The triangle-free process

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The triangle-free process starts with the empty graph G_0 on n vertices. At each step thereafter we set $G_{m+1} = G_m \cup \{e\}$ where e is a selected uniformly at random from the set of potential edges whose addition would not create a triangle. The process ends when we reach a maximal triangle-free graph; we denote by $G_{n,\triangle}$ this (random) final graph. Recently Bohman (2009) proved that $e(G_{n,\triangle}) = \Theta(n^{3/2}\sqrt{\log n})$ edges. His approach was based on differential equations method of Wormald and used martingale inequalities to track various parameters associated with the evolution of the process. We follow the triangle-free process until (almost) its end. We prove that, with high probability,

$$e\big(G_{n,\triangle}\big) \;=\; \left(\frac{1}{2\sqrt{2}} + o(1)\right) n^{3/2}\sqrt{\log n}\,.$$

Furthermore, by bounding the size of independent sets in $G_{n,\triangle}$, we give an improved lower bound on the Ramsey number $R(3,k)$ (the upper bound is due to Shearer):

$$\left(\frac{1}{4} - o(1)\right)\frac{k^2}{\log k} \;\leqslant\; R(3,k) \;\leqslant\; \big(1 + o(1)\big)\frac{k^2}{\log k}\,.$$

These results are based on joint work with Gonzalo Fiz Pontiveros, Robert Morris and Roberto Imbuzeiro Oliveira. Similar results have been proved independently by Bohman and Keevash.