AULA 16

Intercalação

Fonte: http://csunplugged.org/sorting-algorithms

PF 9

http://www.ime.usp.br/~pf/algoritmos/aulas/mrgsrt.html

Intercalação

Problema: Dados $\texttt{v}[\texttt{p}\,..\,\texttt{q}-1]$ e $\texttt{v}[\texttt{q}\,..\,\texttt{r}-1]$ crescentes, rearranjar $\texttt{v}[\texttt{p}\,..\,\texttt{r}-1]$ de modo que ele fique em ordem crescente.

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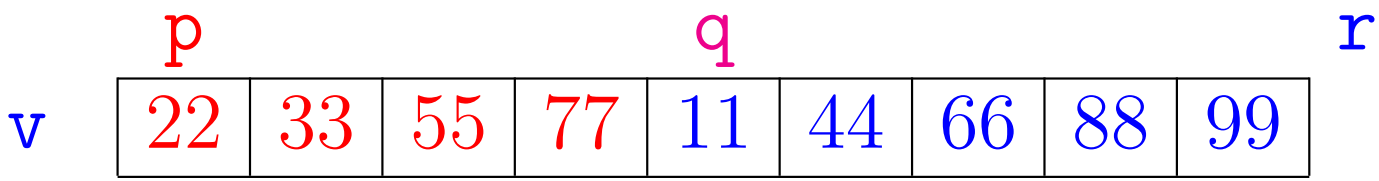

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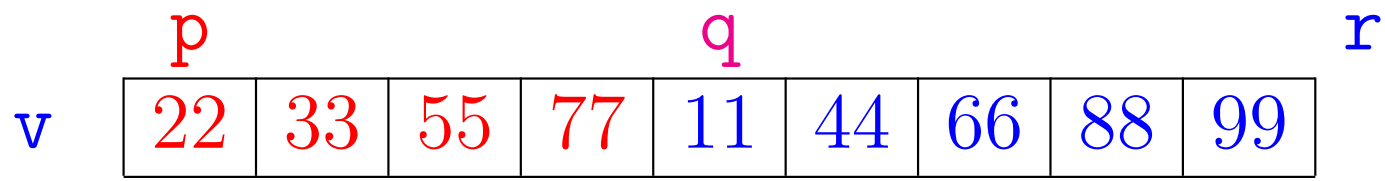
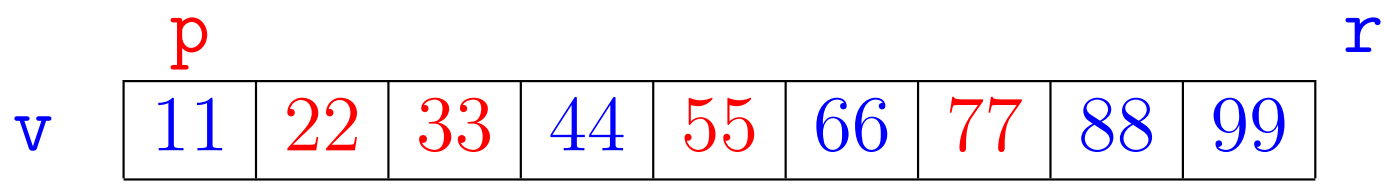


Sai:



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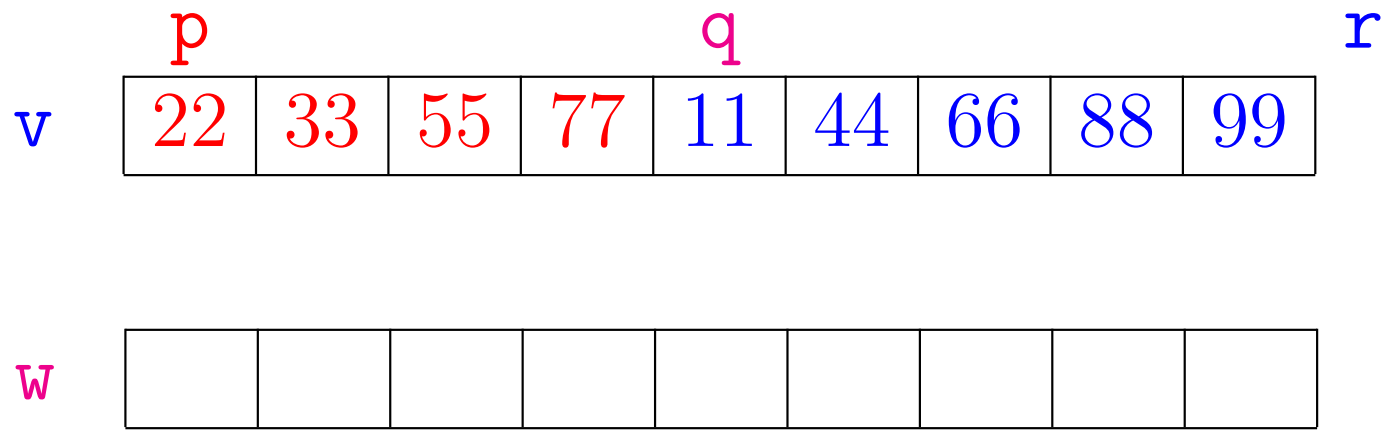

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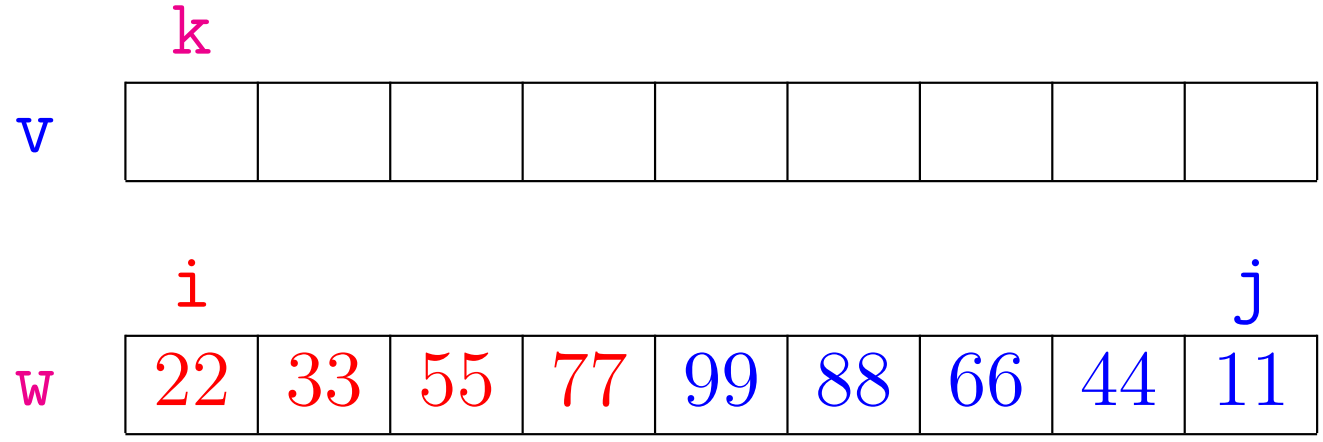

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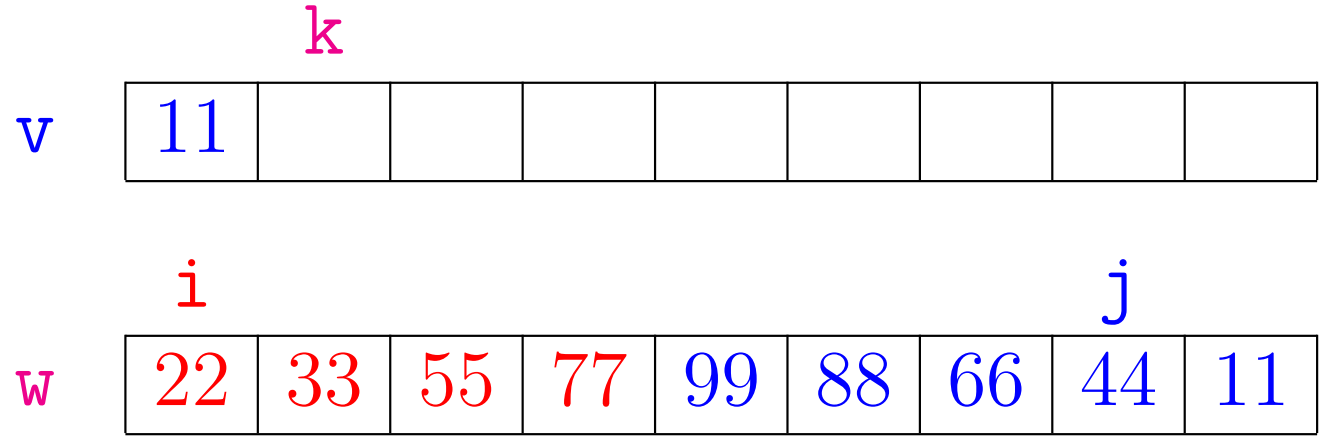

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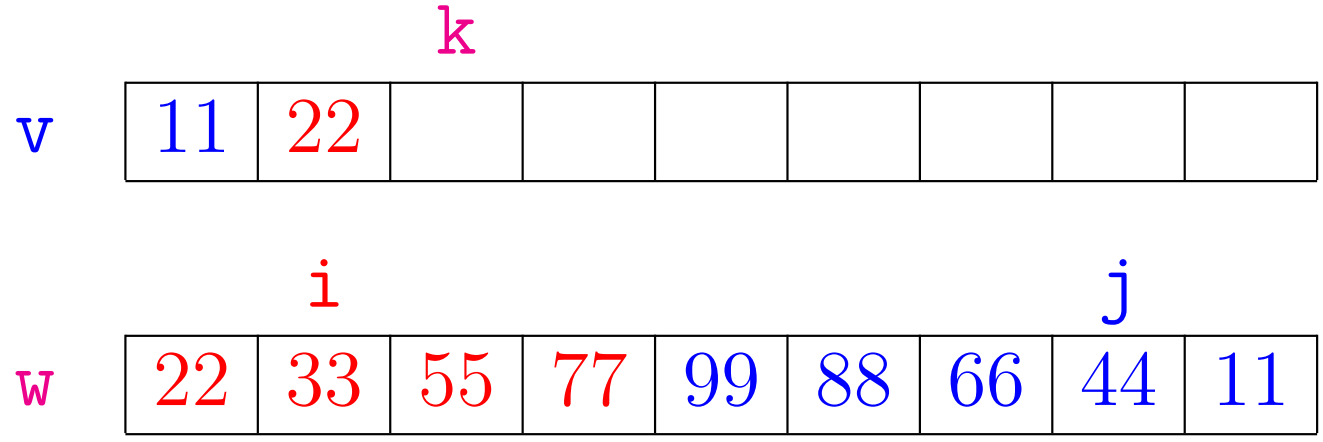

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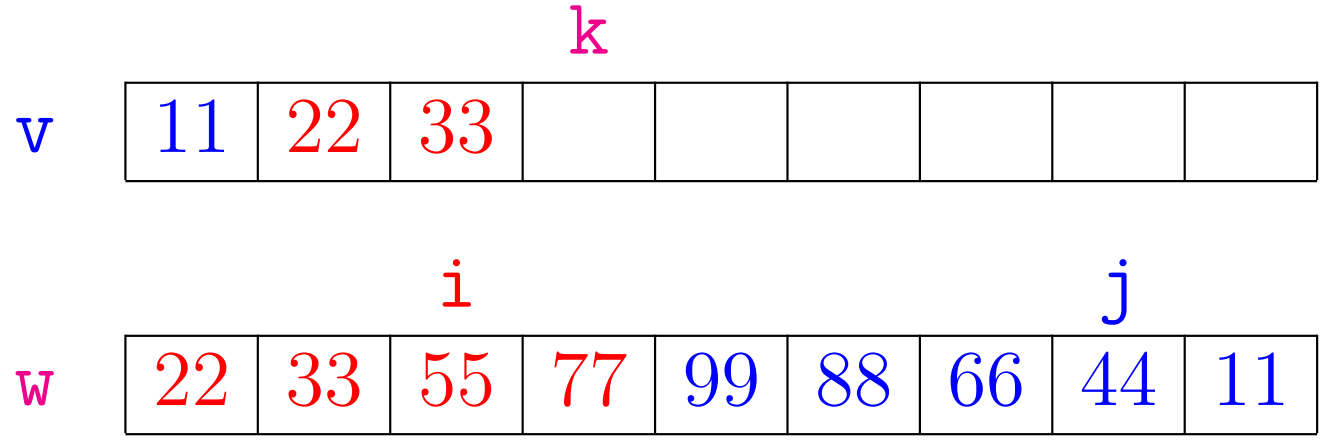

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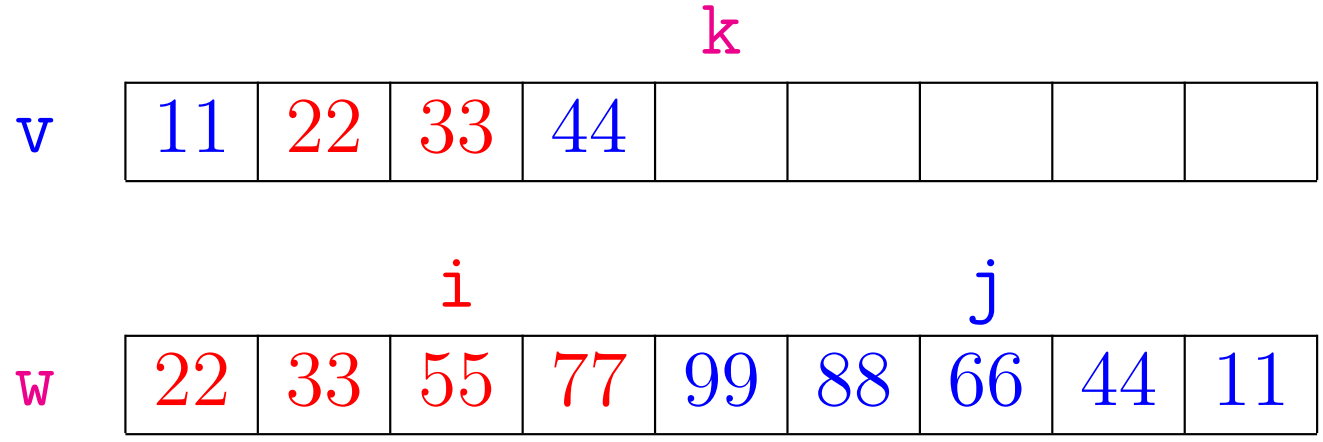

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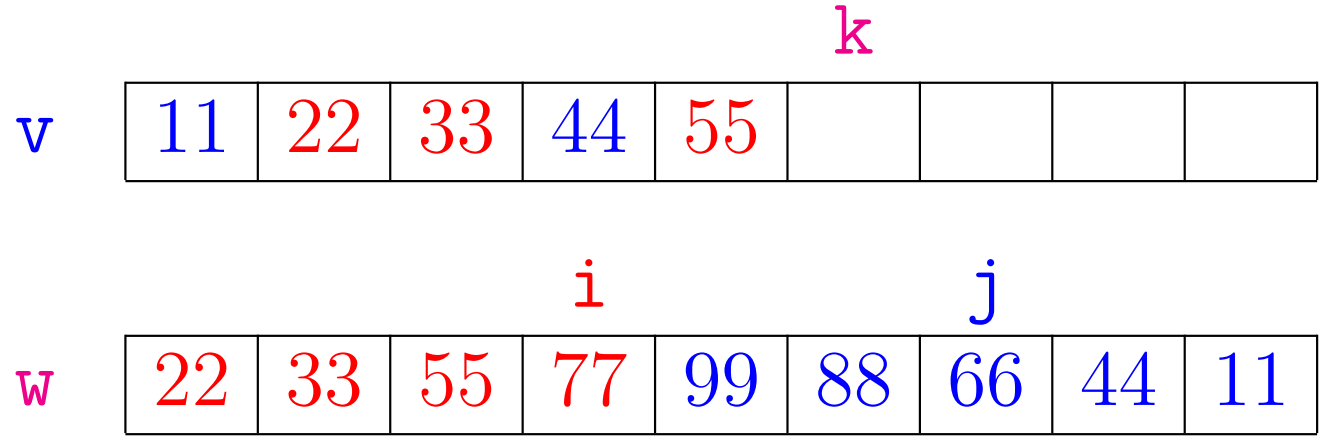

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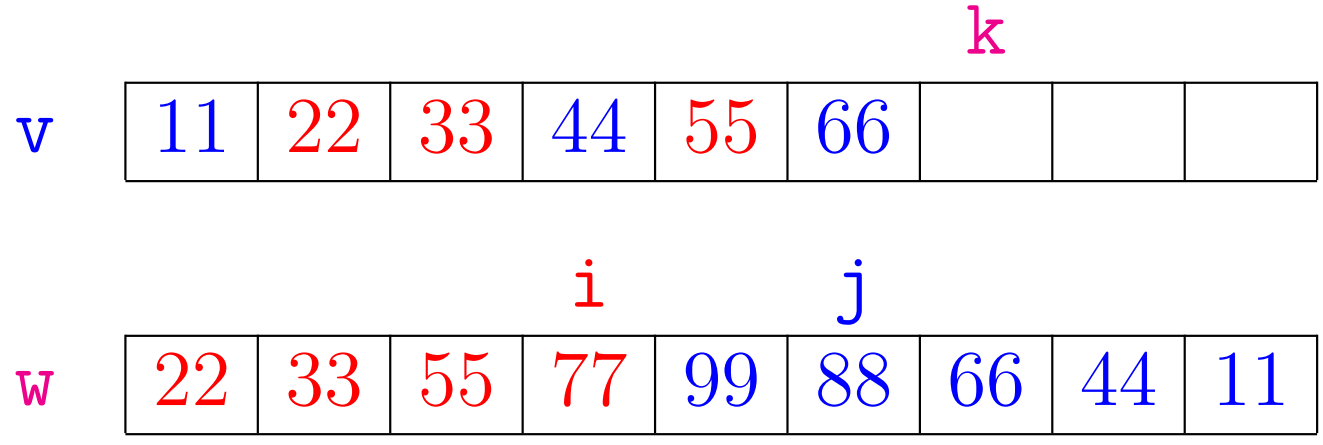

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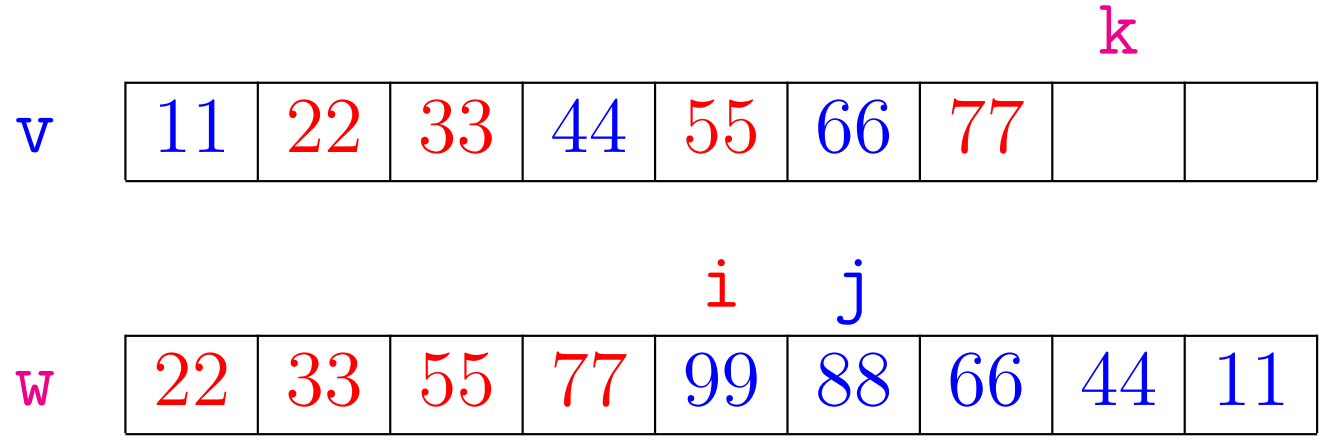

