Questão b2) Sejam $f = f(x, y)$ uma função de classe $\mathcal{C}^2$ em $I\!R^2$ e g a função dada por

$$g(u,v) = f(1 + u\cos v, u^3 - uv^2) + \frac{\partial f}{\partial x}(e^{2u+3v}, vu)\,.$$

(a) Determine $\dfrac{\partial g}{\partial u}$ e $\dfrac{\partial g}{\partial v}$ em função das derivadas parciais de primeira e de segunda ordem de f.

$$\frac{\partial g}{\partial u}(u,v) = \frac{\partial f}{\partial x}(1 + u\cos v, u^3 - uv^2)(\cos v) + \frac{\partial f}{\partial y}(1 + u\cos v, u^3 - uv^2)(3u^2 - v^2)$$

$$+ \frac{\partial^2 f}{\partial x^2}(e^{2u+3v}, vu)e^{2u+3v}2 + \frac{\partial^2 f}{\partial y \partial x}(e^{2u+3v}, vu)v$$

$$\frac{\partial g}{\partial v}(u,v) = \frac{\partial f}{\partial x}(1 + u\cos v, u^3 - uv^2)(-u\,sen\, v) + \frac{\partial f}{\partial y}(1 + u\cos v, u^3 - uv^2)(-2uv)$$

$$+ \frac{\partial^2 f}{\partial x^2}(e^{2u+3v}, vu)e^{2u+3v}3 + \frac{\partial^2 f}{\partial y \partial x}(e^{2u+3v}, vu)u$$

(b) Suponha que

(I) $2x - 4y + 2z = 5$ é o plano tangente ao gráfico de f no ponto $(1, 0, f(1, 0))$.

(II) $\dfrac{\partial^2 f}{\partial x^2}(1,0) = -3$

Determine o valor máximo da derivada direcional de g no ponto $(0, 0)$.

$$2x - 4y + 2z = 5 \;\Rightarrow\; z = -x + 2y + \frac{5}{2} \;\Rightarrow\; \begin{cases} \dfrac{\partial f}{\partial x}(1,0) = -1 \\ \dfrac{\partial f}{\partial y}(1,0) = 2 \end{cases}$$

$$\nabla g(0,0) = \left(\frac{\partial g}{\partial u}(0,0), \frac{\partial g}{\partial v}(0,0)\right) = \left(\frac{\partial f}{\partial x}(1,0) + 2\frac{\partial^2 f}{\partial x^2}(1,0), 3\frac{\partial^2 f}{\partial x^2}(1,0)\right) = (-7, -9)$$

$$\frac{\partial g}{\partial \vec{u}}(0,0) = ||\nabla g(0,0)|| = ||(-7,-9)|| = \sqrt{49 + 81} = \sqrt{130}.$$