

The elusive geometry of the Banach space ℓ_∞/c_0

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Banach spaces and topological preliminaries

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Does every subspace of ℓ_∞/c_0 isomorphic to ℓ_∞/c_0 contains a further subspace isomorphic to ℓ_∞/c_0 which is complemented in the entire ℓ_∞/c_0 ?

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Suppose that X, Y are Banach spaces and

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Theorem (Dow, Shelah - Top. Ap. 2008)

It is consistent that there are two disjoint open subsets $U, V \subseteq N^$ with $\overline{U} \cap \overline{V} = \{x\}$ for some $x \in N^*$ and neither \overline{U} nor \overline{V} is homeomorphic to N^* .*

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Some references:

L. Drewnowski, J. Roberts, *On the primariness of the Banach space ℓ_∞/c_0* . Proc. Amer. Math. Soc. 112 (1991), no. 4, 949–957.

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