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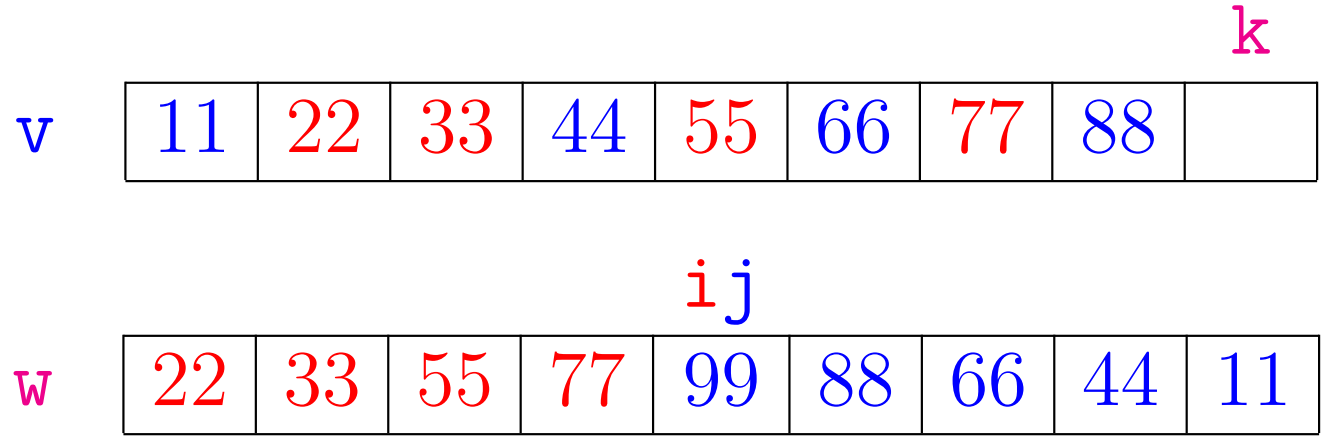

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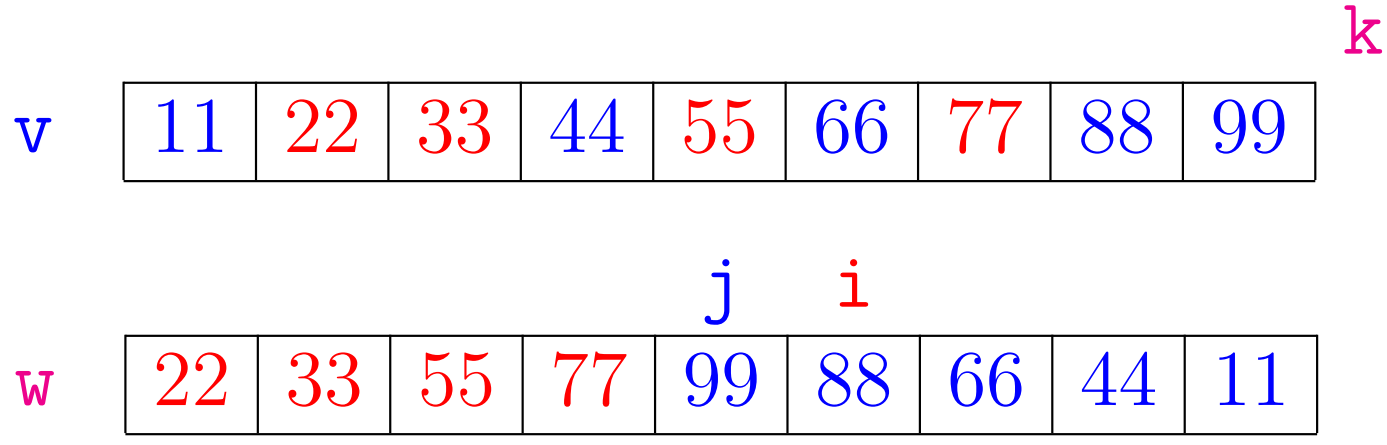

Intercalação

```
void intercala(int p,int q,int r,int v[]) {
    int i, j, k, *w;
    w = mallocSafe((r-p)*sizeof(int));
1   for (i = 0, k = p; k < q; i++, k++)
2      w[i] = v[k];
3   for (j = r-p-1; k < r; j--, k++)
4      w[j] = v[k];
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3   for (j = r-p-1; k < r; j--, k++)
4      w[j] = v[k];
5   i = 0; j = r-p-1;
6   for (k = p; k < r; k++)
7      if (w[i] <= w[j])
8         v[k] = w[i++];
9      else v[k] = w[j--];
    free (w);
}
```

Consumo de tempo

Se a execução de cada linha de código consome 1 unidade de tempo, o consumo total é:

linha	execuções da linha
1	?
2	?
3	?
4	?
5	?
6	?
7	?
8–9	?
total	?

Consumo de tempo

Se a execução de cada linha de código consome 1 unidade de tempo, o consumo total é ($\mathtt{n} := \mathtt{r} - \mathtt{p}$):

linha	execuções da linha		
1	$= \mathtt{q} - \mathtt{p} + 1$		
2	$= \mathtt{q} - \mathtt{p}$		
3	$= \mathtt{r} - \mathtt{q} + 1$		
4	$= \mathtt{r} - \mathtt{q}$		
5	$= 1$		
6	$= \mathtt{r} - \mathtt{p} + 1$	$= \mathtt{n} + 1$	
7	$= \mathtt{r} - \mathtt{p}$	$= \mathtt{n}$	
8–9	$= \mathtt{r} - \mathtt{p}$	$= \mathtt{n}$	
total	$= 5\mathtt{n} + 4$		

Conclusão

A função `intercala` consome $5\mathtt{n} + 4$ unidades de tempo.

O algoritmo `intercala` consome $\mathrm{O}(\mathtt{n})$ unidades de tempo.

Também escreve-se

O algoritmo `intercala` consome tempo $\mathrm{O}(\mathtt{n})$.

Ordenação: algoritmo Mergesort

Fonte: https://www.youtube.com/watch?v=XaqR3G_NVoo

PF 9

http://www.ime.usp.br/~pf/algoritmos/aulas/mrgsrt.html

Ordenação

$\mathtt{v}[0\,..\,\mathtt{n}-1]$ é crescente se $\mathtt{v}[0] \le \cdots \le \mathtt{v}[\mathtt{n}-1]$.

Problema: Rearranjar um vetor $\mathtt{v}[0\,..\,\mathtt{n}-1]$ de modo que ele fique crescente.

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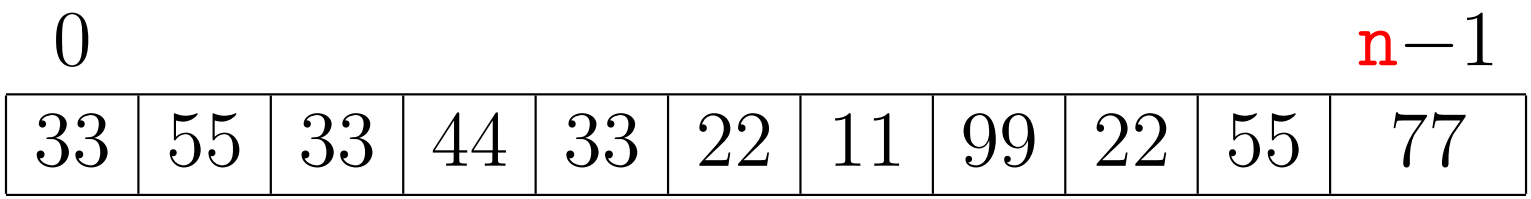

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Entra:

0										$\mathtt{n}-1$
33	55	33	44	33	22	11	99	22	55	77

Sai:

0										$\mathtt{n}-1$
11	22	22	33	33	33	44	55	55	77	99

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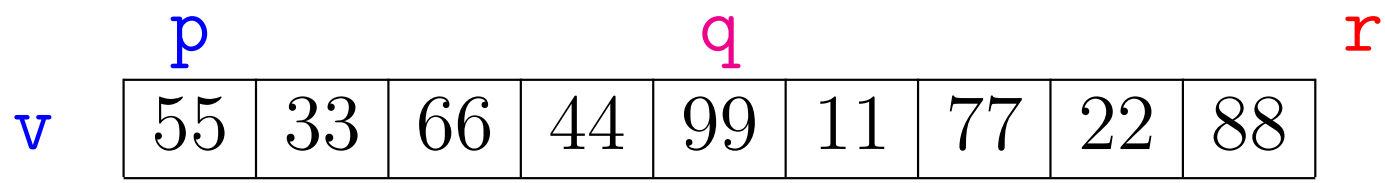

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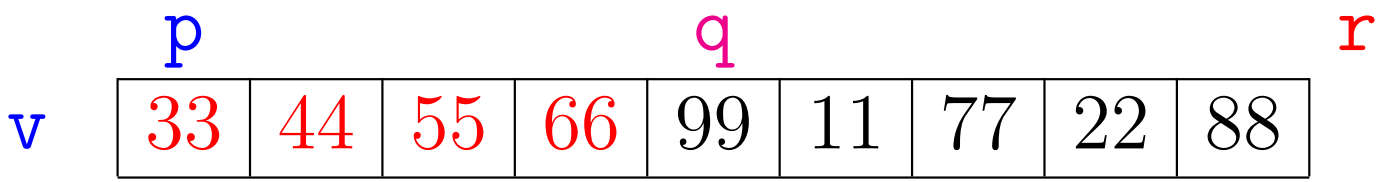

Ordena metade esquerda...

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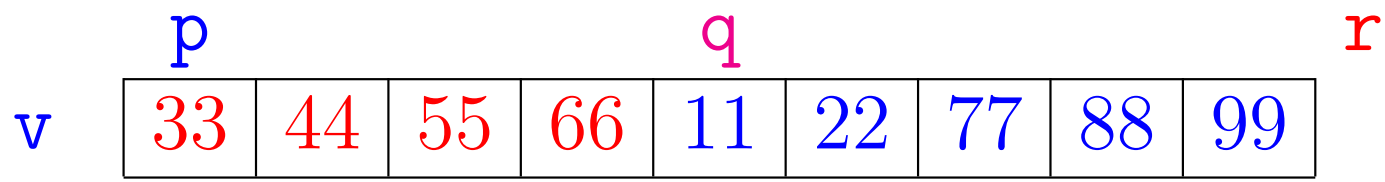

Ordena metade esquerda... Ordena metade direita...

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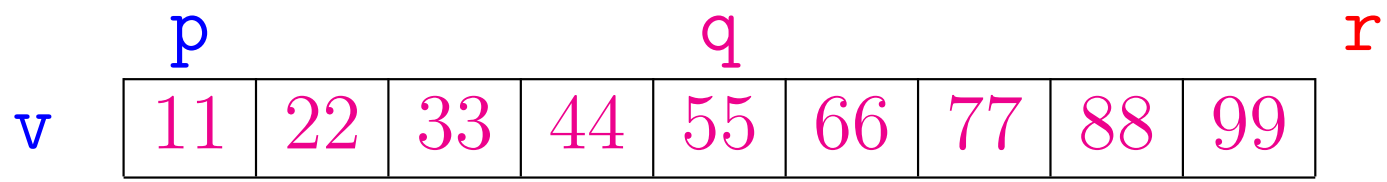
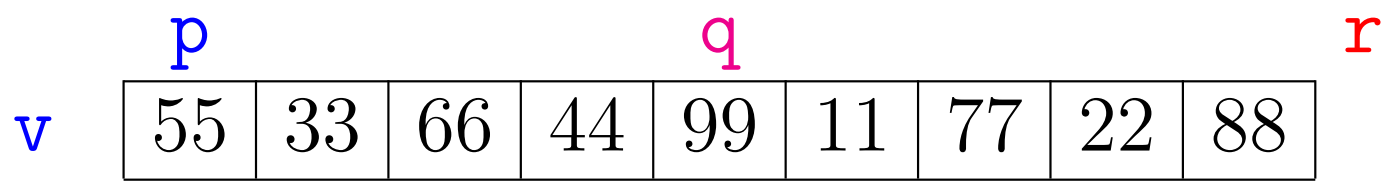


Ordena metade esquerda... Ordena metade direita...
Intercala!

Função mergeSort

Rearranja $v[p\,..\,r-1]$ em ordem crescente.

```
void mergeSort (int p, int r, int v[]) {
1   if (p < r-1) {   /* pelo menos dois elementos */
2      int q = (p + r)/2;
3      mergeSort(p, q, v);
4      mergeSort(q, r, v);
5      intercala(p, q, r, v);
    }
}
```



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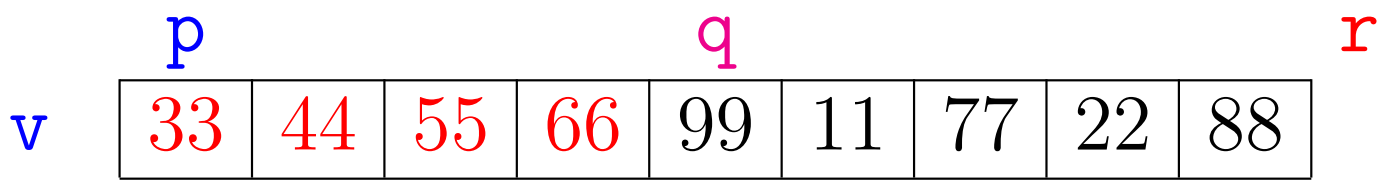

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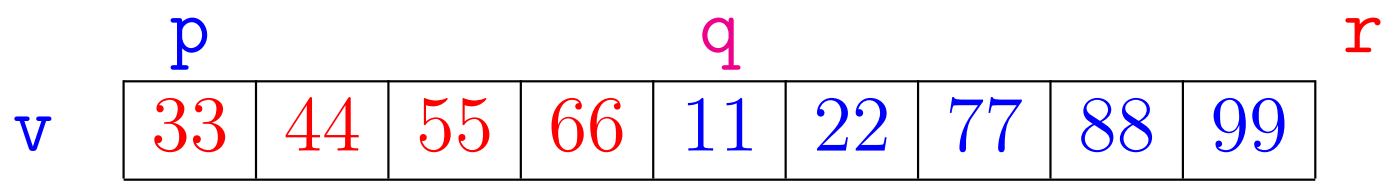

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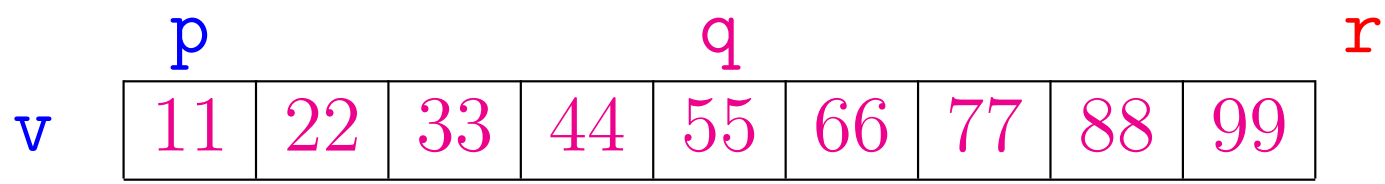

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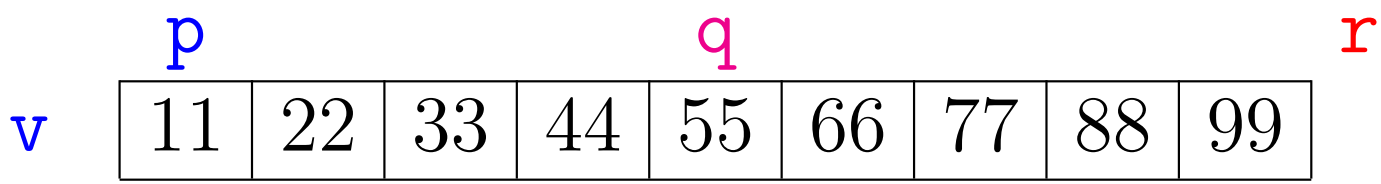
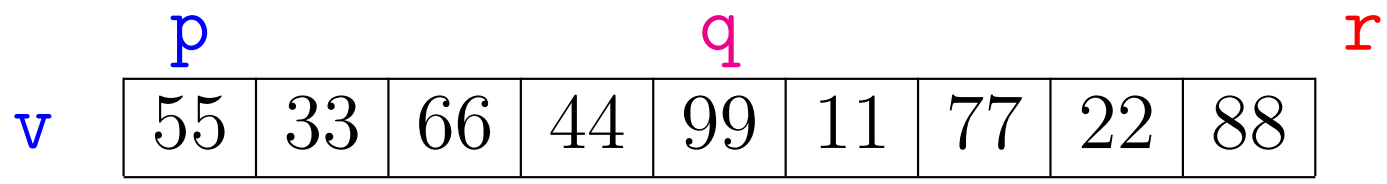


Mergesort



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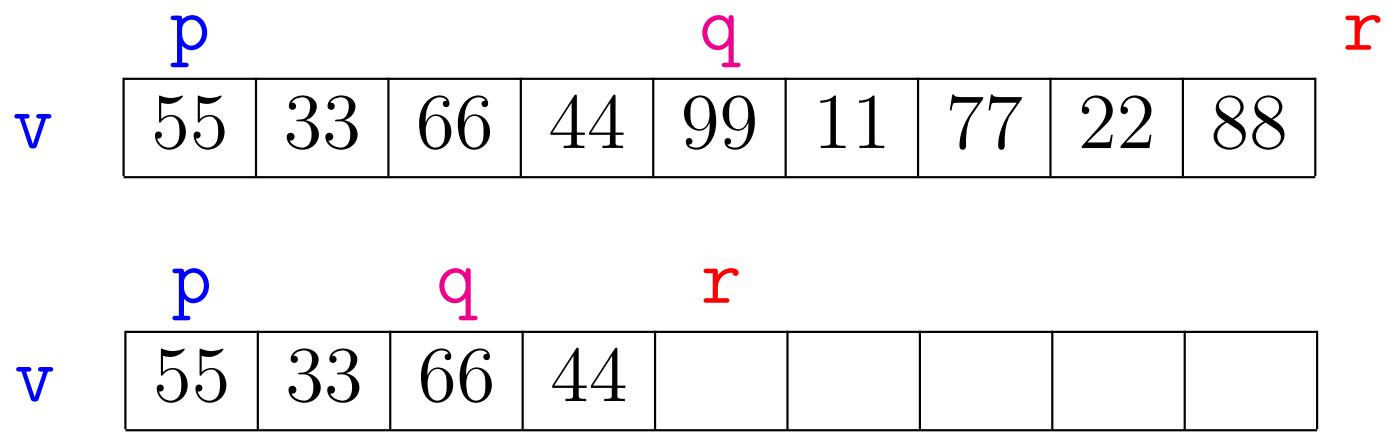

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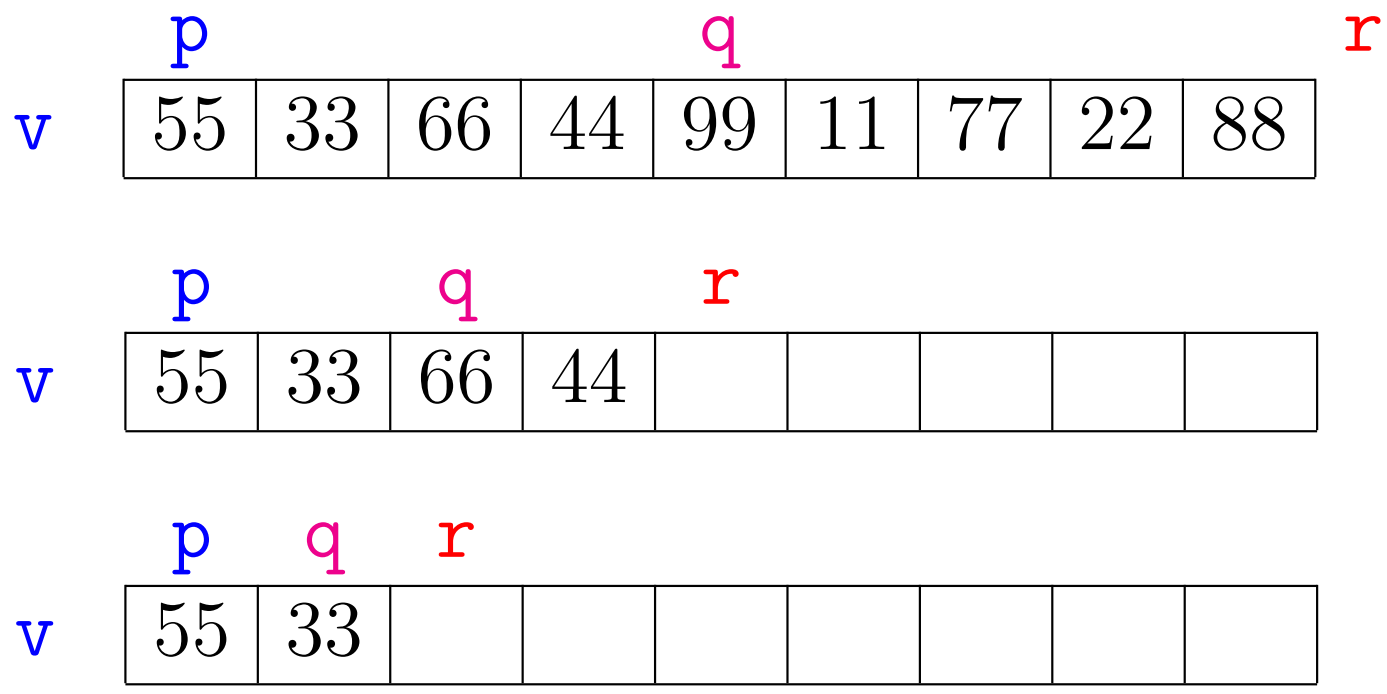

