$  q'
3  se size[p']  $>$  size[q']
4      então pai[q']  $\leftarrow$  p'
5          size[p']  $\leftarrow$  size[p'] + size[q']
6  senão pai[p']  $\leftarrow$  q'
7          size[q']  $\leftarrow$  size[q'] + size[p']
```

# WeightedQuickUnionUF

```
ufInit(10)
```

```
uf
```

```
+-----+
| parent (static)                count: 10 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# WeightedQuickUnionUF

ufFind(3) retorna 3

ufFind(0) retorna 0

uf

```
+-----+
| parent (static)                count: 10 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|      +---+---+---+---+---+---+---+---+---+ |
|          0  1  2  3  4  5  6  7  8  9
|
| rank (static)
| | +---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|      +---+---+---+---+---+---+---+---+---+ |
|          0  1  2  3  4  5  6  7  8  9
|
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# WeightedQuickUnionUF

ufUnion(4, 3)

uf

```
+-----+
| parent (static)                count: 9 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 4 | 4 | 5 | 6 | 7 | 8 | 9 |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# WeightedQuickUnionUF

ufUnion(3, 8)

uf

```
+-----+
| parent (static)                count:  8 (static) |
| |   +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 4 | 4 | 5 | 6 | 7 | 4 | 9 |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| rank (static)
| |   +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9
|
| Métodos:
|   count(), connected(), find(), union(), free()
+-----+
```

# WeightedQuickUnionUF

ufUnion(6, 5)

uf

```
+-----+
| parent (static)                count: 7 (static) |
| | +---+---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 4 | 4 | 6 | 6 | 7 | 4 | 9 | |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
|
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
|
| Métodos:
|   count(), connected(), find(), union(), free() |
+-----+
```

# WeightedQuickUnionUF

ufUnion(9, 4)

uf

```
+-----+
| parent (static)                count: 6 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 2 | 4 | 4 | 6 | 6 | 7 | 4 | 4 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
|
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
|
| Métodos:
|   count(), connected(), find(), union(), free() |
+-----+
```

# WeightedQuickUnionUF

ufUnion(2, 1)

uf

```
+-----+
| parent (static)                count: 5 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 4 | 4 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| | | | | | | | | | | | | | | | | | | | | | | | |
| rank (static)                  |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| | | | | | | | | | | | | | | | | | | | | | | |
| Métodos:                       |
|   count(), connected(), find(), union(), free() |
+-----+
```



# WeightedQuickUnionUF

ufUnion(8, 9)

uf

```
+-----+
| parent (static)                count: 4 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 4 | 4 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| | | | | | | | | | | | | | | | | | | | | | | | |
| rank (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| | | | | | | | | | | | | | | | | | | | | | | |
| Métodos: |
|   count(), connected(), find(), union(), free() |
+-----+
```

# WeightedQuickUnionUF

```
ufUnion(5, 0)
```

```
uf
```

```
+-----+
| parent (static)                count: 3 (static) |
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 6 | 1 | 1 | 4 | 4 | 6 | 6 | 7 | 4 | 4 | |
|       +---+---+---+---+---+---+---+---+---+ |
|       0  1  2  3  4  5  6  7  8  9 |
| |
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+ |
|       0  1  2  3  4  5  6  7  8  9 |
| |
| Métodos:
|   count(), connected(), find(), union(), free() |
+-----+
```



# WeightedQuickUnionUF

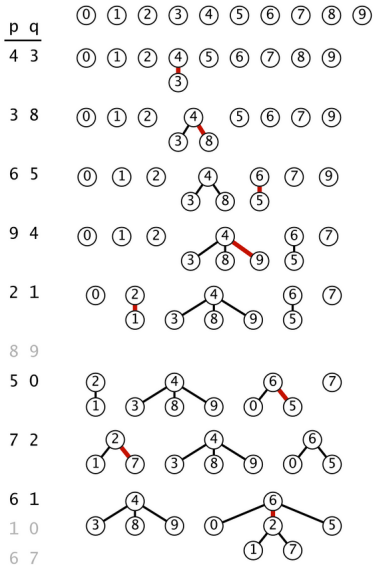
ufUnion(6, 1)

