

# Minioficina de Combinatória

NUMEC/MaCLinC, Núcleo de Modelagem Estocástica e Complexidade  
Instituto de Matemática e Estatística, USP

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**2:00–3:00**

## On Erdős–Ko–Rado type theorems

PETER FRANKL

*Hungarian Academy of Sciences*

The lecture will focus on extremal set theory. The general problem concerns the maximum possible size of a subset of the power set of a finite set  $X$  of  $n$  elements subject to some conditions. The simplest result is probably the following.

**Proposition 0.** If  $F$  is a subset of  $2^X$  such that any two sets in  $F$  have non-empty intersection, then  $|F| \leq 2^{n-1}$ .

One way to achieve equality is by taking all subsets containing a fixed element.

**Erdős–Ko–Rado Theorem.** If  $F$  is a collection of  $k$ -element subsets of  $X$  such that any two sets in  $F$  have non-empty intersection and, moreover,  $2k < n$ , then  $|F| \leq \binom{n-1}{k-1}$ , with equality holding if and only if all subsets in  $F$  contain a fixed element.

We shall discuss various generalisations and extensions of this result, some of which are still unsolved.

**3:20–4:20**

## On two Ramsey type problems for $K_{t+1}$ -free graphs

VOJTĚCH RÖDL

*Emory University*

In 1970, Erdős and Hajnal asked if for any  $r$  and  $t$  there is a  $K_{t+1}$ -free graph  $H$  with the property that any  $r$ -coloring of the edges of  $H$  yields a monochromatic  $K_t$ . This conjecture was resolved positively by Folkman for  $r = 2$  and by Nešetřil and the speaker for  $r$  arbitrary. In this talk we will discuss some old and new results related to this conjecture.

A related question was raised by A. Hajnal. The  $t$ -independence number  $\alpha_t(H)$  of a graph  $H$  is the largest size of a subset of vertices of  $H$  containing no  $K_t$ . Let  $h_t(n)$  be the minimum value of  $\alpha_t(H)$  with the minimum taken over all graphs on  $n$  vertices containing no  $K_{t+1}$ . Hajnal proposed the problem of investigating  $h_t(n)$ . This question was addressed first by Erdős and Rogers, who proved that  $h_t(n)$  is at most  $n^{1-\epsilon}$ , where  $\epsilon = \Theta(1/t^{4 \log t})$ . Recently, jointly with Dudek and Retter, we proved that  $h_t(n) = n^{1/2+o(1)}$  for any given  $t$ .