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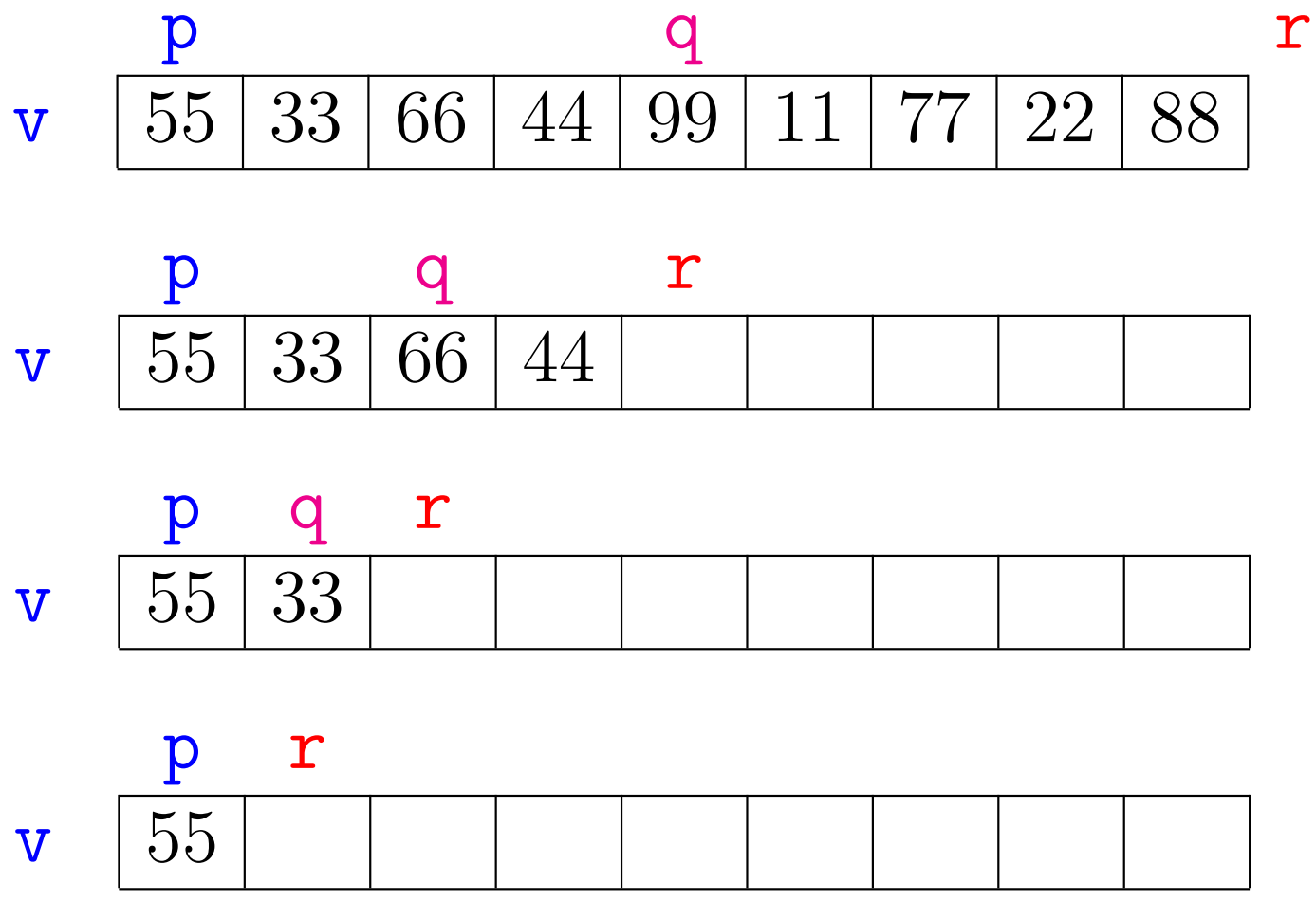

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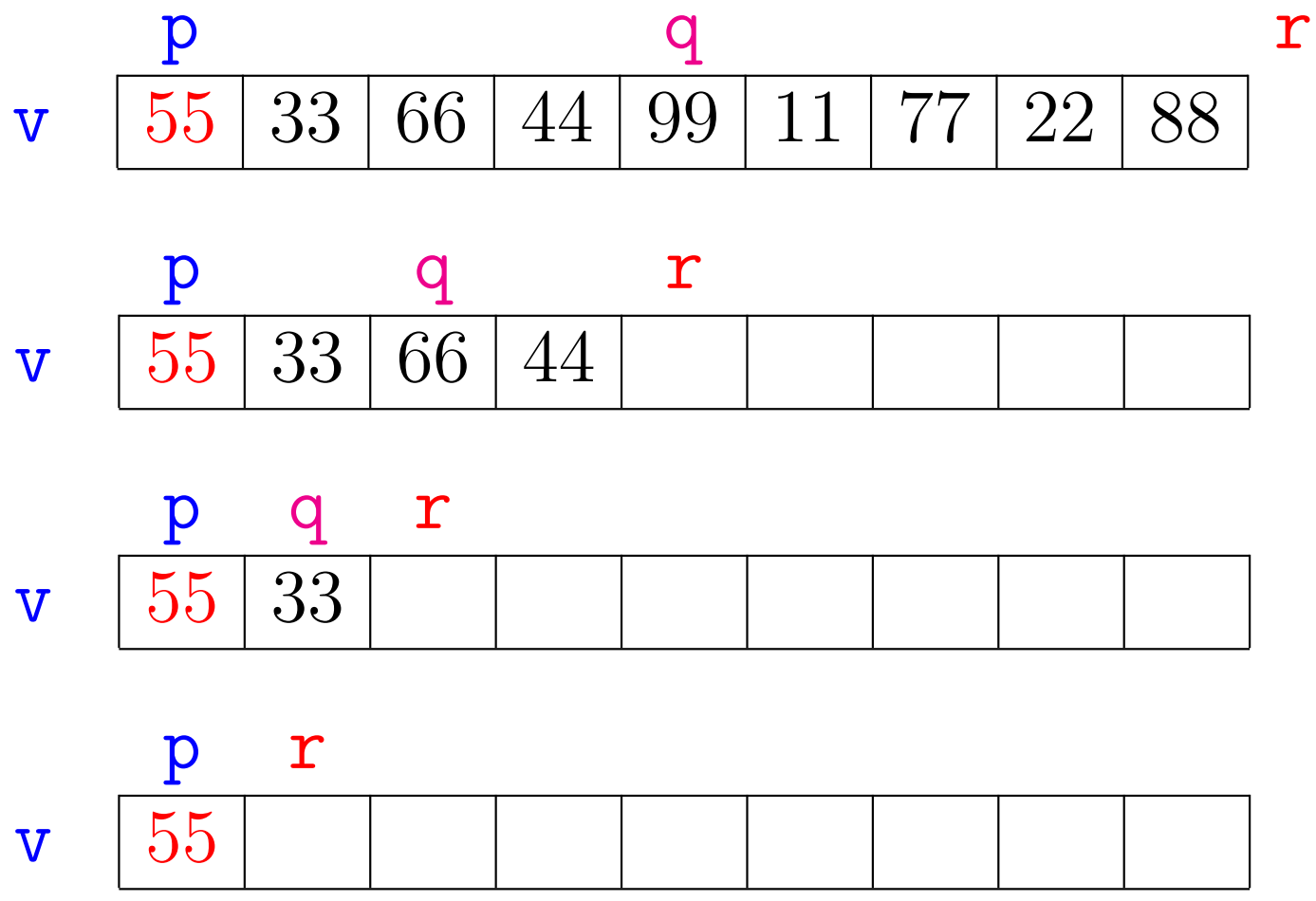

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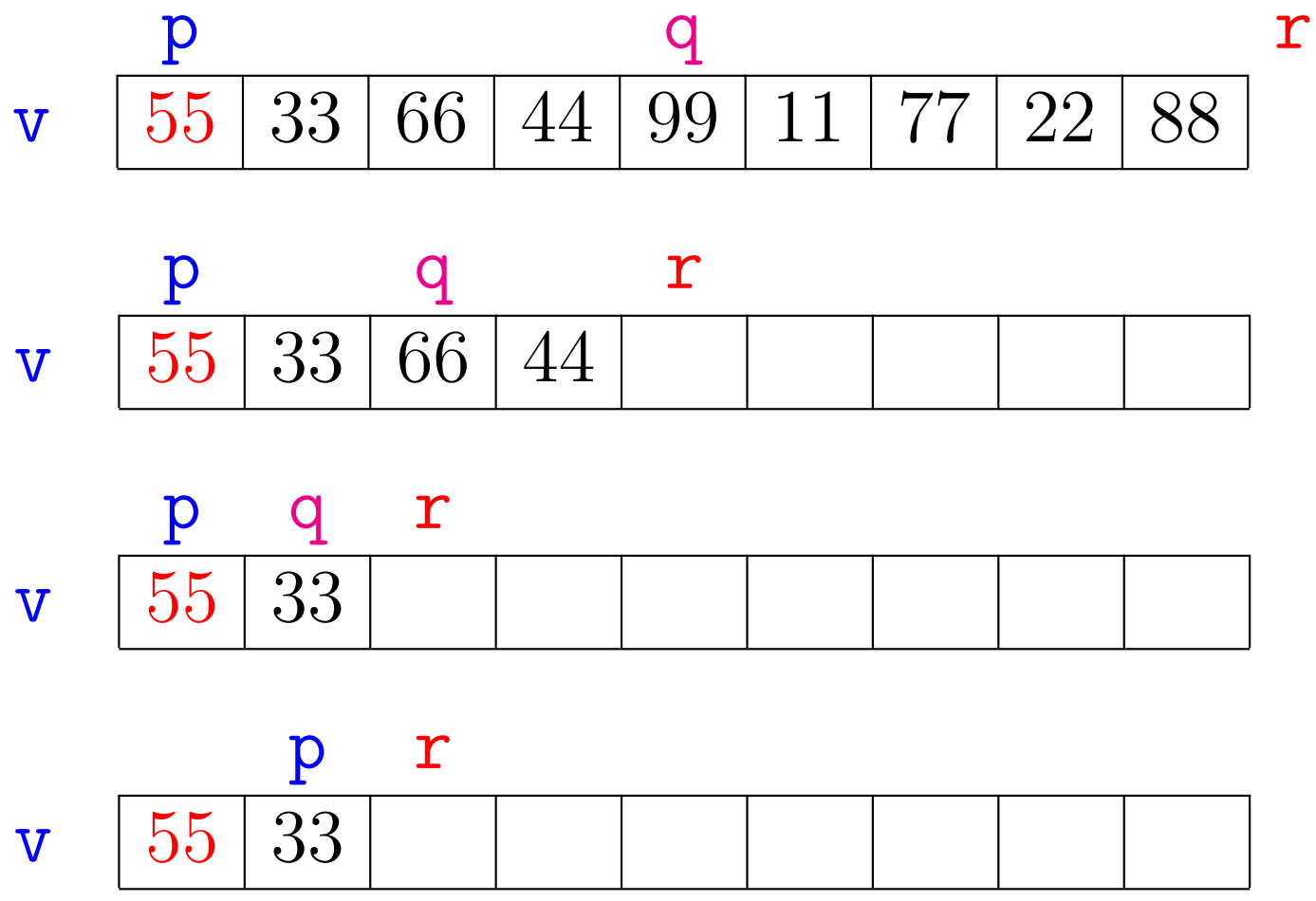

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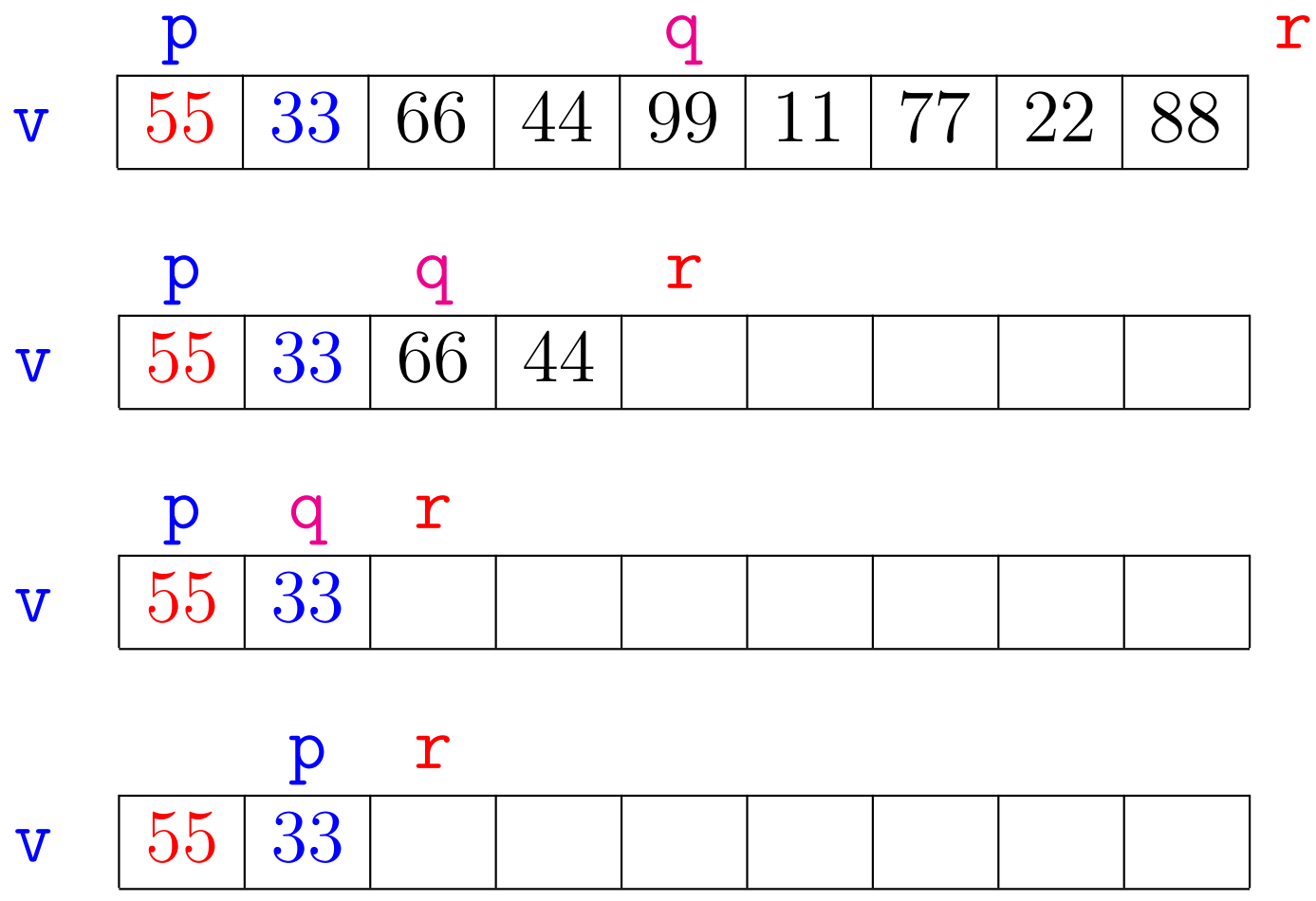

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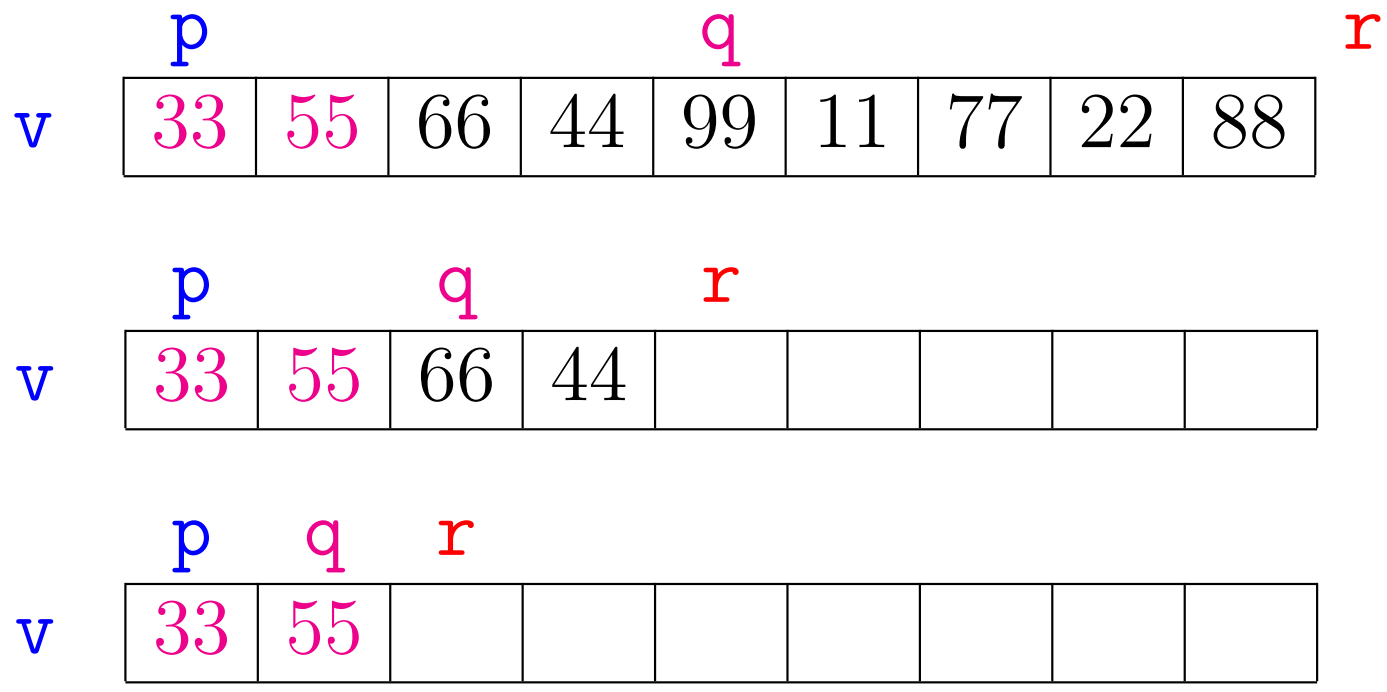

