The stabilized set of p's in Krivine's theorem can be disconnected

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Brazilian Workshop in Geometry of Banach spaces
25 August 2014, Maresias

European Union
European Social Fund

OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS, CULTURE & SPORTS
MANAGING AUTHORITY

NSRF 2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co-financed by Greece and the European Union

The main problem

- **Question:** Let X be a Banach space with a basis, with a stabilized Krivine set F.
- Is F necessarily connected?
- During this lecture we shall demonstrate that this need ***not*** always be the case.

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Preliminaries: finite block representability

- Let X be a Banach space with a Schauder basis $(x_i)_i$.
- Let also $(e_j)_j$ be a Schauder basic sequence, not necessarily in X.
- We say that $(e_j)_j$ *is finitely block represented in* $(x_i)_i$ (or simply in X) if:

  for every natural number n and $\varepsilon > 0$ there exists

  a finite block sequence $(y_j)_{j=1}^n$ of $(x_i)_i$

  that is $(1+\varepsilon)$-equivalent to $(e_j)_{j=1}^n$.

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Preliminaries: spreading models

- Let X be a Banach space and $(x_i)_i$ be a sequence in X.
- Let also $(e_j)_j$ be a sequence in some Banach space, which is not necessarily X.
- We say that *$(x_i)_i$ generates $(e_j)_j$ as a spreading model* if:

  for every natural number n and $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$

  such that for every natural numbers $n_0 \leqslant i_1 < \cdots < i_n$

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- An easy observation: if X is a Banach space with a basis $(x_i)_i$
- and $(y_j)_j$ is a block sequence of $(x_i)_i$ that generates some sequence $(e_j)_j$ as a spreading model,
- then $(e_j)_j$ is finitely block represented in $(x_i)_i$.

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Krivine's theorem

Theorem (J. L. Krivine)

Let X be a Banach space with a Schauder basis. Then there exists a $p \in [1, \infty]$ such that the unit vector basis of ℓ_p is finitely block represented in X (the case $p = \infty$ refers to the unit vector basis of c_0).

- The set of all p's that are finitely block represented in X is called *the Krivine set of X* and is denoted by $K(X)$.
- **Remark:** It follows that if for some p, X admits a spreading model equivalent to the unit vector basis of ℓ_p, then p is in the Krivine set of X.

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Krivine Set

- In his paper on Krivine's theorem, H. P. Rosenthal observed the following:
- On some block subspace Y of X, the Krivine set is stabilized, i.e.
- if Z is a further block subspace of Y then the sets $K(Y)$ and $K(Z)$ coincide.
- Rosenthal concluded his paper with the following question:
- Is such a stabilized Krivine set necessarily always a singleton?

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- E. Odell and Th. Schlumprecht constructed a space X with the property that every 1-unconditional basic sequence is finitely block represented in every block subspace Y of X.
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- **Next Question:** Let X be a Banach space with a basis, with a stabilized Krivine set F.
- Is F necessarily connected?
- This question first appeared in a paper by P. Habala and N. Tomczak-Jaegermann and was also later mentioned by Odell as one of 15 open problems in Banach spaces.
- In their paper Habala and Tomczak-Jaegermann prove the following:
- if $p < q$ are in the stabilized Krivine set of X, then X admits a block quotient Z such that every $r \in [p, q]$ is finitely block represented in Z.

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The main result

Theorem

Let $F \subset [1, \infty]$ be either a finite set or a set consisting of an increasing sequence and its limit. Then there exists a reflexive Banach space X with an unconditional basis such that for every infinite dimensional block subspace Y of X:

(i) *For all $1 \leqslant p \leqslant \infty$, the space ℓ_p is finitely block represented in Y if and only if $p \in F$.*

(ii) *If F is finite then the spreading models admitted by Y are exactly the spaces ℓ_p for $p \in F$.*

(iii) *If F is an increasing sequence with limit p_ω then every spreading model admitted by Y is isomorphic to ℓ_p for some $p \in F$ and for every $p \in F \setminus \{p_\omega\}$ ℓ_p is admitted as a spreading model by Y.*

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Disconnected Krivine Set

- In particular, the stabilized Krivine set of X is F (which is either finite or consists of an increasing sequence and its limit) and hence not connected.
- This space also answers some questions concerning spreading models, which were asked by G. Androulakis, Odell, Schlumprecht and Tomczak-Jaegermann.

