fractal dimension is obtained by analyzing the log-log plot of the area of influence *versus* D: curves with higher slopes are obtained for simple shapes, and the Bouligand-Minkowsky fractal dimension is defined as $d = 2 - slope$, where *slope* is the slope of the just mentioned log-log plot. A technique for fractal dimension estimation based on exact dilations (see Section 5.7) has been recently described [Costa, 1999] that allows the consideration of every possible Minkowsky sausage in an orthogonal grid. Another extension reported recently is the multiscale fractal dimension, which expresses the fractal behaviour of the shape in terms of the spatial scale[2] [Costa *et al.*, 2001].

6.4 CURVATURE

6.5.2 Biological Motivation

The curvature is one of the most important features that can be extracted from contours. Indeed, strong biological motivation has been identified for studying curvature, which is apparently an important clue explored by the human visual system. In this section we discuss some important related aspects, concentrating in the developments by Fred Attneave, which have largely influenced many developments in shape analysis by computers. In one of his seminal works [Attneave, 1954], Attneave emphasized the importance that transient events and asymmetries have in human visual perception. In his work, which considered psychological issues in terms of information processing, it was claimed that the visual information is highly redundant, both in space and time. In one of his reported experiments, Attneave explored the fact that the points in a scene about which the subjects tend to make more mistakes while trying to predict continuation (see below) are precisely those carrying more information. In his experiment, an image containing a small black bottle laid on a brown table in front of a white wall was gradually shown to the subject. This image was divided into 4000 cells, organized as an array of 50 rows and 80 columns and presented to subjects in a cell-by-cell fashion, from left to right and from up to bottom. That experiment was actually a kind of game in which the subject was requested to predict the color of the next hidden cell to be shown (black, brown or white). In the beginning of the experiment, after some mistakes, the subject perceived the background homogeneity and started to correctly guess the white cells until the bottle top has been reached. In fact, this special cell presented a high error rate. After a few more mistakes, the subject perceived the bottle homogeneity, returning to correct guesses. This kind of psychophysical result illustrates the importance of edges or outline contours in images, which are associated with the physical boundaries between the scene objects.

Another type of redundancy explored by the human subjects was related to the straight line segments along the shape edges. More specifically, while the first cell containing the top of the bottle concentrated a large amount of

[2] L. da F. Costa, A. G. Campos, and E. T. M. Manoel, An Integrated Approach to Shape Analysis: Results and Perspectives, in *Proceedings of International Conference on Quality Control by Artificial Vision*, Vol. 1, 23-34, 2001

information (i.e., presented a high error rate), the next cells in the following rows presented less errors, since the subjects started to correctly infer the cells' color along the prolongation of the line segment along the following cells. This experiment demonstrated that since *corners* and *high curvature points* tended to induce higher error probability, they should *concentrate more information.*

Finally, the subjects were verified to be fully capable of correctly extrapolating the bottle *symmetry*, which implied the right side of the bottle to be perceived with fewer errors. Based on these considerations, it is possible to compare the information content of several shape elements, as shown in Table 6.2.

Table 6.2: *Some interesting relationships identified in Attneave's work.*

LESS INFORMATION	MORE INFORMATION
Homogeneous Color Region	Edges
Straight Lines (Null Curvature) and Constant Curvature Segments	Corners and High Curvature Points
(Local) Periodic Patterns	Extremities of Periodic Patterns
Symmetries	Asymmetries

Attneave remarked that even Gestalt principles [Rock and Palmer, 1990] can be interpreted under the viewpoint of information redundancy exploration. Since typical Gestalt images present a high degree of some kind of redundancy, they induce our perceptual system to explore such redundancies in order to achieve enhanced perception.

We conclude the discussion about Attneave's work by briefly describing two other of his psychophysical experiments. In the first, a series of 2D shapes was presented to the subjects, who were requested to represent each contour in terms of a set of 10 points. The results were presented as a histogram superimposed onto the shapes contours, from which it was clear that most subjects preferred to use high curvature points to represent each contour. In another experiment, Attneave created the now-classical picture of a cat by identifying high curvature points in an ordinary snapshot and linking them by line segments, showing that most of the information in the original image concentrated at the curvature extrema. Similar experiments and results have been reported in [Fischler and Wolf, 1994].

To probe further: ***Biological curvature analysis***

The reader interested in further material on the biological importance of curvature analysis is referred to [Biederman, 1985; Blakemore and Over, 1974; Dobbins et al., 1987; Gibson, 1933; Guez et al, 1994; Levine, 1985; Perrett and Oram, 1993; Timney and Macdonald, 1978].

As far as computational shape analysis is concerned, curvature plays an important role in the identification of many different geometric shape primitives. Table 6.3, adapted from [Levine, 1985], summarizes some important geometrical aspects that can be characterized by considering curvature.