uf

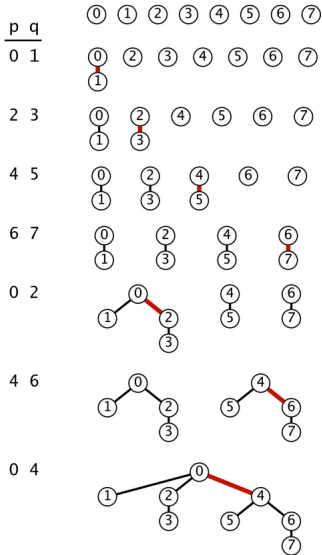
```
+-----+
| parent (static)                count:  2 (static) |
| | +---+---+---+---+---+---+---+---+---+---+---+ |
| +--> | 6 | 6 | 1 | 4 | 4 | 6 | 6 | 1 | 4 | 4 | |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| |
| rank (static)
| | +---+---+---+---+---+---+---+---+---+---+---+ |
| +--> | 0 | 1 | 0 | 0 | 1 | 0 | 2 | 0 | 0 | 0 | |
|       +---+---+---+---+---+---+---+---+---+---+ |
|         0  1  2  3  4  5  6  7  8  9 |
| |
| Métodos:
|   count(), connected(), find(), union(), free() |
+-----+
```

# Simulação

reference input



worst-case input



Weighted quick-union traces (forests of trees)

## Implementação: WeightedQuickUnionUF.c

```
#include "uf.h"  
static int *pai;  
static int *rank;  
static int count;
```

## Implementação: WeightedQuickUnionUF.c

```
#include "uf.h"
static int *pai;
static int *rank;
static int count;

void ufInit(int n) {
    int i;
    pai = mallocSafe(n * sizeof(int));
    rank = mallocSafe(n * sizeof(int));
    for (i = 0; i < n; i++) {
        pai[i] = i;
        rank[i] = 0;
    }
    count = n;
}
```

## Implementação: WeightedQuickUnionUF.c

```
void ufUnion(int p, int q) {
    int p1 = ufFind(p), q1 = ufFind(q);
    if (rank[p1] > rank[q1])
        pai[q1] = p1;
    else {
        pai[p1] = q1;
        if (rank[p1] == rank[q1])
            rank[q1]++;
    }
    count--;
}
```



## Implementação: WeightedQuickUnionUF.c

```
/* retorna o id do componente de p */  
int ufFind(int p) {  
    while (pai[p] != p) p = pai[p];  
    return p;  
}
```

## Implementação: WeightedQuickUnionUF.c

```
/* retorna o id do componente de p */
int ufFind(int p) {
    while (pai[p] != p) p = pai[p];
    return p;
}

/* p e q estão no mesmo componente? */
bool ufConnected(int p, int q) {
    return ufFind(p) == ufFind(q);
}

int ufCount() {
    return count;
}

void ufFree() {
    free(pai);
    free(rank);
    pai = rank = NULL;
    count = 0;
}
```

## Estrutura *disjoint-set forest*

Para verificar que o consumo de tempo de `ufUnion()` e `ufFind()` é não superior a  $\lg n$ , basta demonstrar que

*Na floresta de árvores disjuntas produzida durante uma sequência de operações `ufUnion()`, toda árvore com altura  $h$  tem pelo menos  $2^h$  nós.*

A demonstração é por indução no número de operações `ufUnion()` realizadas.

## Estrutura *disjoint-set forest*

**Base:** Inicialmente nenhuma operação `ufUnion()` foi realizada e toda árvore tem altura zero e possui um nó. Logo vale a afirmação.

**Passo:** Sejam  $p$  e  $q$  elementos e considere a operação `ufUnion( $p$ ,  $q$ )`.

Se  $p$  e  $q$  estão em uma mesma árvore, então não há o que demonstrar.

Portanto, podemos supor que a árvore  $T_p$  que contém  $p$  e a árvore  $T_q$  que contém  $q$  são distintas.

## Estrutura *disjoint-set forest*

Sejam

- ▶  $h_p$  e  $n_p$  a altura e número de nós de  $T_p$  e
- ▶  $h_q$  e  $n_q$  a altura e número de nós de  $T_q$ .

Pela hipótese de indução,  $n_p \geq 2^{h_p}$  e  $n_q \geq 2^{h_q}$ .

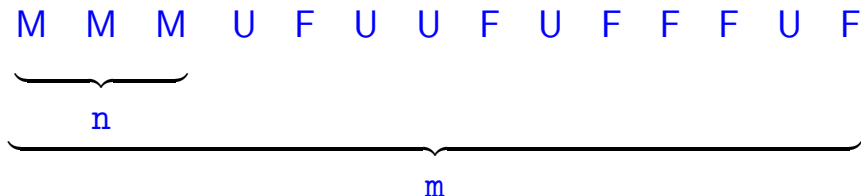
Seja  $T$  a árvore de altura  $h$  resultante da operação  $ufUnion(p, q)$ . Se  $h \leq \max\{h_p, h_q\}$ , não há o que demonstrar. Assim, podemos supor que, digamos,  $n_p \geq n_q$  e  $h = h_q + 1$ . Logo,

$$n = n_p + n_q \geq n_q + n_q \geq 2^{h_q} + 2^{h_q} = 2^{h_q+1} = 2^h,$$

o que encerra este rascunho de demonstração.