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Spreading models

- **Question:** Let n be a natural number. Does there exists a Banach space X such that every subspace admits n-many spreading models?
- **Answer:** Yes, and they can be chosen to be ℓ_p's for $p \in F$ for any n-set $F \subset [1, \infty]$.

- **Question:** Does there exists a Banach space X such that every subspace admits countably infinite many spreading models?
- **Answer:** Yes, for F an increasing sequence the space constructed has this property.

- **Question:** Let X be a Banach spaces such that every subspace admits both ℓ_1 and ℓ_2 spreading models. Does X admit uncountably many spreading models?
- **Answer:** No, for $F = \{1, 2\}$ the space constructed admits only ℓ_1 and ℓ_2 spreading models in every subspace.

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- It is worth pointing out that the previously stated theorem is false if stated for F a decreasing sequence and its limit.
- Indeed, as B. Sari has proved, if a Banach space admits a strictly increasing, with respect to domination, sequence of spreading models, then it admits uncountably many spreading models.

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The construction

- The definition of the norm uses the method of saturation under constraints, a method initialized by Odell and Schlumprecht to construct the earlier mentioned space with $[1, \infty]$ as its stabilized Krivine set.
- The construction method used in the preset case is more related to the one used by S. Argyros, K. Beanland and P. M. to construct Tsirelson like reflexive spaces. Among the properties of these spaces is that they admit only ℓ_1 and c_0 as a spreading model in every subspace.
- Actually, the space X we construct for $F = \{1, \infty\}$ is a slight modification of the simplest case presented in that paper.

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- From now on let us assume that F consists of a strictly increasing sequence $(p_k)_{k=1}^{\infty}$ and its limit p_ω. (The case in which F is finite is the same)
- We fix a constant $0 < \theta \leqslant 1/4$.
- The norm $\|\cdot\|_*$ of the space X satisfies an implicit formula which is based on countably infinite many layers.
- Each layer also comes in various sizes.

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The implicit formula

- The base layer: for $m \in \mathbb{N}$ and $x \in c_{00}(\mathbb{N})$ define

$$\|x\|_{0,m} = \theta \sup \frac{1}{m^{1/p'_\omega}} \sum_{q=1}^{m} \|E_q x\|_*$$

  where p'_ω denotes the conjugate exponent of p_ω and the supremum is taken over all successive subsets of the natural number $E_1 < \cdots < E_m$.

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  where p'_ω denotes the conjugate exponent of p_ω and the supremum is taken over all successive subsets of the natural number $E_1 < \cdots < E_m$.

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We assume that for some k, the layers $0, \ldots, k-1$ have been defined, i.e. for every layer $0 \leqslant i < k$ and every size $m \in \mathbb{N}$, the norm $\| \cdot \|_{i,m}$ has been defined.

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- The k'th layer: for $m \in \mathbb{N}$ and $x \in c_{00}(\mathbb{N})$ define

$$\|x\|_{k,m} = \theta \sup \left(\sum_{q=1}^{d} \|E_q x\|_{i_q,m_q}^{p_k} \right)^{1/p_k}$$

where the supremum is taken over all $d \in \mathbb{N}$, $0 \leqslant i_q < k$ and all admissible and very fast growing $(E_q)_{q=1}^d$, $(m_q)_{q=1}^d$, i.e. they satisfy

$$d \leqslant E_1 < \cdots < E_d, \quad \min E_i > (\max E_{i-1})^2 \text{ and}$$

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The α-indices

- To show that the set F is contained in the Krivine set of every block subspace of X, we show that for every k, ℓ_{p_k} is admitted as a spreading model by all subspaces of X.
- In the present construction we use the α-indices of a block sequence.
- The α-index is a tool that has been used in earlier works related to the method of constraints.
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The α-indices: definition of the $\alpha_{<k}$-index

- Let k be a natural number and $(x_i)_i$ be a block sequence.

