2. Seja $f = f(x, y)$ uma função de classe C^2 em $\mathbb{R}^2$. Seja

$$g(u, v) = uf(2uv, 2u - v) + \frac{\partial f}{\partial x}(2uv, 2u - v).$$

(a) **(1,5)** Encontre $\frac{\partial g}{\partial u}$ e $\frac{\partial g}{\partial v}$ em termos das derivadas parciais de primeira e de segunda ordem de f.

(b) **(1,0)** Sabendo que $3x - 10y + 25z = 6$ é a equação do plano tangente ao gráfico de f no ponto $(2, 1, f(2, 1))$ e que

$$\frac{\partial^2 f}{\partial x \partial y}(2, 1) = -\frac{3}{25} \quad \text{e} \quad \frac{\partial^2 f}{\partial x^2}(2, 1) = \frac{4}{125},$$

calcule $\frac{\partial g}{\partial u}(1, 1)$.

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(2) (a) Seja $x = x(u, v) = 2uv$ e $y = y(u, v) = 2u - v$,

$$\frac{\partial g}{\partial u}(u, v) = f(x, y) + u\left[\frac{\partial f}{\partial x}(x, y) 2v + \frac{\partial f}{\partial y}(x, y) \cdot 2\right]$$

$$+ \frac{\partial^2 f}{\partial x^2}(x, y) \cdot 2v + \frac{\partial^2 f}{\partial y \partial x}(x, y) \cdot 2$$

$$= f(2uv, 2u - v) + 2u\left[v \frac{\partial f}{\partial x}(2uv, 2u - v) + \frac{\partial f}{\partial y}(2uv, 2u - v)\right]$$

$$+ 2\left[v \frac{\partial^2 f}{\partial x^2}(2uv, 2u - v) + \frac{\partial^2 f}{\partial y \partial x}(2uv, 2u - v)\right]$$

$$\frac{\partial g}{\partial v}(u, v) = u\left[\frac{\partial f}{\partial x}(x, y) 2u + \frac{\partial f}{\partial y}(x, y)(-1)\right]$$

$$+ \frac{\partial^2 f}{\partial x^2}(x, y) \cdot 2u + \frac{\partial^2 f}{\partial y \partial x}(x, y) \cdot (-1)$$

$$= u\left[2u \frac{\partial f}{\partial x}(2uv, 2u - v) - \frac{\partial f}{\partial y}(2uv, 2u - v)\right] +$$

$$2u \frac{\partial^2 f}{\partial x^2}(2uv, 2u - v) - \frac{\partial^2 f}{\partial y \partial x}(2uv, 2u - v) \cdot$$

(b) Sendo $3x - 10y + 25z = 6$ a

equação do plano tangente ao gráfico

de f no ponto $(2, 1, f(2,1))$ obtemos que

$$6 - 10 + 25 f(2,1) = 6$$

$$\text{Logo } f(2,1) = \frac{10}{25}$$

Também $(3, -10, 25) = \lambda \left(\frac{\partial f}{\partial x}(2,1), \frac{\partial f}{\partial y}(2,1), -1 \right)$

$$\text{Logo } \lambda = -25 \quad e$$

$$\frac{\partial f}{\partial x}(2,1) = -\frac{3}{25} \quad e \quad \frac{\partial f}{\partial y}(2,1) = \frac{10}{25}.$$

Quando $(u, v) = (1, 1)$, $(x, y) = (2, 1)$ e

do item (a) temos então que

$$\frac{\partial g}{\partial u}(1,1) = \frac{10}{25} + 2 \left[-\frac{3}{25} + \frac{10}{25} \right] + 2 \left[\frac{4}{25} \quad -\frac{3}{25} \right]$$