## Consumo de tempo

<code>ufInit(n)</code>	$\Theta(n)$
<code>ufFind(p)</code>	$O(\lg n)$
<code>ufUnion(p, q)</code>	$O(\lg n)$

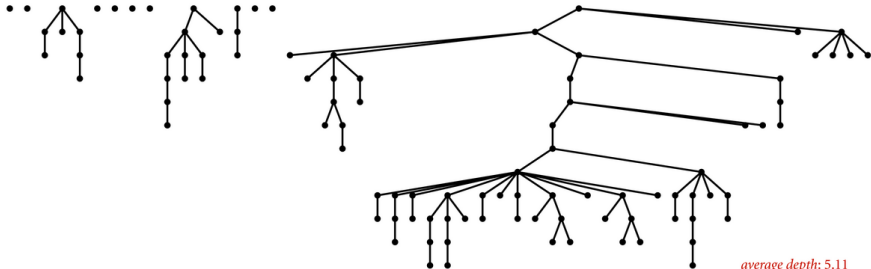


Custo total da sequência:

$$\Theta(n) + m O(\lg n) + n O(\lg n) = O(m \lg n)$$

# Ilustração

quick-union



weighted



Quick-union and weighted quick-union (100 sites, 88 union() operations)

# Experimentos

```
% time client < tinyUF.txt
```

```
2 components
```

```
0.0003seg
```

```
% time client < mediumUF.txt
```

```
3 components
```

```
0.005seg
```

```
% time client < largeUF.txt
```

```
6 components
```

```
3.726seg
```



## Encurtamento de caminhos

Acrescentando uma linha a `ufFind()`  
encurtamos o comprimento do caminho à metade.

```
int ufFind(int p) {
    while (p != pai[p]) {
        /* encurta caminho à metade */
        pai[p] = pai[pai[p]];
        p = pai[p];
    }
    return p;
}
```

## Mais experimentos

```
% time client < tinyUF.txt
```

```
2 components
```

```
0.0003seg
```

```
% time client < mediumUF.txt
```

```
3 components
```

```
0.004seg
```

```
% time client < largeUF.txt
```

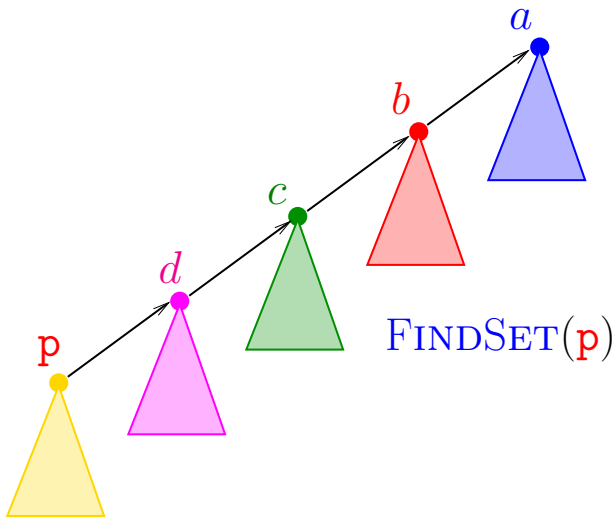
```
6 components
```

```
3.511seg
```

## Path compression

**Ideia:**

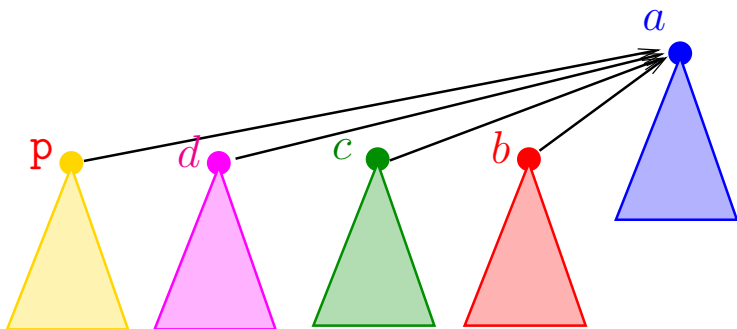
encurtar os caminhos durante cada `ufFind()`.



## Path compression

**Ideia:**

encurtar os caminhos durante cada `ufFind()`.



`FINDSET(p)`