1. **(2,5 pontos)** Seja $f(x,y) = \sqrt[3]{5x^4 + 3y^4}$.

   (a) Calcule $\dfrac{\partial f}{\partial x}(x,y)$ para todo $(x,y) \in \mathbb{R}^2$.

   (b) Verifique se a função $\dfrac{\partial f}{\partial x}$ é ou não contínua em $(0,0)$.
   (Justifique todas as passagens.)

(a) $\dfrac{\partial f}{\partial x}(x,y) = \dfrac{1}{3}(5x^4+3y^4)^{-2/3} \cdot 20x^3$ desde que $5x^4+3y^4 \neq 0$, isto é, desde que $(x,y) \neq (0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x-0} = \lim_{x\to 0} \frac{\sqrt[3]{5x^4}}{x} = \lim_{x\to 0} \sqrt[3]{5}\, x^{1/3} = 0$$

Assim:
$$\frac{\partial f}{\partial x}(x,y) = \begin{cases} \dfrac{20}{3} \dfrac{x^3}{\sqrt[3]{(5x^4+3y^4)^2}} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$$

(b) Verificar se $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial x}(0,0)$.

$$\lim_{(x,y)\to(0,0)} \frac{20}{3} \frac{x^3}{\sqrt[3]{(5x^4+3y^4)^2}} = \lim_{(x,y)\to(0,0)} \frac{20}{3} \sqrt[3]{\frac{x^9}{(5x^4+3y^4)^2}}$$

$$= \lim_{(x,y)\to(0,0)} \frac{20}{3} \sqrt[3]{\underbrace{\left(\frac{x^4}{5x^4+3y^4}\right)^2}_{\text{limitada } (*)} \underbrace{x}_{\to 0}} = 0 = \frac{\partial f}{\partial x}(0,0).$$

$(*)\ 0 \leq x^4 \leq 5x^4+3y^4 \implies 0 \leq \dfrac{x^4}{5x^4+3y^4} \leq 1 \quad \forall (x,y) \neq (0,0).$

Portanto $\dfrac{\partial f}{\partial x}$ é contínua em $(0,0)$.