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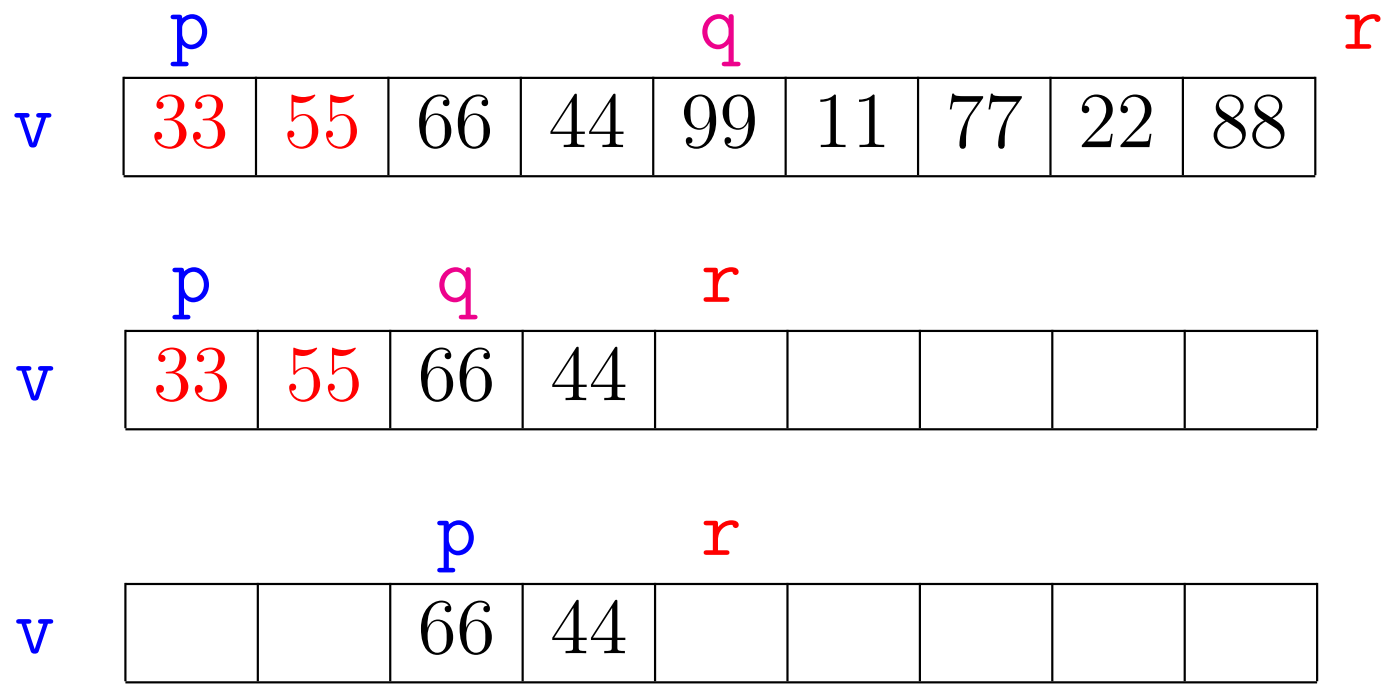

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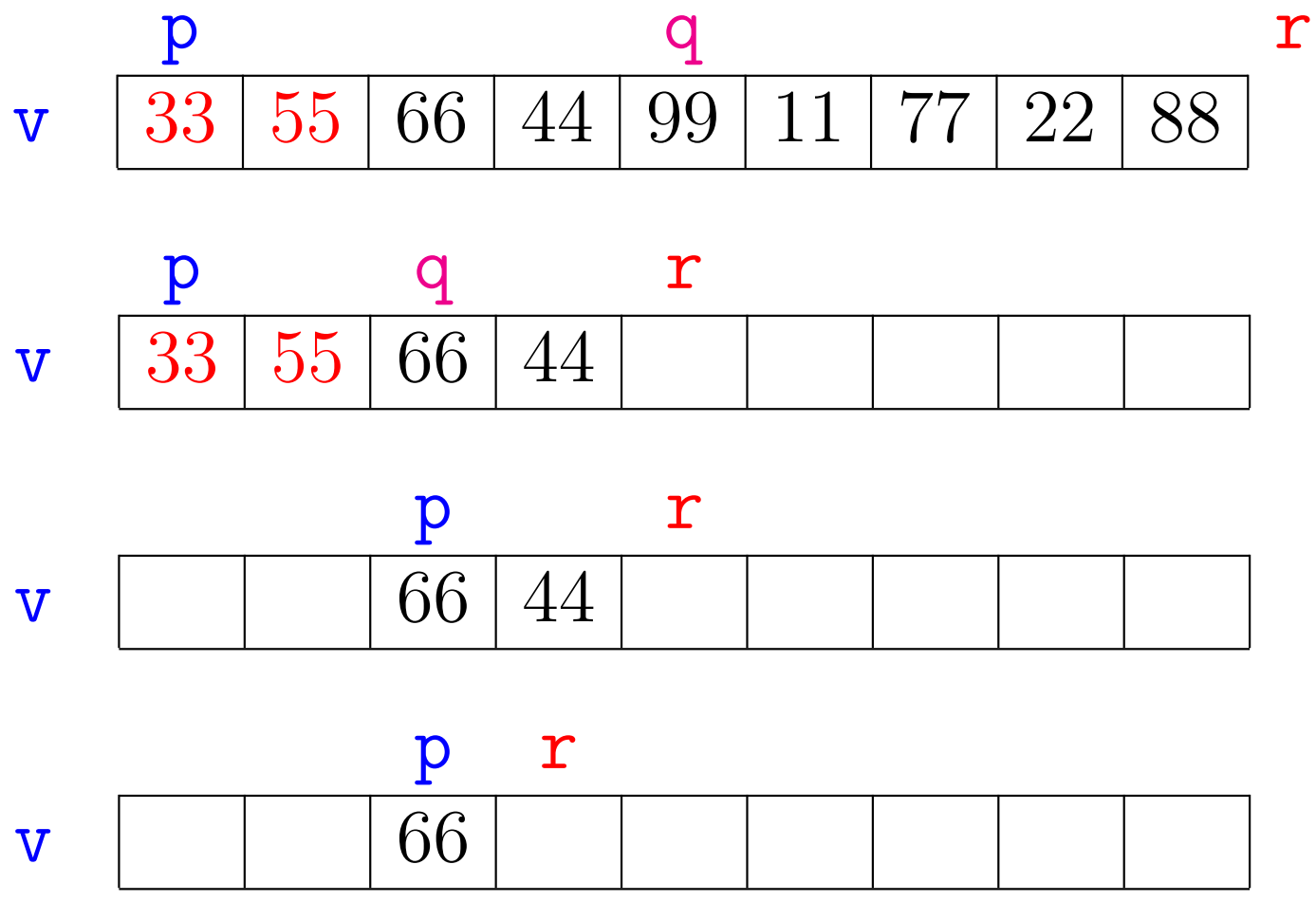

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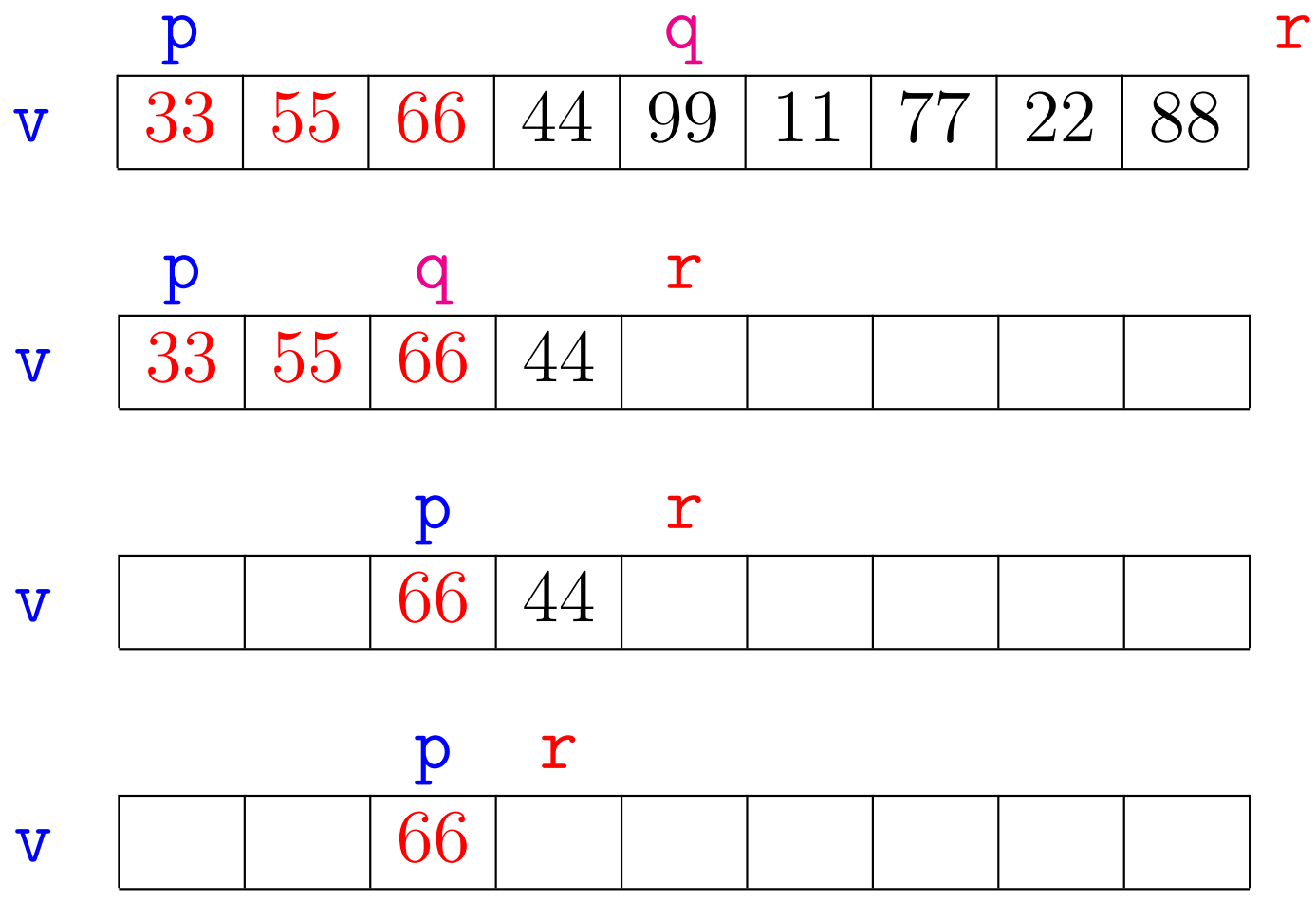

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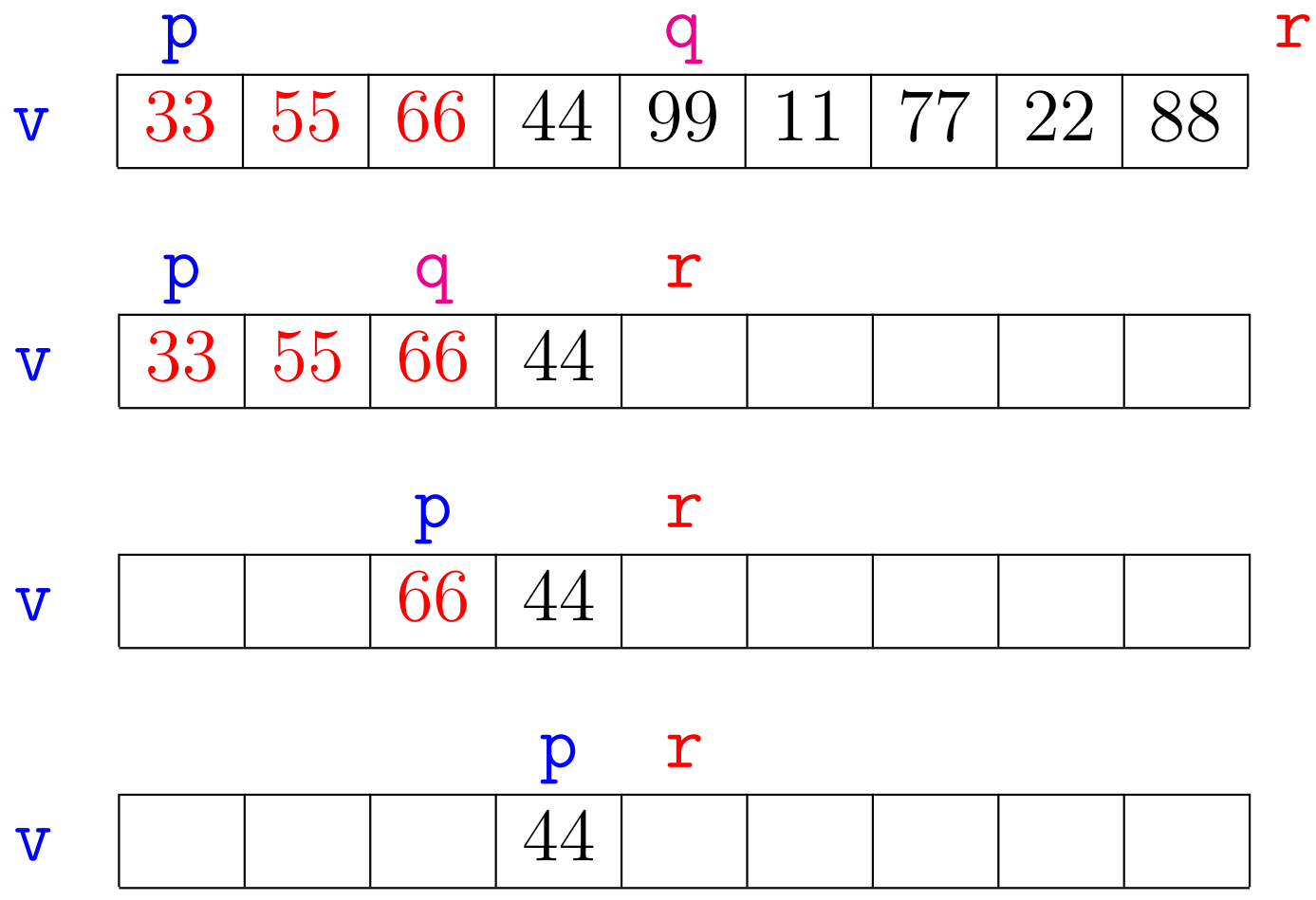

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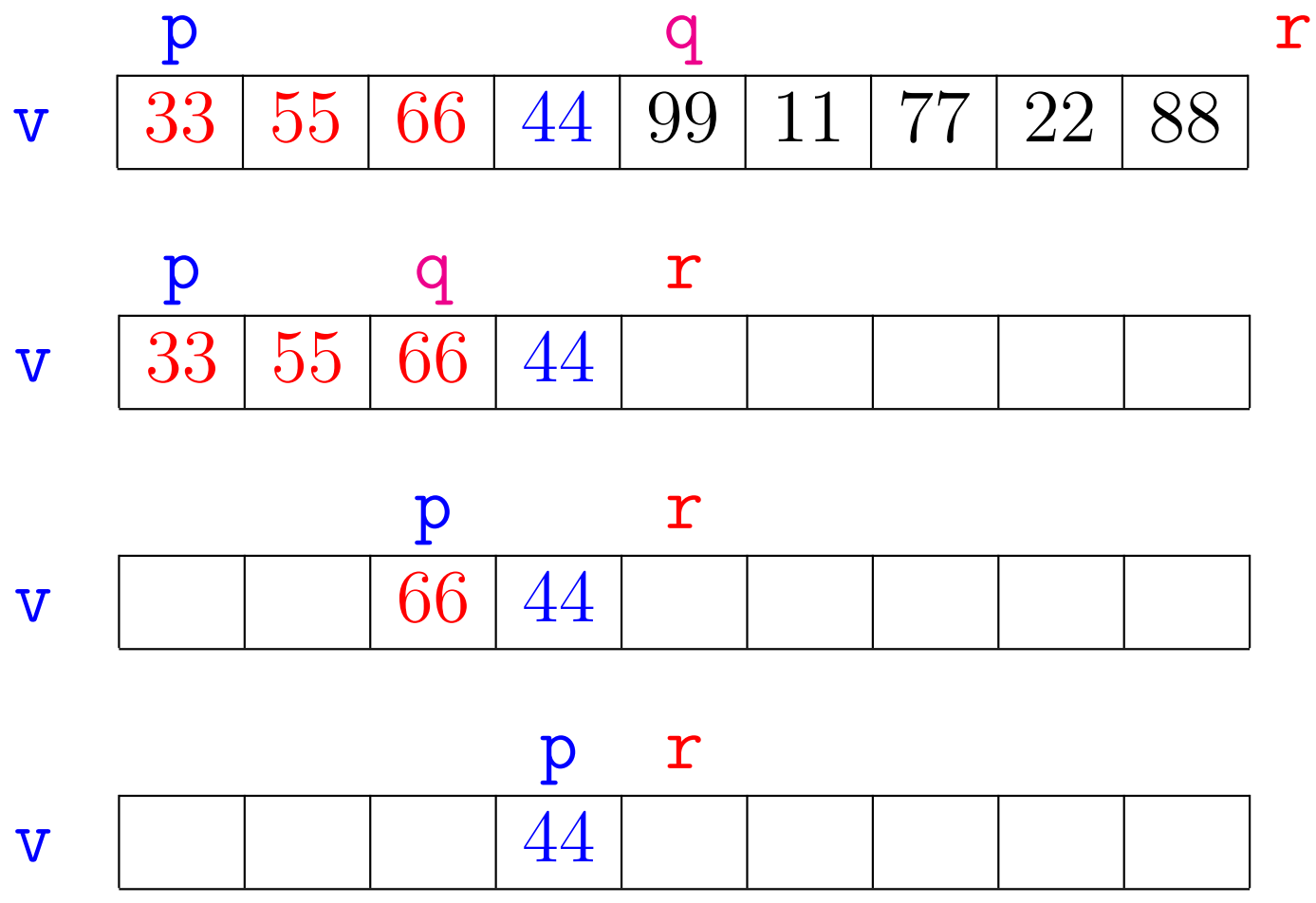

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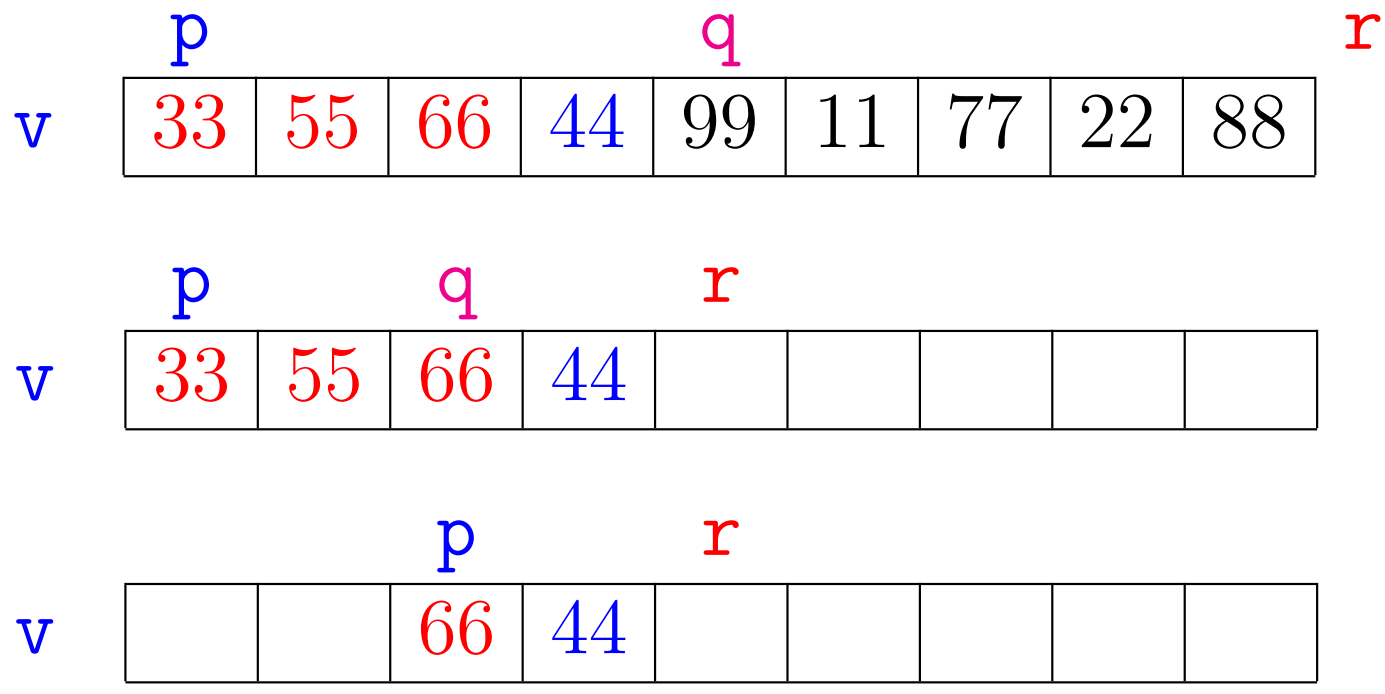

