MAT2456 - Cálculo Diferencial e Integral para Engenharia IV

2o. Semestre de 2013 - 2a. Lista de exercícios

Respostas

1. a) $R = 1/4$; b) $R = 1/2$; c) $R = 1$; d) $R = e$; e) $R = \sqrt[3]{e}$; f) $R = 1$.

2. a) $]-1, 1[$; b) $]-5/3, 5/3[$; c) $]-3/2, 3/2[$; d) $[-1, 1[$; e) $]-b-1, b-1[$.

3 (a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{5^{2n}},\ -1/5 < x < 1/5$ (b) $\sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}}{2n+1}x^{2n+1},\ -1/2 \le x <\le 1/2$

(c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(n+1)x^n,\ -1 < x < 1$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n(n+2)(n+1)}{2}\ x^n, -1 < x < 1$

(e) $\displaystyle 2\left(\sum_{n=0}^{\infty}(-1)^n x^{4n+1}\right), -1 < x < 1$ (f) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}, -1 < x \le 1$

(g) $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^n}{n}\ x^{2n},\ \frac{-1}{\sqrt{3}} \le x \le \frac{1}{\sqrt{3}}$ (h) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1-(-1)^n 2^n}{3}\right)x^n,\ \frac{-1}{2} < x < \frac{1}{2}$

(i) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{5n+2}\ x^{5n+2},\ -1 < x \le 1$ (j) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n 2^n}{(2n+1)^2}x^n,\ -1 \le x \le 1$

5. a) $-\ln(1-x)$ b) $\ln(1+x)$ c) $\ln\sqrt{\dfrac{1+x}{1-x}}$ d) $\operatorname{arctg} x$ e) $\dfrac{1}{(1-x)^2}$ f) $\dfrac{x}{(1-x)^2}$

g) $\dfrac{x}{(1-x^2)^2}$ h) $(1+x)\ln(1+x)-x$ i) $\dfrac{1+x}{(1-x)^3}$ j) $\dfrac{x+4x^2+x^3}{(1-x)^4}$ k) $\dfrac{4-3x}{(1-x)^2}$ l) $\frac{-1}{4}\ln(1-x^4)$.

6. $\ln 2$; 26; $\frac{6}{5}\ln\frac{6}{5} - \frac{1}{5}$.

9. 0 e $(320)!$.

10 .

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{n+2}}{n!},\ x \in \mathbb{R}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{(2n)!},\ x \ge 0$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{(2n+1)!},\ x \in \mathbb{R}$ (d) $\displaystyle 1 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n} x^{2n}}{(2n)!},\ x \in \mathbb{R}$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{(2x)^{2n+1}}{n!},\ x \in \mathbb{R}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+2}}{(2n+1)},\ -1 \ge x \ge 1$

11 .

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)(2n+1)!},\ x \in \mathbb{R}$ (b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)n!},\ x \in \mathbb{R}$

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n^2},\ -1 < x \le 1$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)!},\ x \in \mathbb{R}$

12. (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6!}$, se $\alpha = 6$; 0 , se $\alpha < 6$; ∞ , se $\alpha > 6$

(e) $-\frac{1}{7!}$, se $\alpha = 7$; 0 , se $\alpha < 7$; ∞ , se $\alpha > 7$

13 .

(a) $\frac{a+b}{2} + \frac{2}{\pi}(b-a)\sum_{n=1}^{\infty} \frac{\operatorname{sen}((2n-1)x)}{2n-1}$.

soma: a, se $(2k-1)\pi < x < 2k\pi$; b, se $2k\pi < x < (2k+1)\pi$; $\frac{a+b}{2}$, se $x = k\pi$, $k \in \mathbb{Z}$

(b) $\frac{b-a}{4}\pi + \frac{2}{\pi}(a-b)\sum_{n=1}^{\infty} \frac{\cos((2n-1)x)}{(2n-1)^2} - (a+b)\sum_{n=1}^{\infty}(-1)^n \frac{\operatorname{sen}(nx)}{n}$.

soma: ax, se $(2k-1)\pi < x \leq 2k\pi$; bx, se $2k\pi \leq x < (2k+1)\pi$; $\frac{b-a}{2}\pi$, se $x = (2k+1)\pi$, $k \in \mathbb{Z}$.

(c) $\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos((2n-1)x)}{(2n-1)^2}$

soma: $|x|$, se $-\pi \leq x \leq \pi$ e sua extensão periódica para $x \in \mathbb{R}$.

(d) $\frac{\operatorname{sinh}(a\pi)}{a\pi} + \frac{2\operatorname{sinh}(a\pi)}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+a^2}(a\cos(nx) - n\operatorname{sen}(nx))$.

soma: e^{ax}, se $-\pi < x < \pi$; $\cosh(a\pi)$ se $x = \pm\pi$, e a sua extensão periódica para $x \in \mathbb{R}$

(e) $\frac{2\operatorname{sen}(a\pi)}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n^2-a^2}\operatorname{sen}(nx)$

soma: $\operatorname{sen}(ax)$, para $-\pi < x < \pi$; 0 para $x = \pm\pi$ e a sua extensão periódica para $x \in \mathbb{R}$.

