Type of multilinear operators and polynomials

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1.1 - Introduction

The concepts of type and cotype of Banach spaces were introduced mainly by J. Hoffmann-Jørgensen and by B. Maurey in the study of Banach spaces-valued random variables.

Since then the theory of type and cotype have found several applications and became a central part of the geometry of Banach spaces, and of the linear and multilinear operator theory and operator ideals.

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1.2 - Our goals

Our main goals throughout this recent research has been to define and study these concepts in a multilinear scenario setting up the relationships of new definitions with the linear and multilinear theory already established.

We introduce the class of multilinear operators of type $(p_1, \ldots, p_n)$ and polynomials of type p and show some relationships between the first one with the multi-ideals constructed by composition and linearization methods from the class of linear operator of type p.

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2.1 - Linear operators of type p

Let the sequence $(r_j)_{j=1}^{\infty}$ of the Rademacher functions. We say that a sequence $(x_j)_{j=1}^{\infty}$ in E is *almost unconditionally summable* if the series $\sum_j r_j(t)x_j$ converges in $L_2([0,1];E)$.

The set of these sequences, denoted $\mathrm{Rad}(E)$, is a Banach space equipped with the norm

$$||(x_i)_{i=1}^{\infty}||_{\mathrm{Rad}(E)} = \left\|\sum_{j=1}^{\infty} r_j x_j\right\|_{L_2(E)} = \left(\int_0^1 \left\|\sum_{j=1}^{\infty} r_j(t) x_j\right\|^2 dt\right)^{1/2}.$$

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We say that an operator $T \in \mathcal{L}(E;F)$ is of *type p*, $1 \leq p \leq 2$, if there is some constant $C > 0$ such that

$$\left(\int_0^1 \left\| \sum_{j=1}^{k} r_j(t) T(x_j) \right\|^2 dt \right)^{1/2} \leq C \left(\sum_{j=1}^{k} \|x_j\|^p \right)^{1/p}, \qquad (1)$$

for any $k \in \mathbb{N}$ and for all $x_1, ..., x_k \in E$. A normed space E is called of type p if id_E has type p.

It is a folklore that the set of all linear operators from E into F of type p, denoted by $\tau_p(E;F)$, is a Banach space with usual operations and under the norm $||\cdot||_{\tau_p} := \inf\{C > 0, \text{ such that } (1) \text{ holds}\}$.

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2.1 - Multilinear and polynomial definitions

Let $p, p_1, ..., p_n \in (0, +\infty)$.

Definition: A continuous n-linear operator $T \in \mathcal{L}(E_1, ..., E_n; F)$ has *type* $(p_1, ..., p_n)$, $\frac{1}{2} \leq \frac{1}{p_1} + \cdots + \frac{1}{p_n} \leq 1$, if there is a constant $C > 0$ such that, however we choose finitely many vectors $\left(x_j^{(1)}, ..., x_j^{(n)}\right)$ in $E_1 \times \cdots \times E_n$, $j \in \{1, ..., k\}$,

$$\left(\int_0^1 \left\|\sum_{j=1}^k r_j(t) T\left(x_j^{(1)}, ..., x_j^{(n)}\right)\right\|^2 dt\right)^{1/2} \leq C \prod_{i=1}^n \left\|\left(x_j^{(i)}\right)_{j=1}^k\right\|_{p_i}. \qquad (2)$$

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Definition: A continuous n-homogeneous polynomial $P \in \mathcal{P}(^nE; F)$ has *type p*, $n \leq p \leq 2n$, if there is a constant $C > 0$ for which

$$\left(\int_0^1 \left\| \sum_{j=1}^{k} r_j(t) P(x_j) \right\|^2 dt \right)^{1/2} \leq C \cdot \left(\sum_{j=1}^{k} \|x_j\|^p \right)^{n/p}, \qquad (3)$$

regardless of the choice of finitely many vectors $x_1, ..., x_k$ in E.

2.1 - Multilinear and polynomial definitions

The sets of all n-linear operators of type $(p_1, ..., p_n)$ from $E_1 \times \cdots \times E_n$ into F and all n-homogeneous polynomials of type p from E into F are denoted by $\tau^n_{(p_1,\ldots,p_n)}(E_1, ..., E_n; F)$ and by $\mathcal{P}_{\tau^n_{(p_1,\ldots,p_n)}}(^nE; F)$. These sets, provided with the usual operations, are a Banach subspaces of $\mathcal{L}(E_1, ..., E_n; F)$ and $\mathcal{P}(^nE; F)$ equipped with the norms

$$||\cdot||_{\tau^n_{(p_1,\ldots,p_n)}} := \inf\{C > 0, \text{ such that } (2) \text{ holds}\}$$

and

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2.2 - Examples

Let us show some examples:

- The continuous 2-linear operator

$$T : l_1 \times l_1 \longrightarrow l_1$$
$$\left((x_j)_{j=1}^{\infty}, (y_j)_{j=1}^{\infty}\right) \mapsto (x_j y_j)_{j=1}^{\infty}$$

does not have any proper type (p_1, p_2);

- Let $E_1, ..., E_n$ be Banach spaces such that E_n has type p_n, $\varphi_i \in E_i^{'}$, $i = 1, ..., n-1$, and the continuous n-linear operator T defined by

$$T : E_1 \times \cdots \times E_n \longrightarrow E_n$$
$$(x_1, ..., x_n) \mapsto \varphi_1(x_1) \cdots \varphi_{n-1}(x_{n-1}).x_n.$$

Then T has type $(p_1, ..., p_n)$ for any $p_1, ..., p_n$ such that $\frac{1}{2} \leq \frac{1}{p_1} + \cdots + \frac{1}{p_n} \leq 1$;