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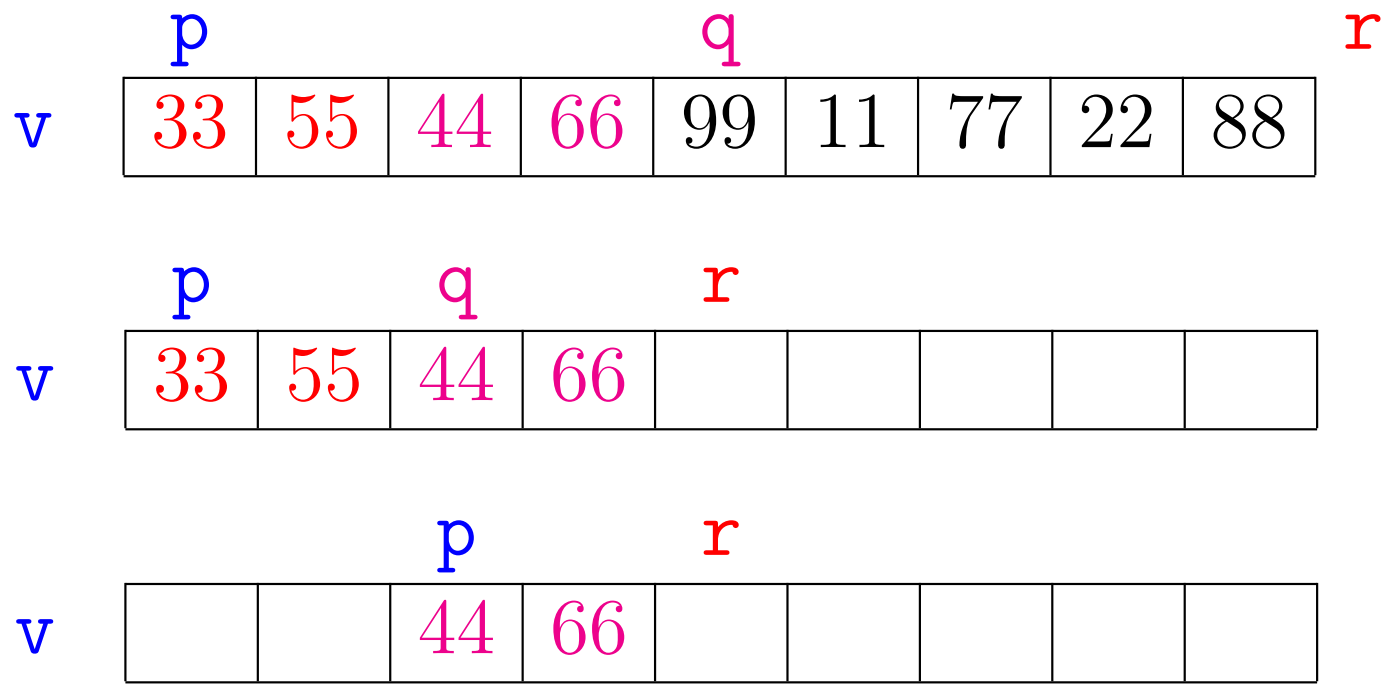

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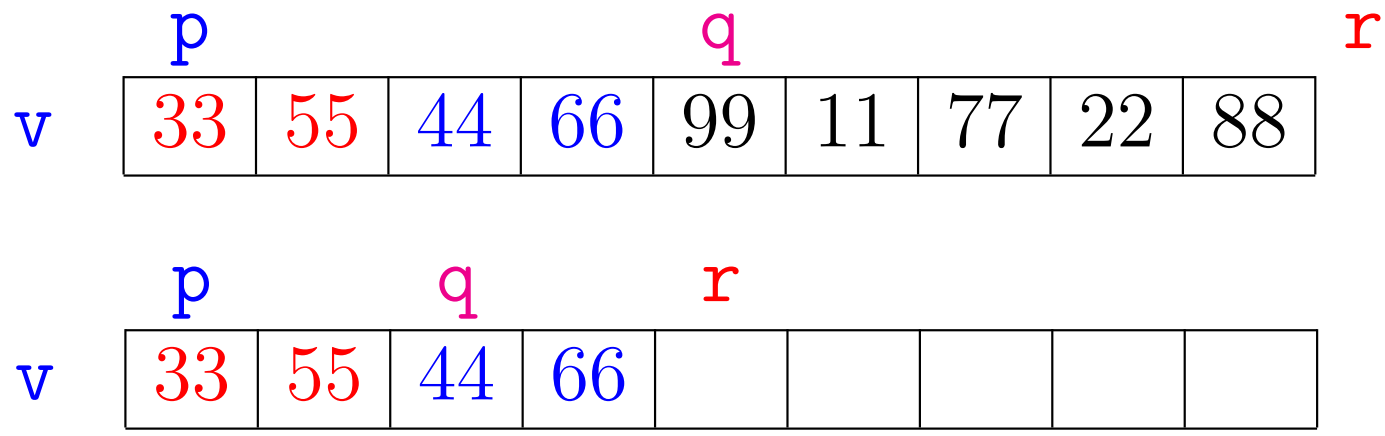

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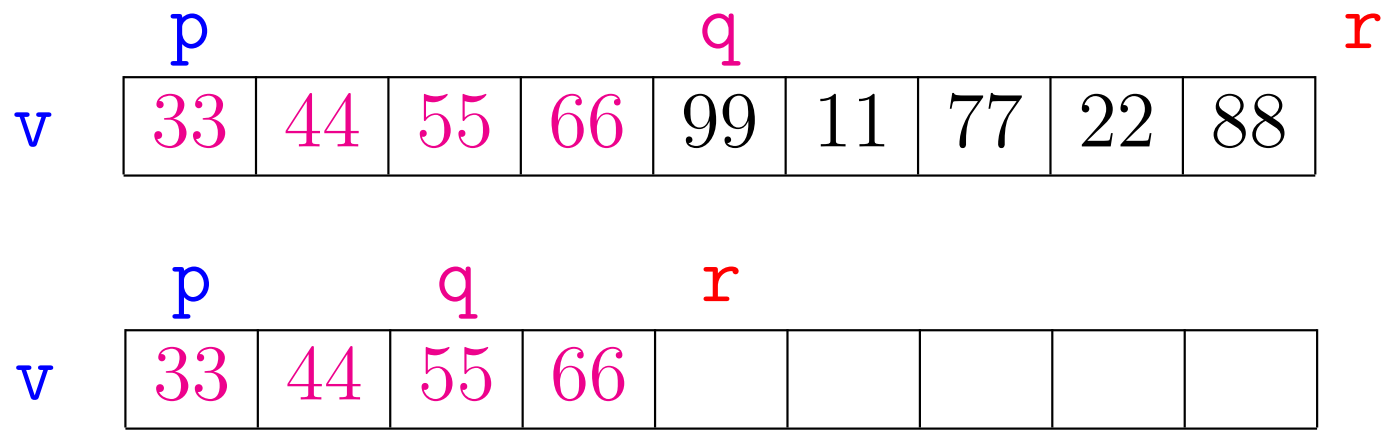

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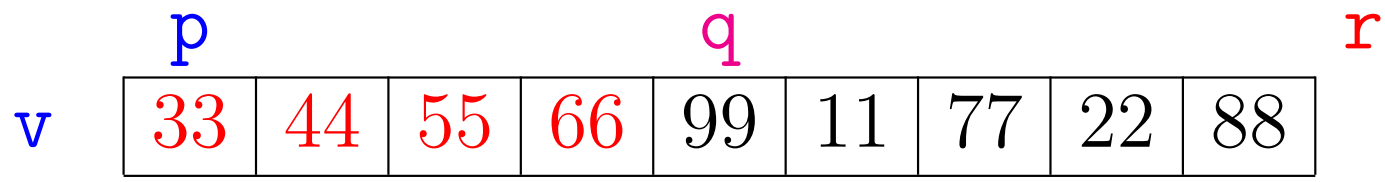
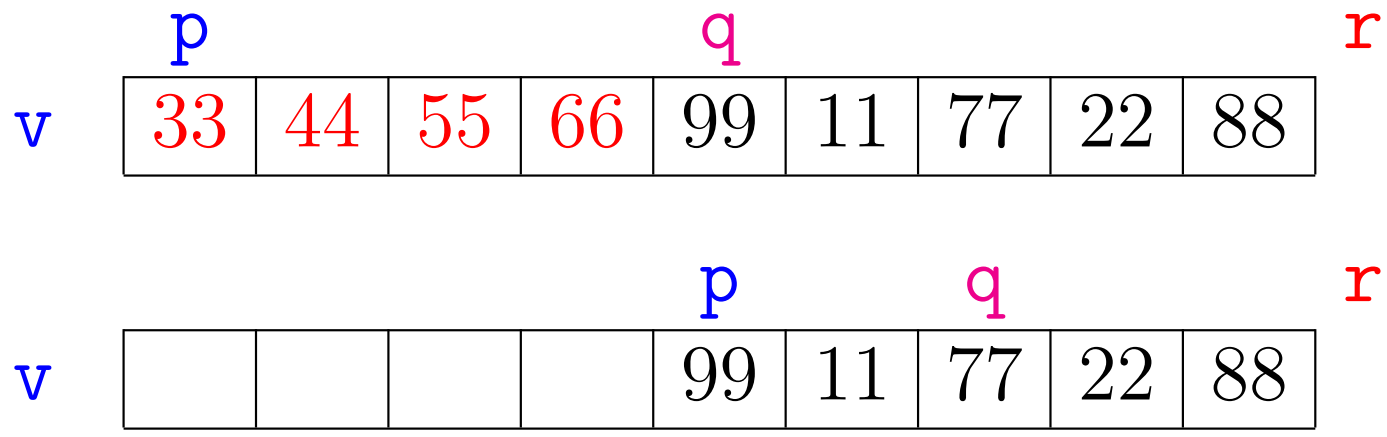


Mergesort



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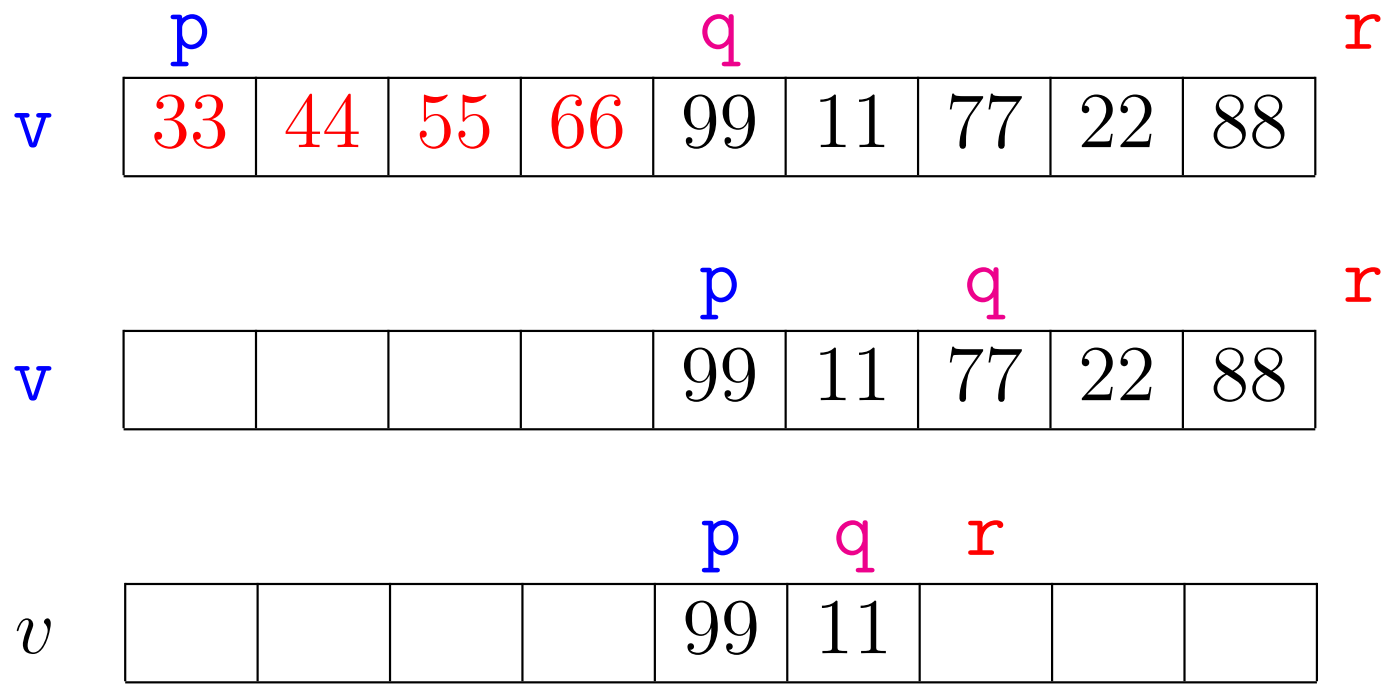

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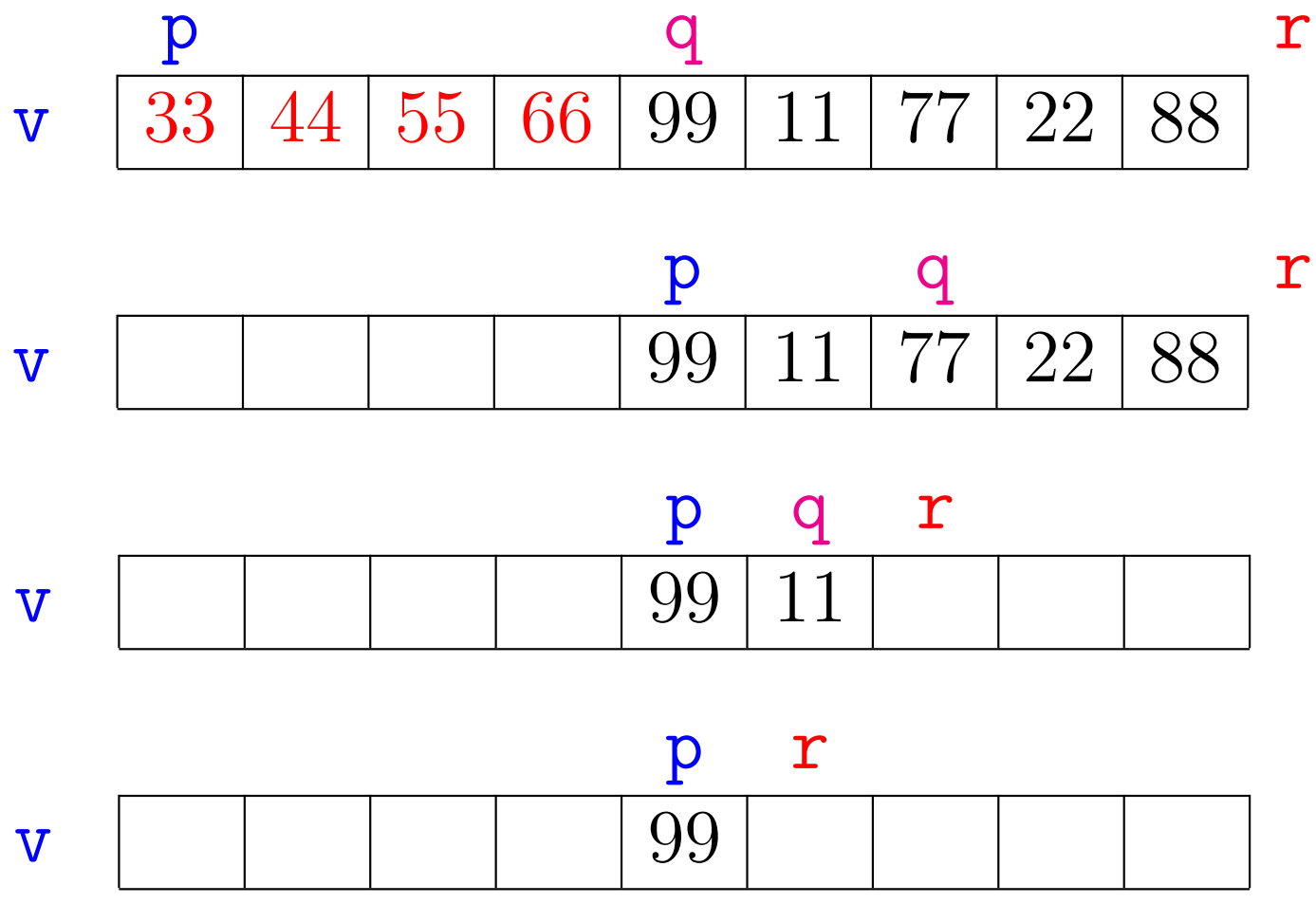

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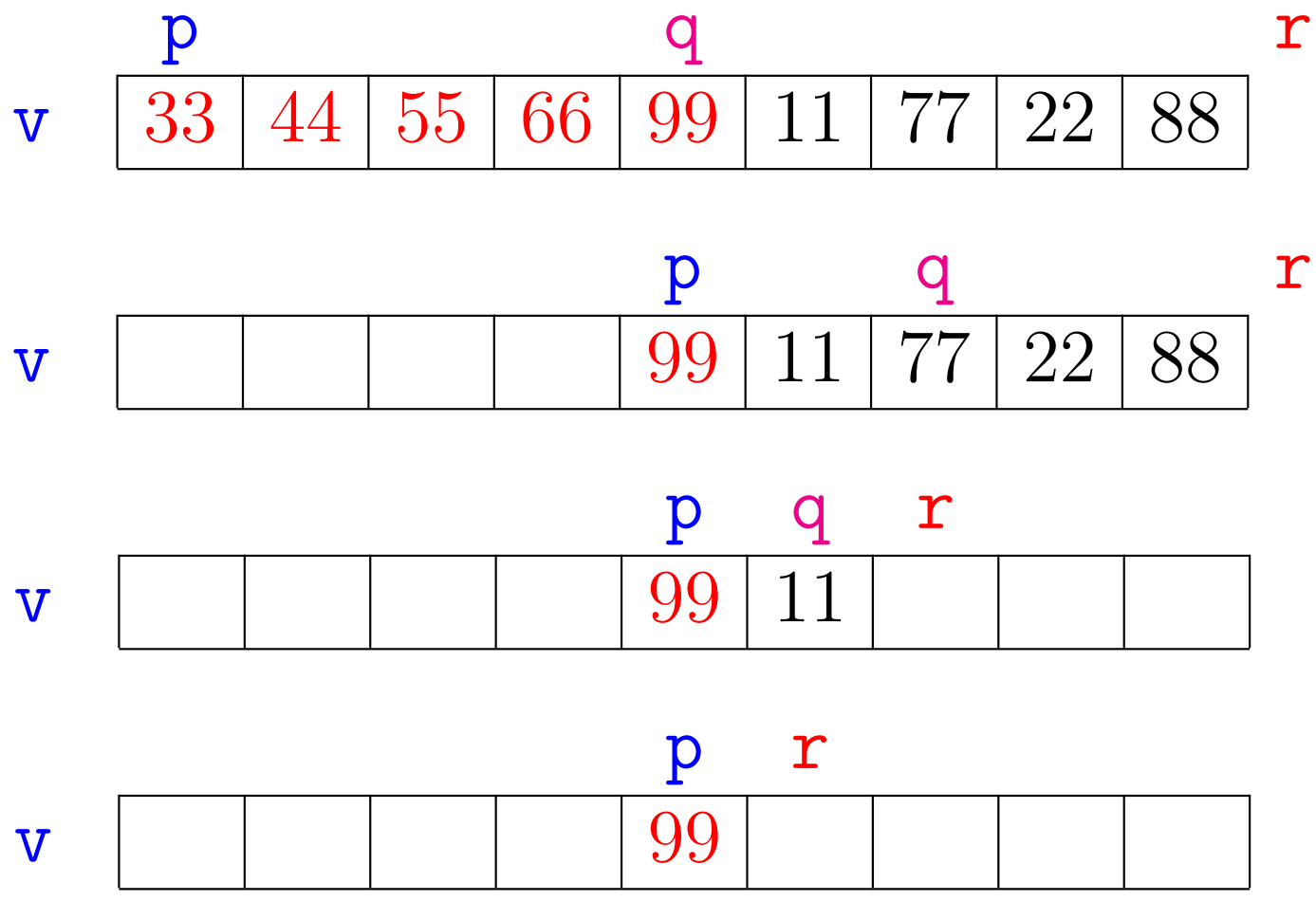

