Questão 4. Calcule, caso exista. Se não existir, explique por quê.

a) (1,5) $\lim_{(x,y)\to(0,0)} \operatorname{sen}(3x^2+4y^2)\ln(x^2+y^2)$ b) (1,0) $\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4-y^6}$

a) 1) $\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(3x^2+4y^2)}{3x^2+4y^2} = 1$ pois $\lim_{t\to 0} \frac{\operatorname{sen} t}{t} = 1$

e $\lim_{(x,y)\to(0,0)} (3x^2+4y^2) = 0$

2) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0$ pois

$$\lim_{t\to 0^+} t\ln t = \lim_{t\to 0^+} \frac{\overset{\to -\infty}{\ln t}}{\underset{+\infty}{1/t}} \overset{L'H}{=} \lim_{t\to 0^+} \frac{1/t}{-1/t^2} = \lim_{t\to 0^+}(-t) = 0$$

e $\lim_{(x,y)\to(0,0)} (x^2+y^2) = 0$

3) $0 \leq \frac{3x^2+4y^2}{x^2+y^2} \leq \frac{4(x^2+y^2)}{x^2+y^2} = 4$ se $(x,y)\neq(0,0)$

De 2) e 3) segue que $\lim_{(x,y)\to(0,0)} \underbrace{\frac{3x^2+4y^2}{x^2+y^2}}_{\text{limitada}} \underbrace{(x^2+y^2)\ln(x^2+y^2)}_{\downarrow\, 0} = 0$

Logo

$$\lim_{(x,y)\to(0,0)} \underbrace{\frac{\operatorname{sen}(3x^2+4y^2)}{3x^2+4y^2}}_{\downarrow\, 1} \cdot \underbrace{\left(\frac{3x^2+4y^2}{x^2+y^2}\right)(x^2+y^2)\ln(x^2+y^2)}_{\downarrow\, 0} = 0,$$

Portanto

$$\lim_{(x,y)\to(0,0)} \operatorname{sen}(3x^2+4y^2)\ln(x^2+y^2) = 0$$

A

4b) Seja $f(x,y) = \dfrac{x^2 y^3}{x^4 - y^6}$

Sejam $\gamma_1(t) = (t, 0)$, $t \neq 0$

e $\gamma_2(t) = (t^6, 2t^4)$, $t \neq 0$

Temos que $\gamma_1(t) \xrightarrow[t \to 0]{} (0,0)$ e $\gamma_2(t) \xrightarrow[t \to 0]{} (0,0)$

Mas $\lim\limits_{t \to 0} f(\gamma_1(t)) = \lim\limits_{t \to 0} \dfrac{0}{t^4} = 0 = L_1$

e $\lim\limits_{t \to 0} f(\gamma_2(t)) = \lim\limits_{t \to 0} \dfrac{(t^6)^2 (2t^4)^3}{(t^6)^4 - (2t^4)^6} =$

$= \lim\limits_{t \to 0} \dfrac{2^3 t^{24}}{(1 - 2^6) t^{24}} = \dfrac{2^3}{1 - 2^6} = L_2$

Como $L_1 \neq L_2$, não existe $\lim\limits_{(x,y) \to (0,0)} f(x,y)$