(f) $b + 2a\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n}$.

soma: $ax + b$, para $-\pi < x < \pi$; b, para $x = \pm\pi$, e sua extensão periódica, para $x \in \mathbb{R}$.

(g) $\frac{2}{\pi}\left(1 + 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos(2nx)}{4n^2-1}\right)$

soma : $|\cos x|$, para $x \in \mathbb{R}$

14 .

(a) $2a\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n}$

soma : ax, para $-\pi < x < \pi$; 0 para $x = \pm\pi$, e sua extensão periódica para $x \in \mathbb{R}$.

$\frac{a}{2}\pi - \frac{4a}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$,

soma : $a|x|$, para $-\pi \leq x \leq \pi$ e sua extensão periódica para $x \in \mathbb{R}$.

(b) $2\pi\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\operatorname{sen}(nx)}{n} - \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\operatorname{sen}((2n-1)x)}{(2n-1)^3}$.

soma : x^2 para $0 \leq x < \pi$; $-x^2$, para $-\pi \leq x \leq 0$; 0 para $x = \pm\pi$ e sua extensão periódica para $x \in \mathbb{R}$

$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{\cos(nx)}{n^2}$.

soma : x^2, para $-\pi \leq x \leq \pi$ e sua extensão periódica para $x \in \mathbb{R}$

(c) $\sum_{n=1}^{\infty}\frac{2}{n\pi}(b - (a\pi + b)(-1)^n)\operatorname{sen}(nx)$.

soma : $ax - b$, para $-\pi < x \leq 0$; $ax + b$, para $0 < x < \pi$, 0, para $x = \pm\pi$, 0 e sua extensão periódica para $x \in \mathbb{R}$.

$\frac{a\pi}{2} + b - \frac{4a}{\pi}\sum_{n=1}^{\infty}\frac{\cos((2n-1)x)}{(2n-1)^2}$.

soma : $a|x| + b$, para $-\pi \leq x \leq \pi$ e sua extensão periódica de $a|x| + b$ para $x \in \mathbb{R}$.

(d) $\operatorname{sen} x$,
soma : $\operatorname{sen} x$, para $x \in \mathbb{R}$;
$\frac{2}{\pi}(1 - 2\sum_{n=1}^{\infty} \frac{\cos(2nx)}{4n^2-1})$,
soma : $|\operatorname{sen} x|$ para $x \in \mathbb{R}$

(e) $\frac{e}{\pi} \operatorname{sen}\ x + \sum_{n=2}^{\infty} \frac{2n-1+(-1)^n}{\pi n(n-1)} \operatorname{sen}((2n-1)x)$,
soma : $-|\cos x|$ para $x \leq 0$ e $|\cos x|$ para $x \geq 0,\ x \neq k\pi$ e 0 para $x = k\pi,\ k \in \mathbb{Z}$.
$\frac{2}{\pi}(1 + 2\sum_{n=1}^{\infty}(-1)^{n-1} \frac{\cos(2nx)}{4n^2-1}$,
soma : $|\cos x|$, para $x \in \mathbb{R}$

16 .

(a) usar 15a) em $x = \pi/2$ (b) usar 15c) em $x = \pi$
(c) usar 15e) em $x = \pi/2$ (d) usar 15e) em $x_0 = \pi/4$
(e) usar 15b) em $x = \pi/4$

17. (a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{12}$

18. .

(a) $c_1 = 2,\ c_2 = -1,\ c_3 = \frac{2}{3}$ (b) $c_1 = \frac{2\pi^2}{3}$
(c) $c_1 = \frac{4}{\pi},\ c_2 = 0,\ c_3 = 0$

19. $\frac{1}{3} + \sum_{n=1}^{\infty} \left(\frac{1}{\pi^2 n^2} \cos(\frac{n\pi x}{4}) - \frac{1}{\pi n} \operatorname{sen}(\frac{n\pi x}{2}) \right)$ $S(999) = 1/2$ e $S(999/2) = 1/4$.

20. .

(a) $\frac{8}{\pi^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} \operatorname{sen}(\frac{(2k+1)\pi x}{2})$. (b) $b_n = \frac{8}{\pi^2} \frac{(-1)^{(n-1)}}{(2n-1)^2}$ para $n \geq 1$.
(c) $c_n = \frac{4}{(2n-1)\pi} + \frac{(-1)^n 8}{(2n-1)^2\pi^2}$, para $n \geq 1$. (d) $S(200) = S(0) = 0$; $S(201) = S(1) = 0$

21. $a = -\pi^2$, $b = 12$

22. (a) Use $f(x) = x^2$ (b) Use exercício 21.

23. (a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^2}{16} - \frac{1}{2}$.