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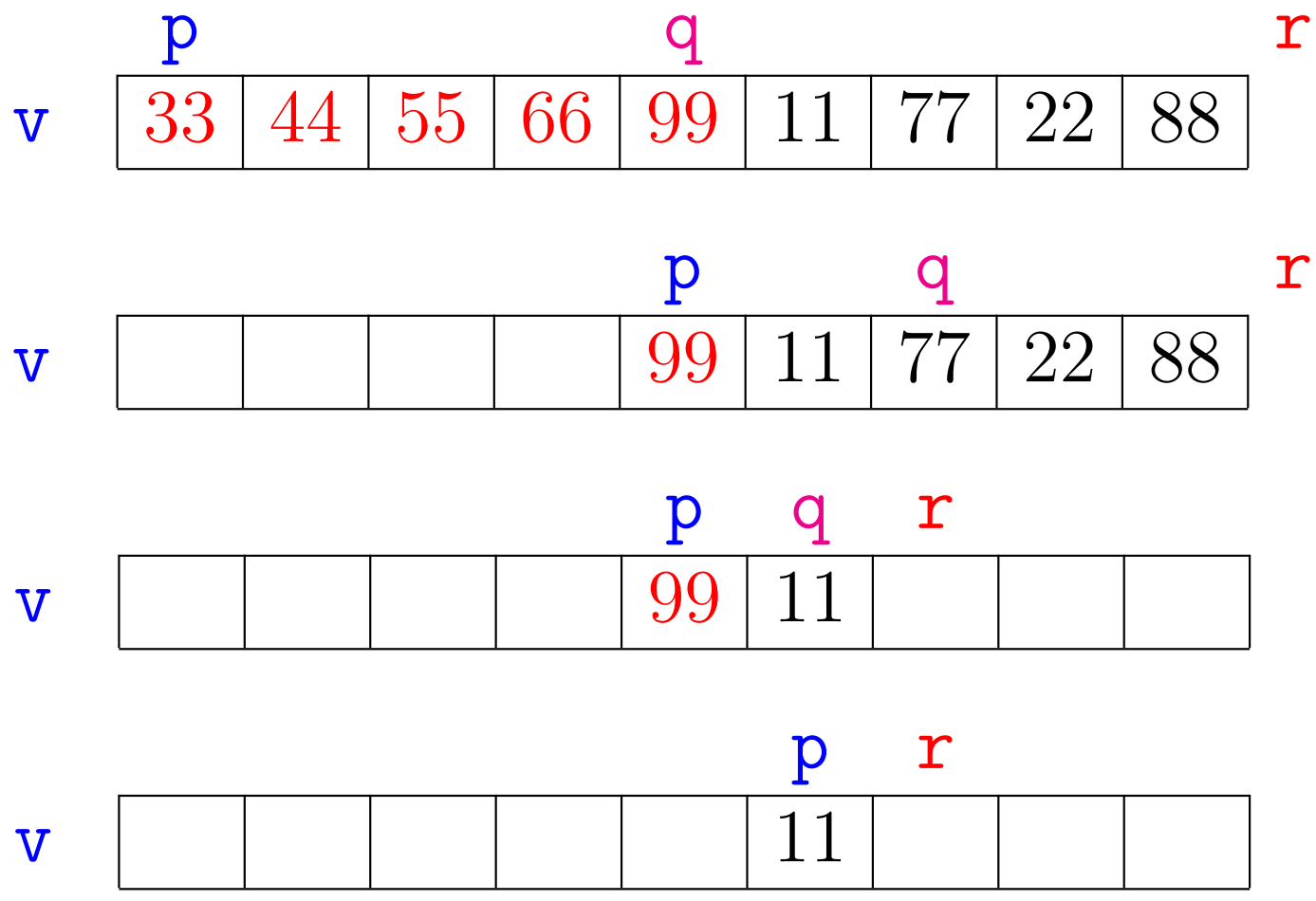

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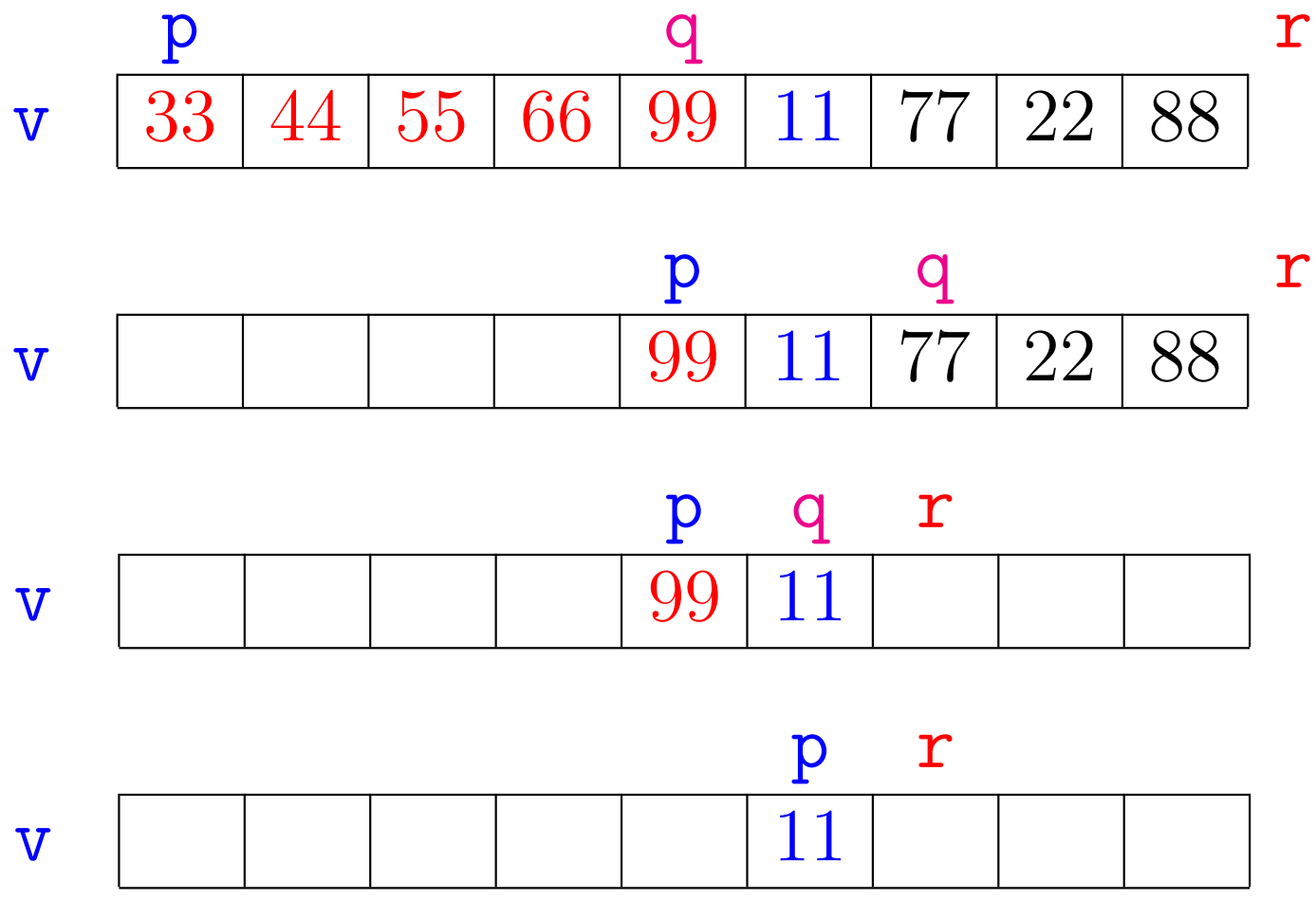

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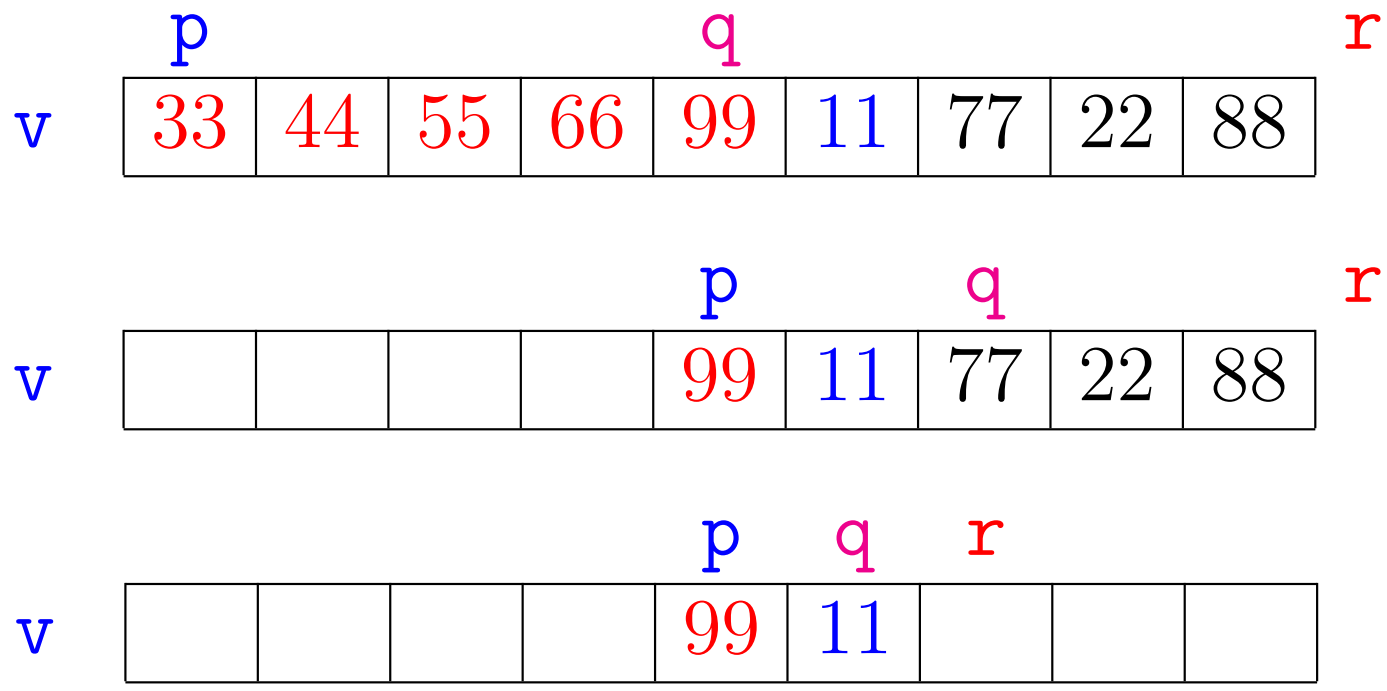

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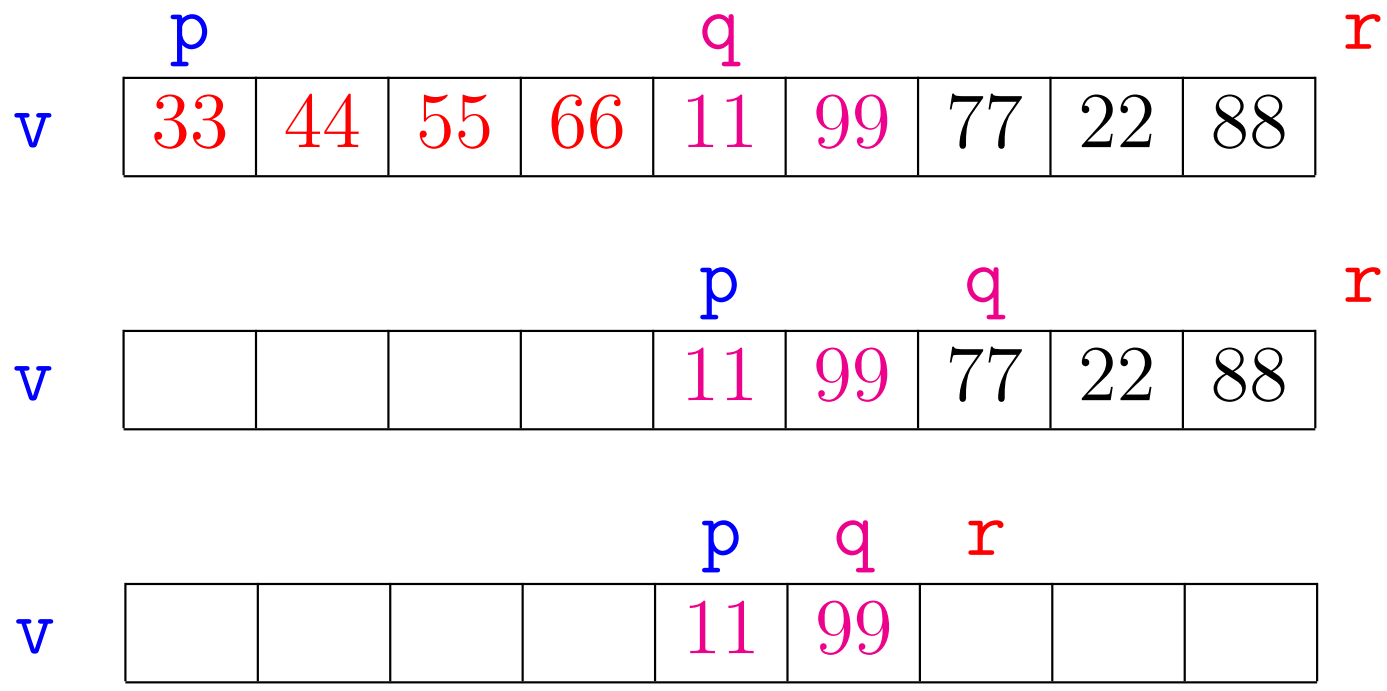

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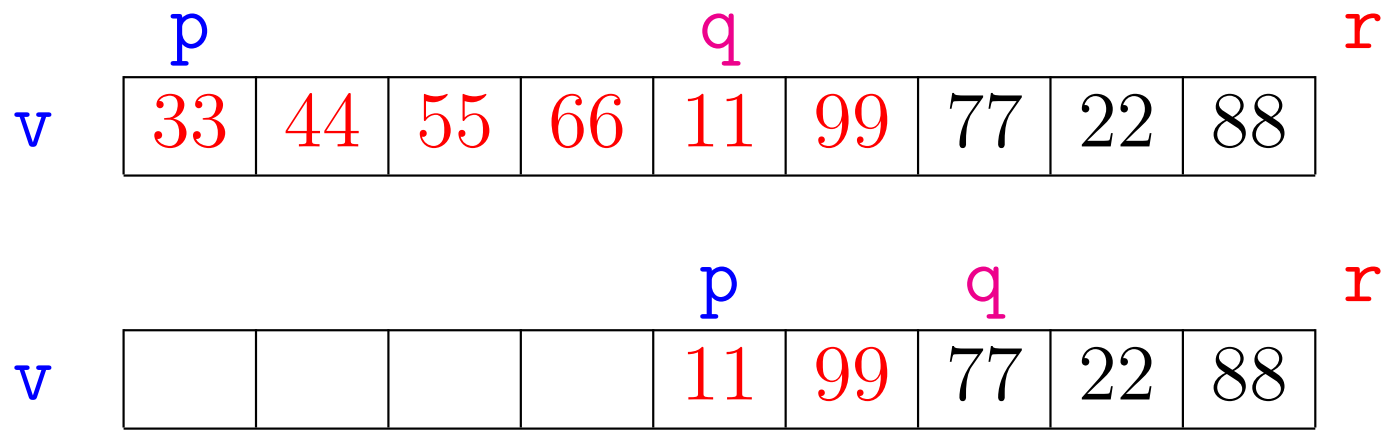

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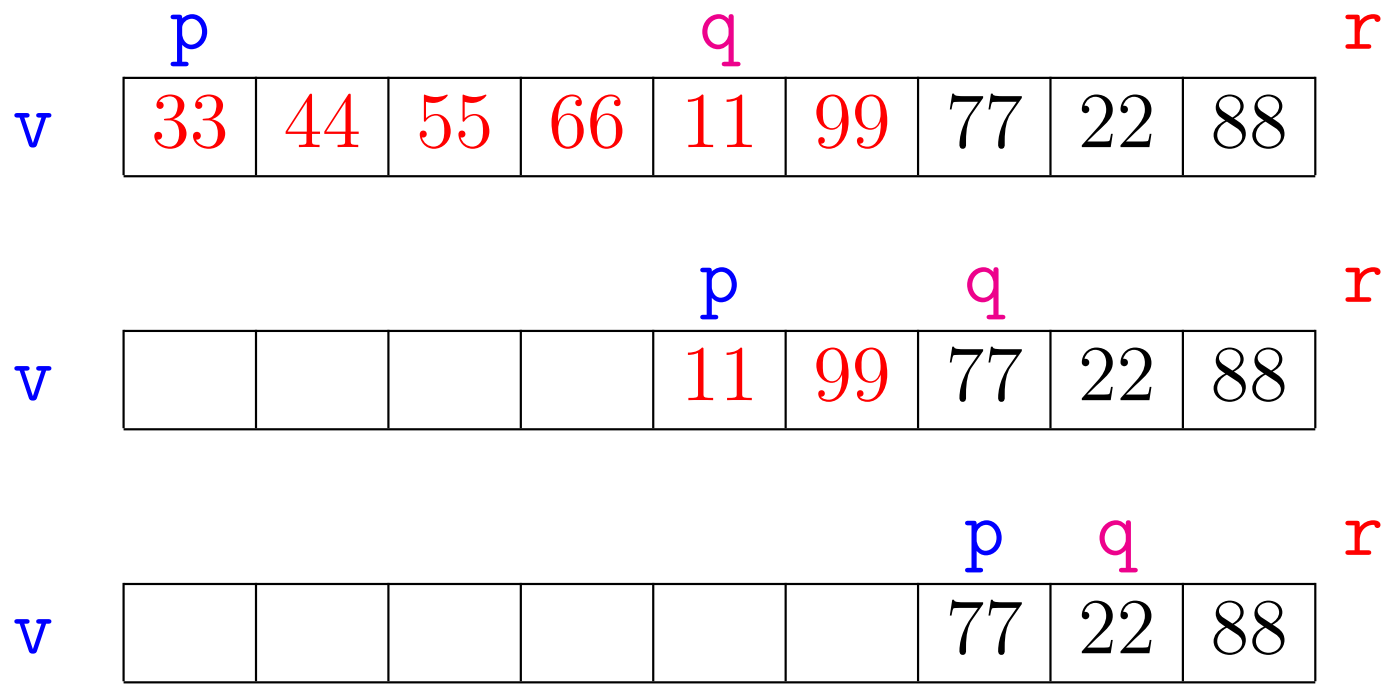

