

**MAT2454 - Cálculo Diferencial e Integral para Engenharia II**  
**Mais algumas respostas da 1ª lista de exercícios - 2010**

19. (a)  $\gamma : \mathbb{R} \rightarrow \mathbb{R}^2, \gamma(t) = (t, \frac{1}{2}(1-t))$

Reta tangente:  $X = (\frac{1}{2}, \frac{1}{4}) + \lambda(2, -1), \lambda \in \mathbb{R}$

(b)  $\gamma : [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow \mathbb{R}^2, \gamma(t) = (5 + \text{sen}(t), \frac{1}{\sqrt{2}} \cos(t))$

Reta tangente:  $X = (6, 0) + \lambda(1, 0), \lambda \in \mathbb{R}$

(c)  $\gamma_1 : ]-\frac{\pi}{2}, \frac{\pi}{2}[ \rightarrow \mathbb{R}^2, \gamma_1(t) = (\sec(t), \text{tg}(t))$  parametriza um ramo da hipérbole

e

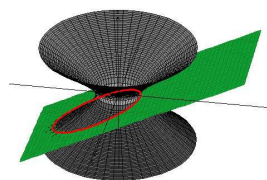
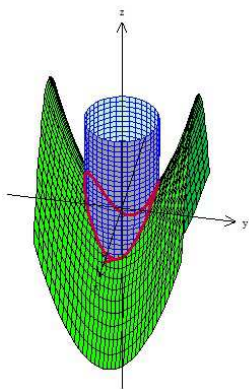
$\gamma_2 : ]\frac{\pi}{2}, \frac{3\pi}{2}[ \rightarrow \mathbb{R}^2, \gamma_2(t) = (\sec(t), \text{tg}(t))$  parametriza o outro ramo.

Reta tangente:  $X = (\sqrt{2}, 1) + \lambda(\sqrt{2}, 1), \lambda \in \mathbb{R}$

20.

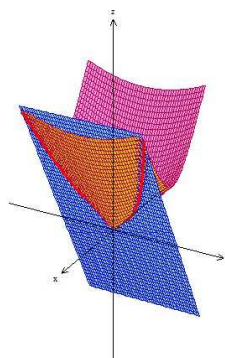
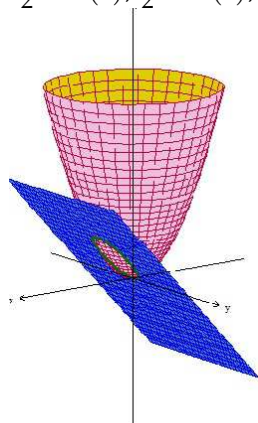
(a)  $\gamma : [0, 2\pi[ \rightarrow \mathbb{R}^3,$   
 $\gamma(t) = (\cos(t), \text{sen}(t), -\cos(2t))$

(b)  $\gamma : [0, 2\pi[ \rightarrow \mathbb{R}^3,$   
 $\gamma(t) = (\sqrt{2} \cos(t), 2 \text{sen}(t) - 1, \text{sen}(t) - 1)$



(c)  $\gamma : [0, 2\pi[ \rightarrow \mathbb{R}^3,$   
 $\gamma(t) = (\frac{1}{2} + \frac{1}{2} \cos(t), \frac{1}{2} \text{sen}(t), \frac{1}{2} + \frac{1}{2} \cos(t))$

(d)  $\gamma : \mathbb{R} \rightarrow \mathbb{R}^3,$   
 $\gamma(t) = (\frac{1}{4}(t^2 - 1), t, \frac{1}{2}(t^2 + 1))$

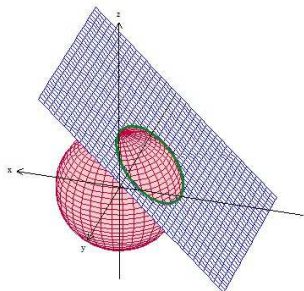


21.

$$(a) \gamma : [0, 2\pi[ \rightarrow \mathbb{R}^3, \gamma(t) = \left(\frac{1}{2}(\cos(t) - 1), \frac{1}{\sqrt{2}} \sin(t), \frac{1}{2}(\cos(t) + 1)\right)$$

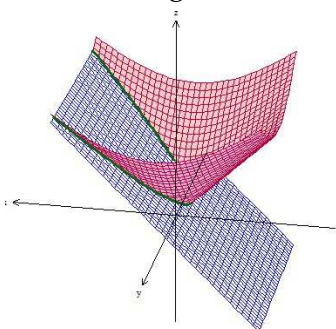
Nessa parametrização,  $\gamma\left(\frac{\pi}{2}\right) = \left(-\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ , assim o vetor tangente à trajetória de  $\gamma$  nesse ponto é paralelo a  $\vec{\gamma}'\left(\frac{\pi}{2}\right)$ .

$$\text{Reta tangente: } X = \left(-\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right) + \lambda(-1, 0, -1), \lambda \in \mathbb{R}$$



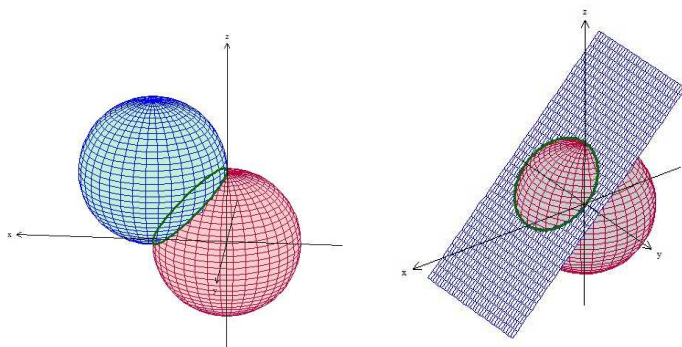
$$(b) \gamma : \mathbb{R} \rightarrow \mathbb{R}^3, \gamma(t) = \left(\frac{1}{2}(t^2 - 1), t, \frac{1}{2}(t^2 + 1)\right)$$

$$\text{Reta tangente: } X = (0, 1, 1) + \lambda(1, 1, 1), \lambda \in \mathbb{R}$$



$$(c) \gamma : [0, 2\pi[ \rightarrow \mathbb{R}^3, \gamma(t) = \left(\frac{1}{2}(1 - \cos(t)), \frac{1}{\sqrt{2}} \sin(t), \frac{1}{2}(\cos(t) + 1)\right)$$

$$\text{Reta tangente: } X = \left(\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{1}{2}\right) + \lambda(1, 0, -1), \lambda \in \mathbb{R}$$



22. Veja a solução na P1 de 2009.