If for every layer $0 \leqslant k' < k$ and strictly increasing sequence of sizes $(m_q)_q$, for every $(x_{i_q})_q$ subsequence of $(x_i)_i$ we have that

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Proposition

Let $(x_i)_i$ be a seminormalized block sequence in X generating a spreading model $(y_j)_j$.

- *The spreading model $(y_j)_j$ of $(x_i)_i$ is equivalent to the unit vector basis of ℓ_{p_ω} if and only if the $\alpha_{<k}$ index of $(x_i)_i$ is zero for every k.*
- *For every $k \in \mathbb{N}$, the spreading model $(y_j)_j$ of $(x_i)_i$ is equivalent to the unit vector basis of ℓ_{p_k} if and only if the $\alpha_{<k}$ index of $(x_i)_i$ is not zero, while the $\alpha_{<k'}$ index is zero for all $k' < k$.*

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- We conclude that every spreading model admitted by X has to be ℓ_p, for some $p \in F$.
- It is also shown that all p's in F, with the possible exception of p_ω, occur as spreading models in every block subspace of X.

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- Since for every $p \in F \setminus \{p_\omega\}$, every block subspace of X admits an ℓ_p spreading model,

  we conclude that $F \setminus \{p_\omega\}$ and hence also F, is in the Krivine set of every block subspace of X

- We also show, by contradiction, that for every $p \notin F$, the unit vector basis of ℓ_p is not finitely block represented in X.
- We derive that the Krivine set of every block subspace of X is precisely F.
- Let us note that it is not known to us whether every block subspace of X admits an ℓ_{p_ω} spreading model.

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- Some words on how to obtain the desired spreading models in a a block subspace.
- We use the following: if a block sequence generates an ℓ_{p_k} spreading model, then an appropriate blocking of this sequence generates an $\ell_{p_{k+1}}$ spreading model. This blocking can be chosen to be increasing p_k-averages.
- It is therefore sufficient to prove that every block subspace of X admits an ℓ_{p_1} spreading model.
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- Some words on how to prove that for $p \notin F$, ℓ_p is not finitely block represented in X.
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- If $p \in [p_1, p_\omega] \setminus F$ then the proof is more technical.

  Roughly speaking, let us assume that k is such that $p_k < p < p_{k+1}$, N is sufficiently large, ε is sufficiently small and

  $(x_i)_{i=1}^N$ is a block sequence $(1+\varepsilon)$-equivalent to the unit vector basis of ℓ_p^N,

- The $k+1$ layer of the norm provides $\ell_{p_{k+1}}$ structure to the space and hence

  the k'th level is the one that has to be used to provide the ℓ_p estimate on some vectors.

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The Krivine set of BLock subspaces of X.

- If $p \in [p_1, p_\omega] \setminus F$ then the proof is more technical.

  Roughly speaking, let us assume that k is such that $p_k < p < p_{k+1}$, N is sufficiently large, ε is sufficiently small and

  $(x_i)_{i=1}^N$ is a block sequence $(1+\varepsilon)$-equivalent to the unit vector basis of ℓ_p^N,

- The $k+1$ layer of the norm provides $\ell_{p_{k+1}}$ structure to the space and hence

  the k'th level is the one that has to be used to provide the ℓ_p estimate on some vectors.

- It turns out however that the ℓ_{p_k} structure imposed by the k'th level demolishes the ℓ_p one of the sequence.

Thank you!