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3.1 - Some definitions and notation required

A Banach multi-ideal $(\mathcal{M}, \|\cdot\|_{\mathcal{M}})$ is a subclass of the all class of continuous multilinear operators between Banach spaces such that for any $n \in \mathbb{N}$ and Banach spaces $E_1, ..., E_n$ and F, the components $\mathcal{M}(E_1, ..., E_n; F) = \mathcal{L}(E_1, ..., E_n; F) \cap \mathcal{M}$ satisfy:

i) $\mathcal{M}(E_1, ..., E_n; F)$ is a complete subspace of $\mathcal{L}(E_1, ..., E_n; F)$ which contains the n-linear finite type operators;

ii) The ideal property: if $A \in \mathcal{M}(E_1, ..., E_n; F)$, $u_j \in \mathcal{L}(G_j, E_j)$ for $j = 1, ..., n$ and $t \in \mathcal{L}(F; H)$, then $tA(u_1, ..., u_n) \in \mathcal{M}(G_1, ..., G_n; H)$ and

$$\|tA(u_1, ..., u_n)\|_{\mathcal{M}} \leq \|t\| \, \|A\|_{\mathcal{M}} \|u_1\| \cdots \|u_n\|.$$

iii) the operator $\|id_{\mathbb{K}^n} : \mathbb{K}^n \longrightarrow \mathbb{K} : id_{\mathbb{K}^n}(x_1, ..., x_n) = x_1 \cdots x_n\|_{\mathcal{M}} = 1$, for all $n \in \mathbb{N}$;

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We notice that

- $\left(\tau_p, ||\cdot||_{\tau_p}\right)$ is a Banach Ideal of linear operators; (this is well known)
- $\left(\tau^n_{(p_1,\ldots,p_n)}, ||\cdot||_{\tau^n_{(p_1,\ldots,p_n)}}\right)$ is a Banach multi-ideal (we proved this in our work).

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3.1 - Some definitions and notation required

We will consider two methods to build multi-ideals from ideals of linear operators. Given $\mathcal{I}_1, ..., \mathcal{I}_n$ ideals of linear operators:

1) An n-linear mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ is said to be of type $[\mathcal{I}_1, ..., \mathcal{I}_n]$, in symbols $T \in [\mathcal{I}_1, ..., \mathcal{I}_n](E_1, ..., E_n; F)$, if $I_j(T) \in \mathcal{I}_j\left(E_j; \mathcal{L}\left(E_1, \overset{[j]}{\dots}, E_n; F\right)\right)$, for all $j \in \{1, ..., n\}$, where the operator $I_j : \mathcal{L}(E_1, ..., E_n; F) \to \mathcal{L}\left(E_j; \mathcal{L}\left(E_1, \overset{[j]}{\dots}, E_n; F\right)\right)$ is defined by

$$I_j(T)(x_j)(x_1, \overset{[j]}{\dots}, x_n) = T(x_1, ..., x_n),$$

and $\overset{[j]}{\dots}$ means that the j-th coordinate is not involved.

If $\mathcal{I}_1, ..., \mathcal{I}_n$ are normed ideals, we define

$$||T||_{[\mathcal{I}_1, ..., \mathcal{I}_n]} = \max\left\{||I_1(T)||_{\mathcal{I}_1} \cdots ||I_n(T)||_{\mathcal{I}_n}\right\},$$

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2) Let $\mathcal{I}$ be an ideal of linear operators. A mapping $T \in \mathcal{L}(E_1, ..., E_n; F)$ belongs to $\mathcal{I} \circ \mathcal{L}$, denoted by $T \in \mathcal{I} \circ \mathcal{L}(E_1, ..., E_n; F)$, if there are a Banach space G, an operator $u \in \mathcal{I}(G; F)$ and an n-linear mapping $B \in \mathcal{L}(E_1, ..., E_n; F)$ such that $T = u \circ B$.

If $\mathcal{I}$ is a normed ideal, for $T \in \mathcal{I} \circ \mathcal{L}(E_1, ..., E_n; F)$, we define

$$||T||_{\mathcal{I} \circ \mathcal{L}} = \inf \{||B||||u||_{\mathcal{I}} : T = u \circ B, \text{ with } u \text{ and } B \text{ as above}\}.$$

It is well-known that if $\mathcal{I}$ and $\mathcal{I}_1, ..., \mathcal{I}_n$ are normed ideals then $([\mathcal{I}_1, ..., \mathcal{I}_n], ||\cdot||_{[\mathcal{I}_1, ..., \mathcal{I}_n]})$ and $(\mathcal{I} \circ \mathcal{L}, ||\cdot||_{\mathcal{I} \circ \mathcal{L}})$ are normed ideals of multilinear mappings.

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3.1 - Relationship with the ideals $\tau_p \circ \mathcal{L}$ and $[\tau_{q_1}, ..., \tau_{q_n}]$

So, we prove that

Theorem

If $1 < p \leq 2$ *and* $\frac{1}{p} = \frac{1}{p_1} + \cdots + \frac{1}{p_n}$ *then* $\tau_p \circ \mathcal{L} \subseteq \tau^n_{(p_1,\ldots,p_n)}$*, for all* $n \in \mathbb{N}$ *and*

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and, for instance, that

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If $1 < q_1, ..., q_n \leq 2$ *and* $\frac{1}{2} \leq \frac{1}{p_1} + \cdots + \frac{1}{p_n} < 1$ *then*

$$\tau^n_{(p_1,\ldots,p_n)} \not\subseteq [\tau_{q_1}, ..., \tau_{q_n}],$$

for all $n \in \mathbb{N}$.

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Thank you very much!