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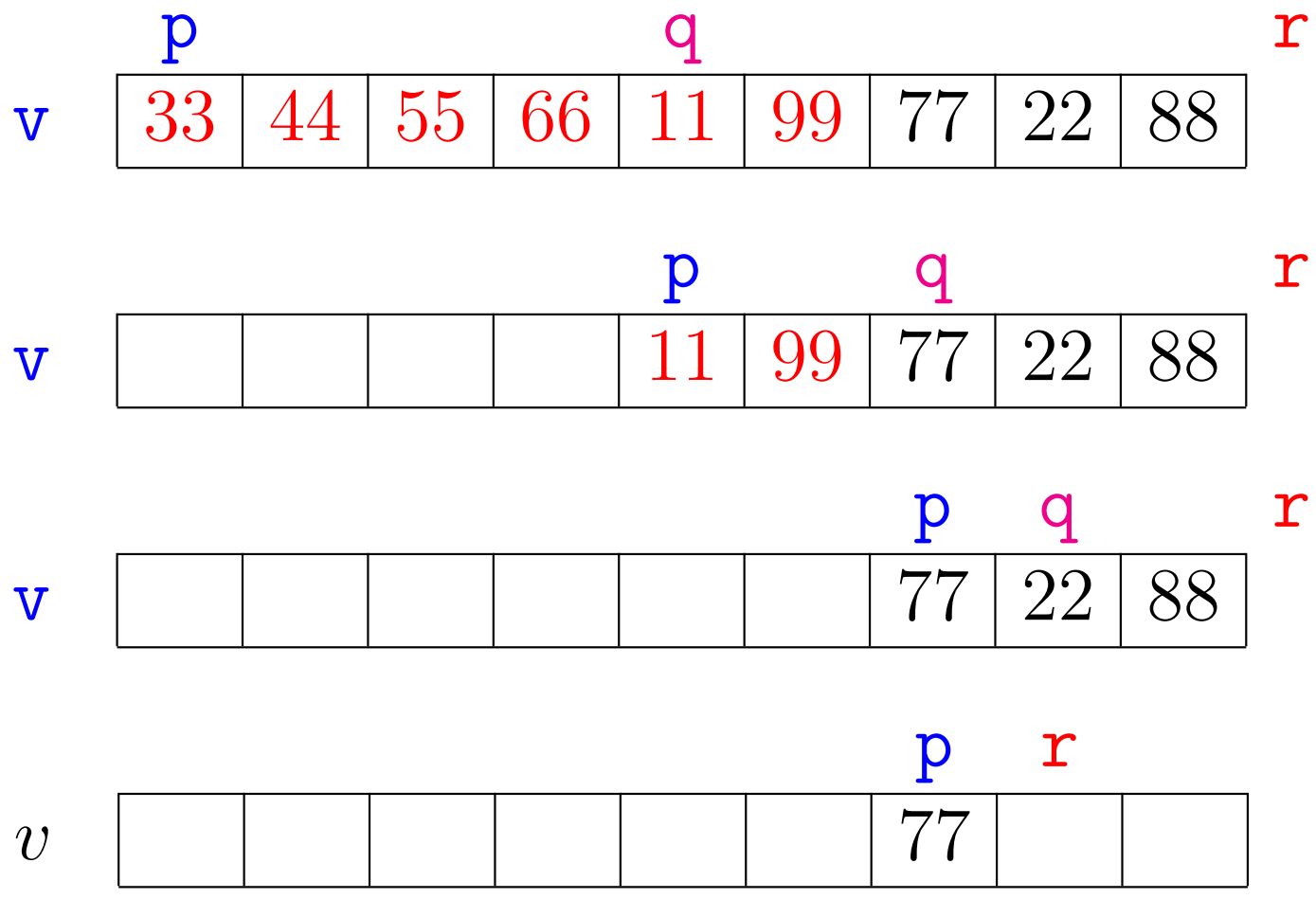
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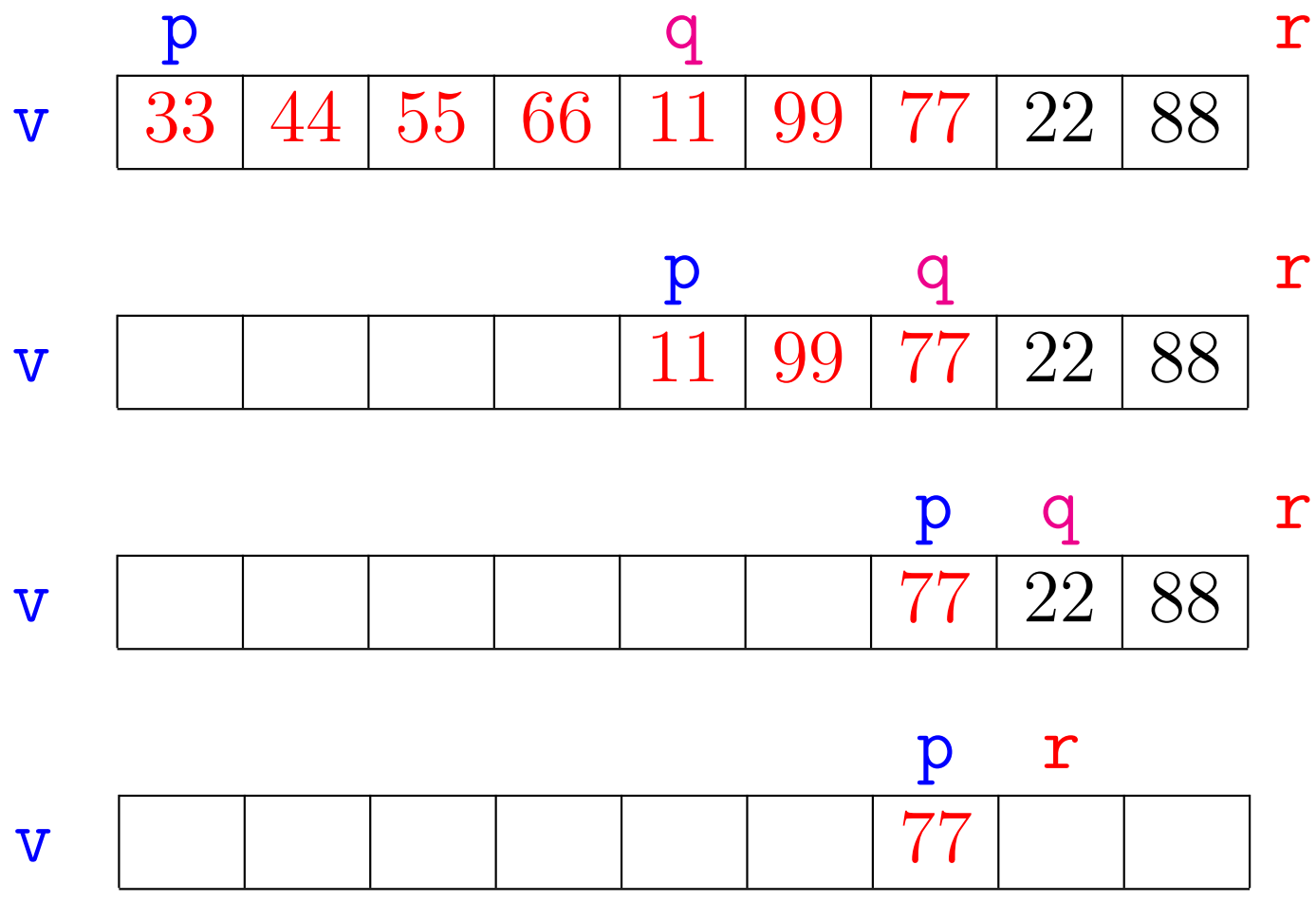
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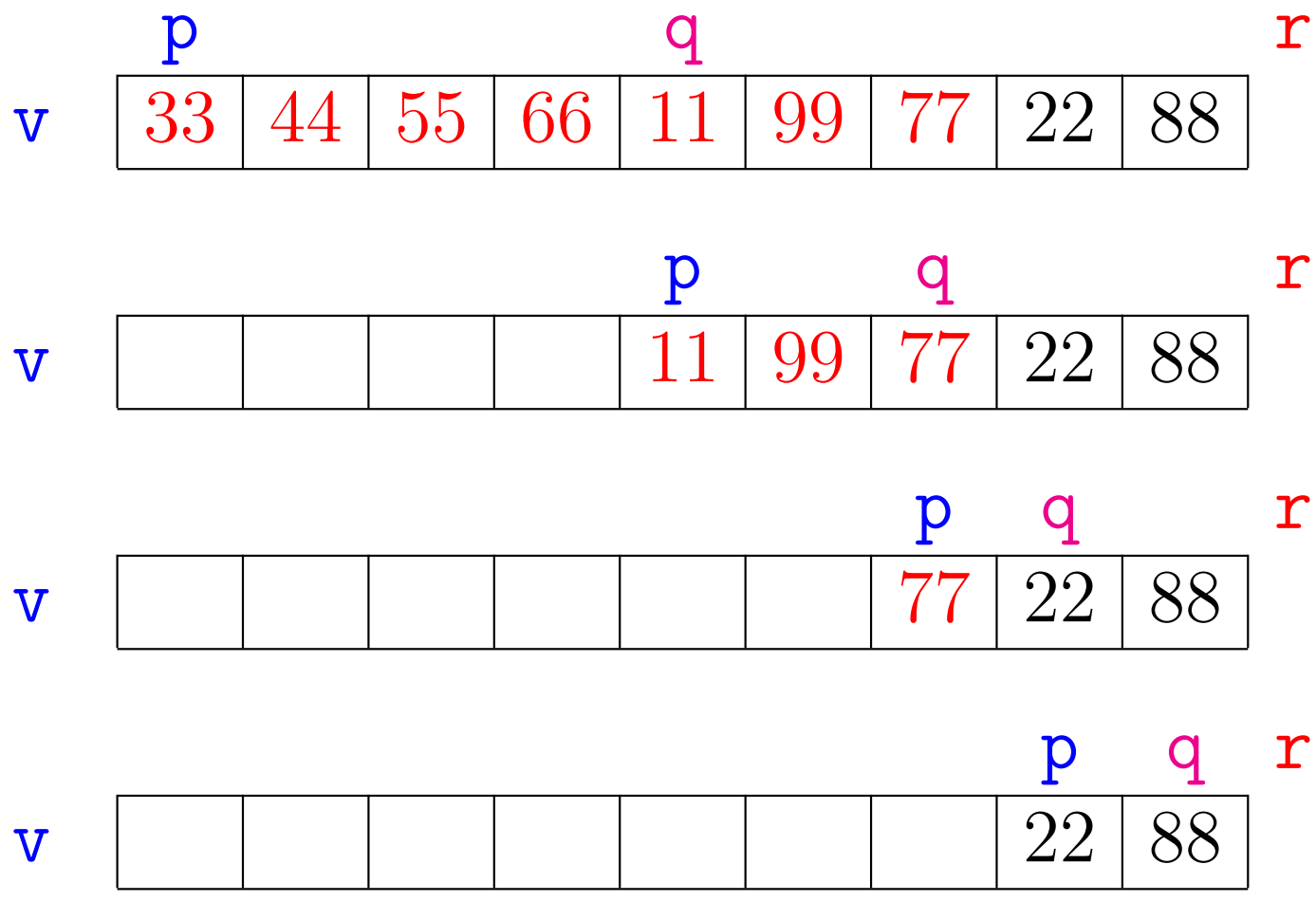
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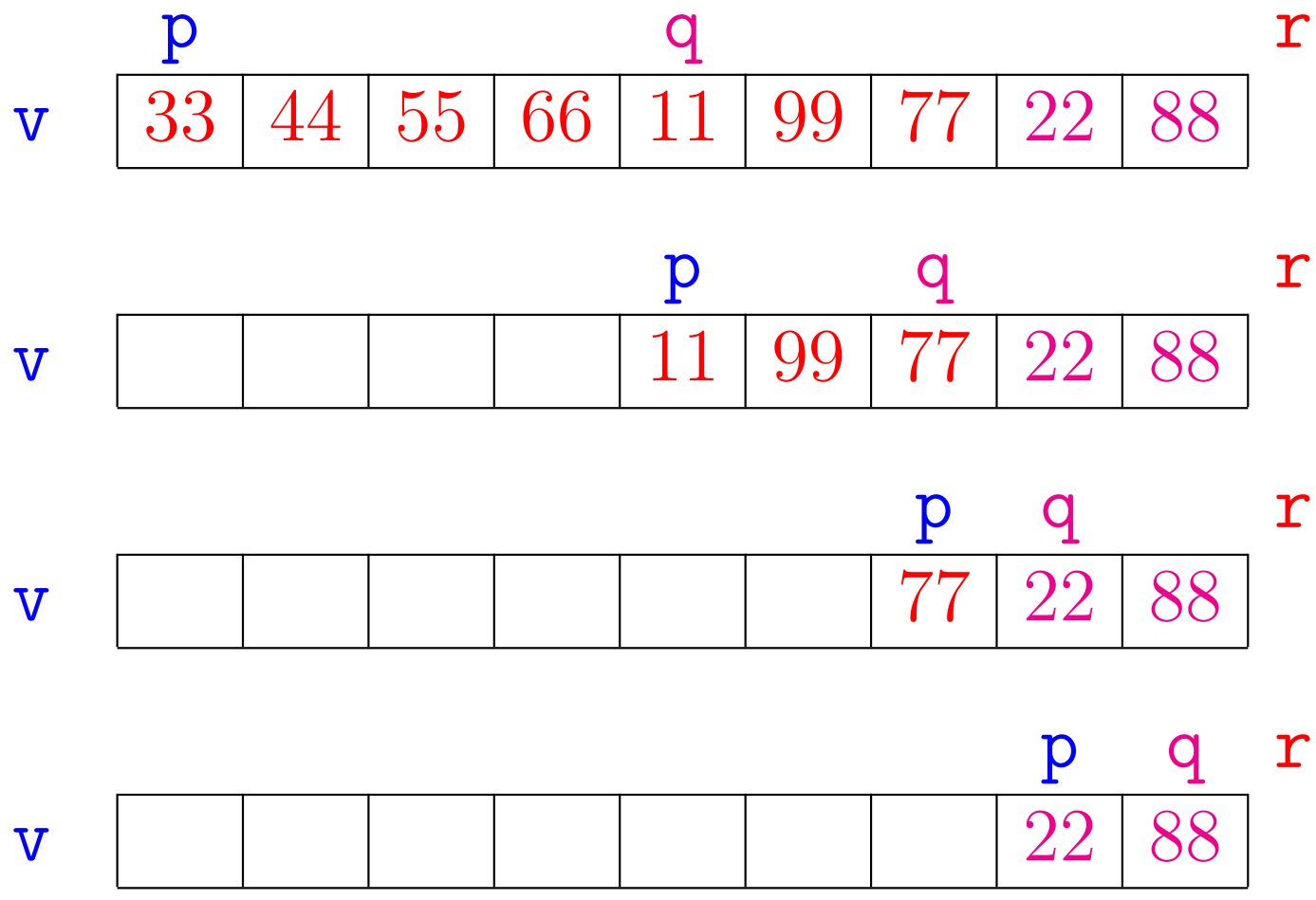
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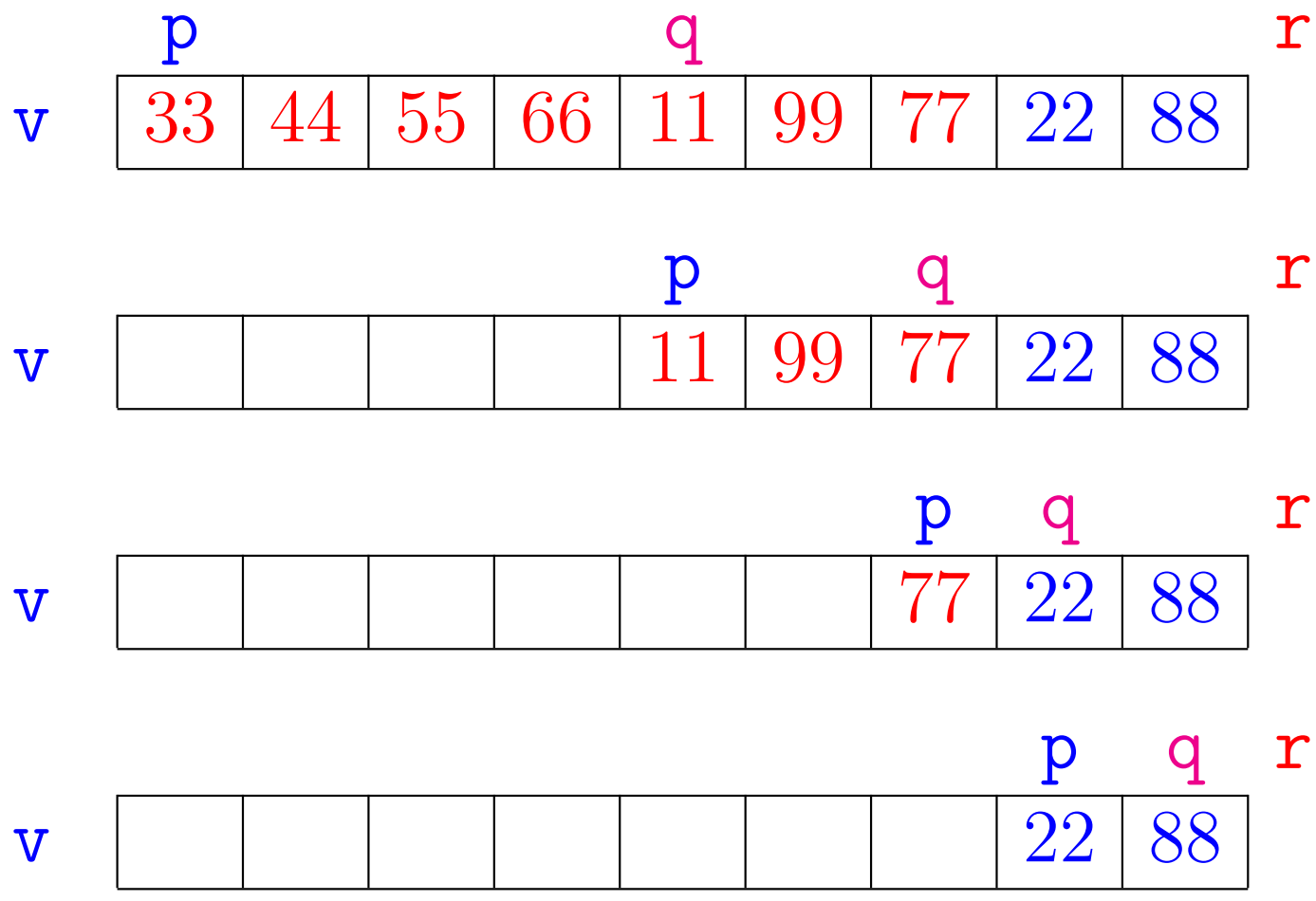
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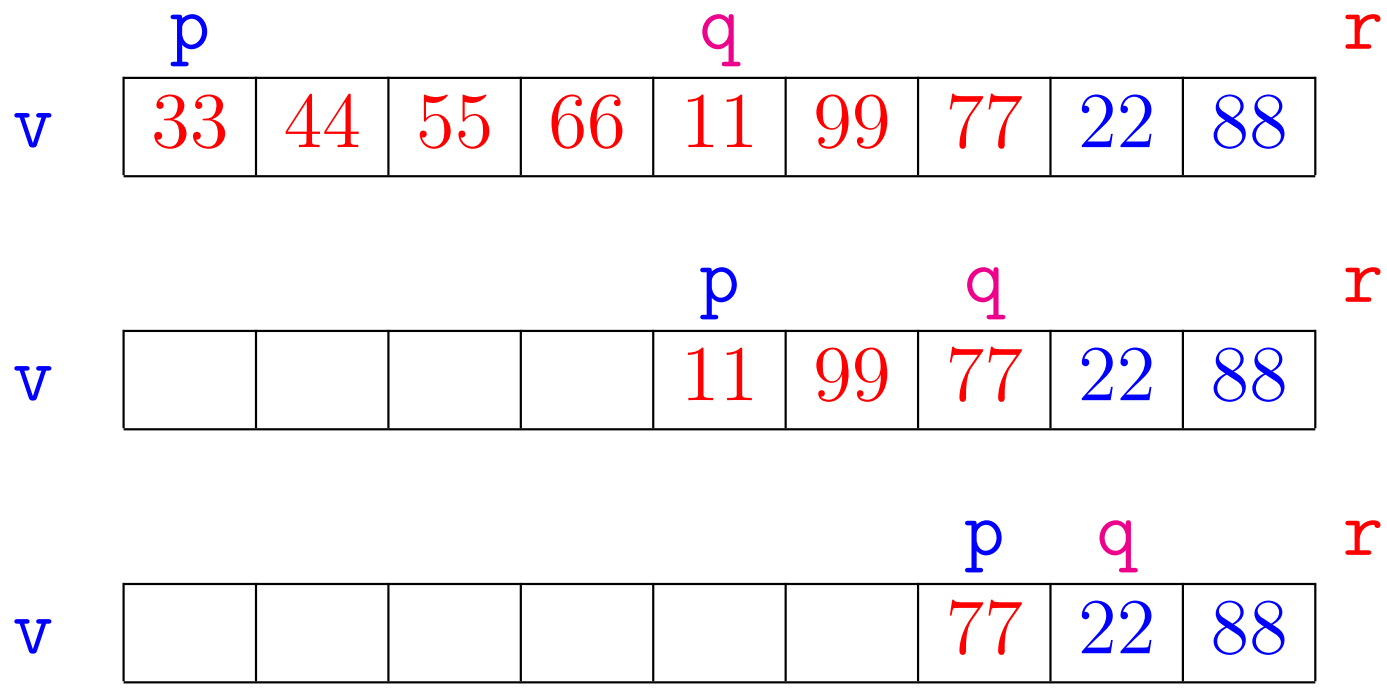

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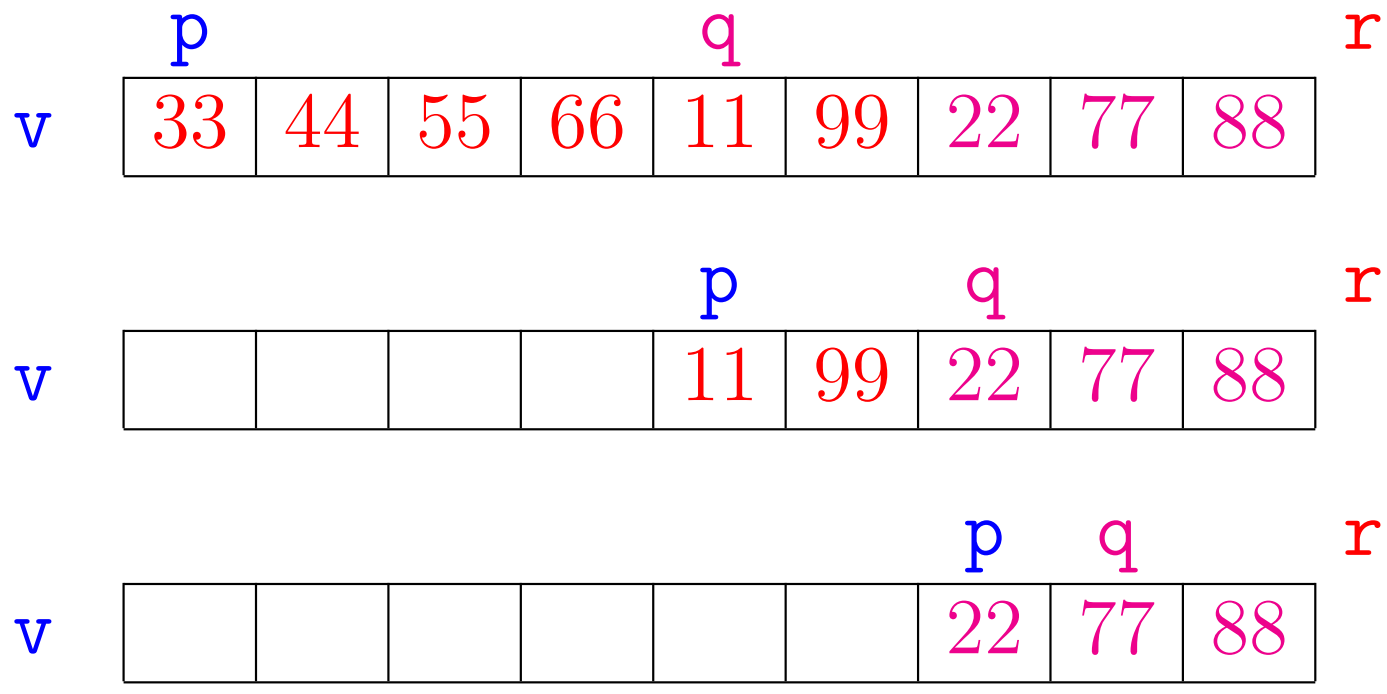

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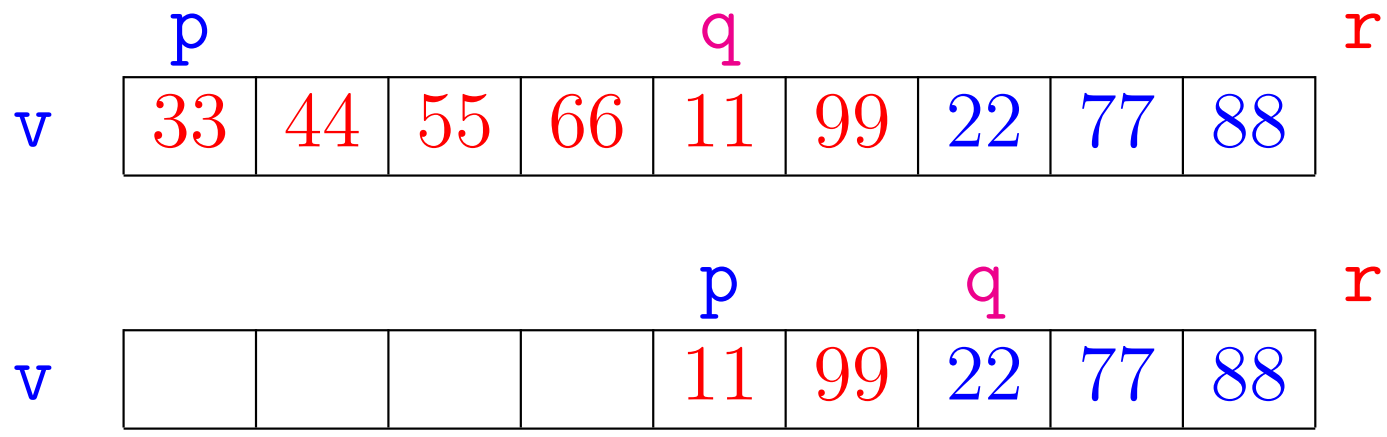

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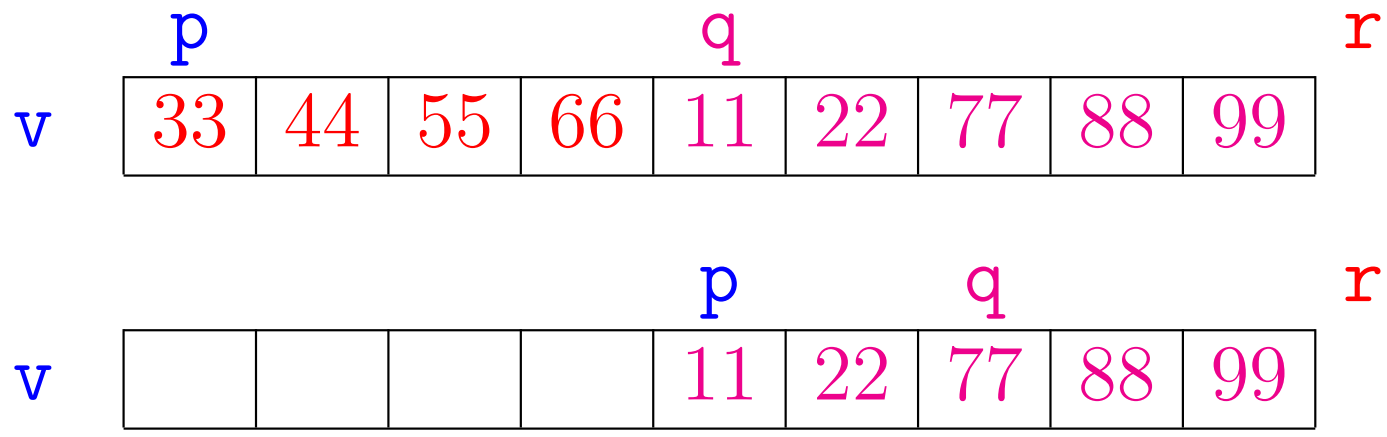

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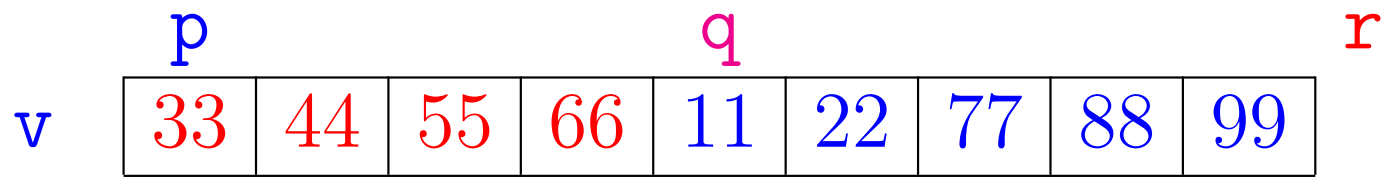

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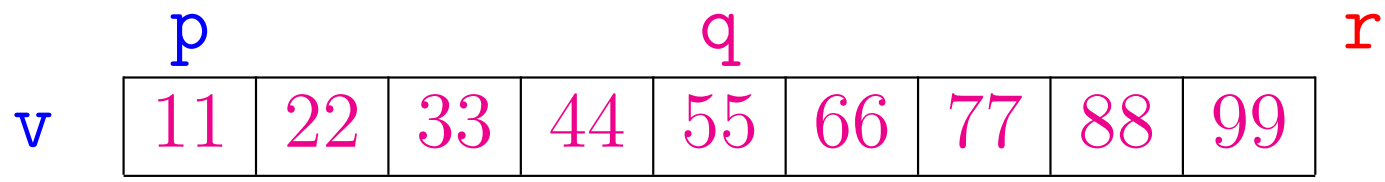

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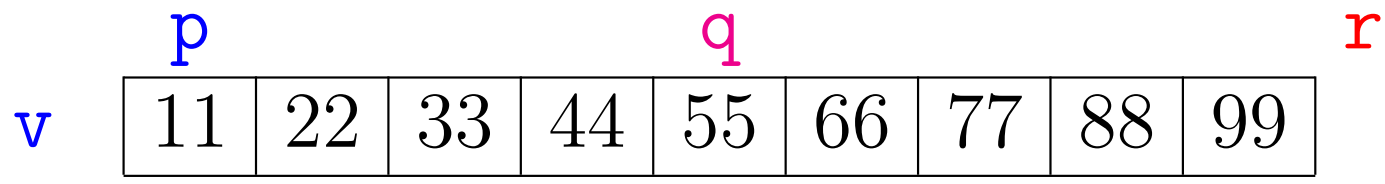
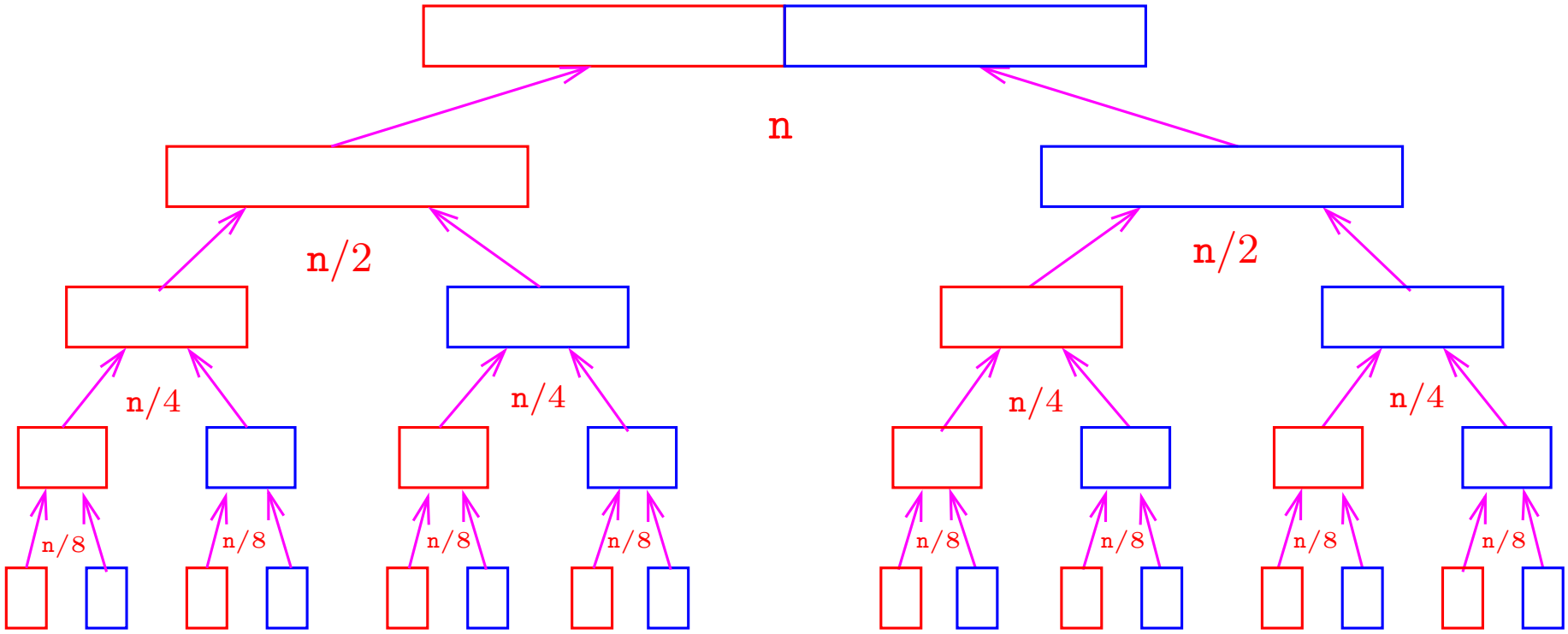


Correção

```
void mergeSort (int p, int r, int v[]) {
1   if (p < r-1) {
2      int q = (p + r)/2;
3      mergeSort(p, q, v);
4      mergeSort(q, r, v);
5      intercala(p, q, r, v);
    }
}
```

A função está correta?
A correção da função, que se apoia na correção do `intercala`, pode ser demonstrada por indução em $n := r - p$.

Consumo de tempo: versão MAC0122



Consumo de tempo: versão MAC0122

O consumo de tempo em cada nível da recursão é proporcional a n.

Há cerca de $\lg n$ níveis de recursão.

nível	consumo de tempo (proporcional a)	
1	$\approx n$	$= n$
2	$\approx n/2 + n/2$	$= n$
3	$\approx n/4 + n/4 + n/4 + n/4$	$= n$
$\ldots$	$\ldots$	
$\lg n$	$\approx 1 + 1 + 1 + 1 \cdots + 1 + 1$	$= n$
Total	$\approx n \lg n = O(n \lg n)$	

Consumo de tempo: outra versão

```
void mergeSort (int p, int r, int v[]) {
1   if (p < r-1) {
2      int q = (p + r)/2;
3      mergeSort(p, q, v);
4      mergeSort(q, r, v);
5      intercala(p, q, r, v);
    }
}
```

Consumo de tempo?

$T(n) :=$ consumo de tempo quando $n = r - p$

Consumo de tempo: outra versão

```
void mergeSort (int p, int r, int v[]) {
1   if (p < r-1) {
2       int q = (p + r)/2;
3       mergeSort(p, q, v);
4       mergeSort(q, r, v);
5       intercala(p, q, r, v);
    }
}
```

linha	consumo na linha (proporcional a)
1	?
2	?
3	?
4	?
5	?

$T(n) = ?$

Consumo de tempo: outra versão

```
void mergeSort (int p, int r, int v[]) {
1   if (p < r-1) {
2       int q = (p + r)/2;
3       mergeSort(p, q, v);
4       mergeSort(q, r, v);
5       intercala(p, q, r, v);
    }
}
```

linha	consumo na linha (proporcional a)
1	$= 1$
2	$= 1$
3	$= T(\lfloor n/2 \rfloor)$
4	$= T(\lceil n/2 \rceil)$
5	$= n$
$T(n)$	$= T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + n + 2$

Consumo de tempo: outra versão

$\mathtt{T}(\mathtt{n})$:= consumo de tempo quando $\mathtt{n} = \mathtt{r} - \mathtt{p}$

$$\mathtt{T}(1) = 1$$
$$\mathtt{T}(\mathtt{n}) = \mathtt{T}(\lceil \mathtt{n}/2 \rceil) + \mathtt{T}(\lfloor \mathtt{n}/2 \rfloor) + \mathtt{n} \quad \text{para } \mathtt{n} = 2, 3, 4, \ldots$$

Solução: $\mathtt{T}(\mathtt{n})$ é $\mathrm{O}(\mathtt{n} \log \mathtt{n})$.

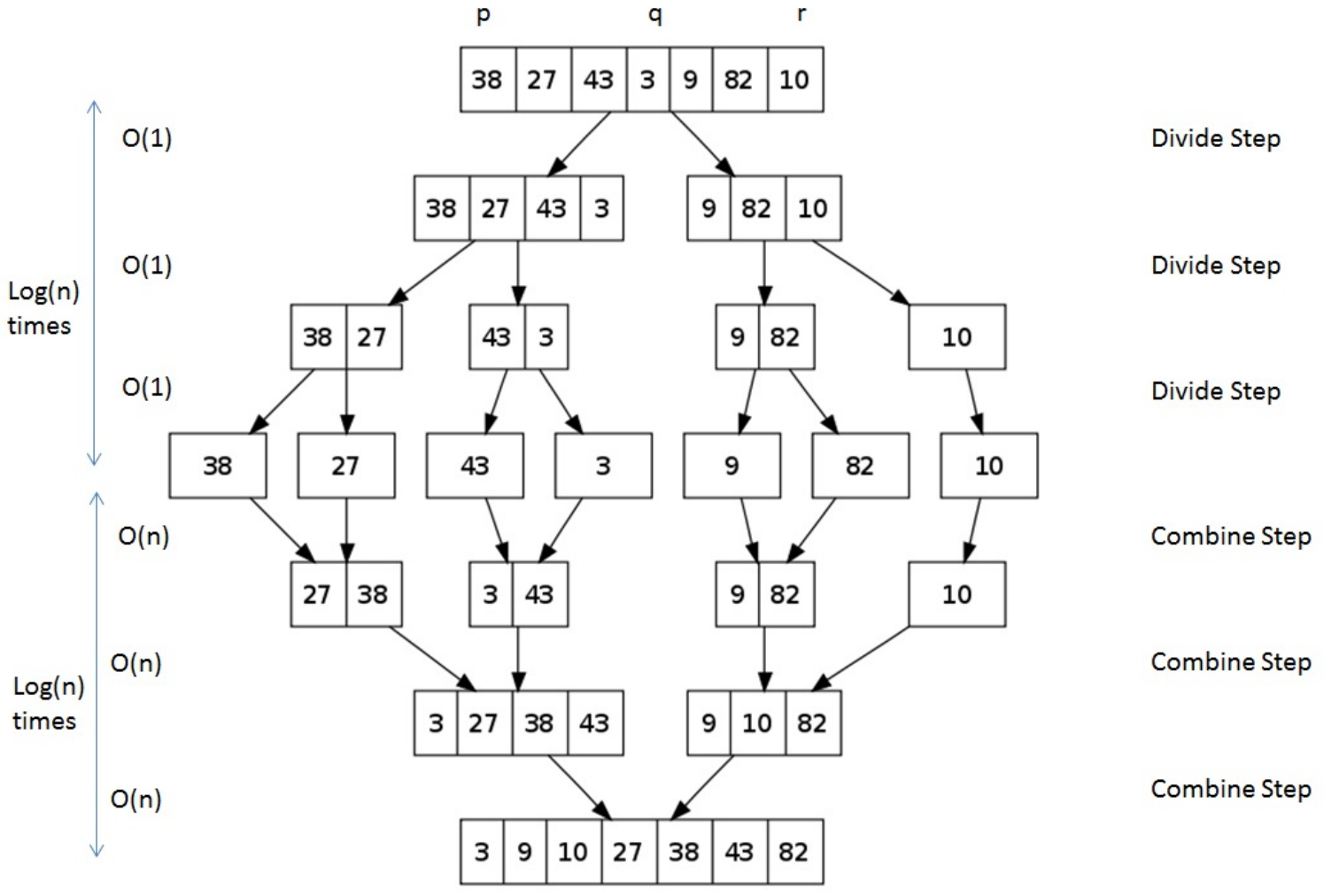
Demonstração: ...

Conclusão

O consumo de tempo da
função `mergeSort` é proporcional a $n \lg n$.

O consumo de tempo da
função `mergeSort` é $O(n \lg n)$.

Consumo de tempo



Total Runtime = Total time required in Divide + Total time required in Combine
= 1 * Log(n) + n * Log(n) = n Log(n).

Fonte: http://images.1233.tw/in-place-quicksort-algorithm/

Divisão e conquista

Algoritmos por divisão-e-conquista têm três passos em cada nível da recursão:

Dividir: o problema é dividido em subproblemas de tamanho menor;

Conquistar: os subproblemas são resolvidos recursivamente e subproblemas "pequenos" são resolvidos diretamente;

Combinar: as soluções dos subproblemas são combinadas para obter uma solução do problema original.

Exemplo: ordenação por intercalação (`mergeSort`).

mergeSort: versão iterativa

```
void mergeSort (int n, int v[]) {
   int p, r, b = 1;
   while (b < n) {
      p = 0;
      while (p + b < n) {
         r = p + 2*b;
         if (r > n) r = n;
         intercala(p, p+b, r, v);
         p = p + 2*b;
      }
      b = 2*b;
   }
}
```

mergeSort: versão iterativa

```
void mergeSort (int n, int v[]) {
   int p, r, b = 1;
   while (b < n) {
      p = 0;
      while (p + b < n) {
         r = p + 2*b;
         if (r > n) r = n;
         intercala(p, p+b, r, v);
         p = p + 2*b;
      }
      b = 2*b;
   }
}
```

Consumo de tempo: $\mathrm{O}(n \lg n)$