Questão a2) Sejam $f = f(x,y)$ uma função de classe $\mathcal{C}^2$ em $I\!R^2$ e g a função dada por

$$g(u,v) = f(u^3 - uv^2, 1 + u\cos v) + \frac{\partial f}{\partial y}(vu, e^{2u+3v}).$$

(a) Determine $\frac{\partial g}{\partial u}$ e $\frac{\partial g}{\partial v}$ em função das derivadas parciais de primeira e de segunda ordem de f.

$$\frac{\partial g}{\partial u}(u,v) = \frac{\partial f}{\partial x}(u^3 - uv^2, 1 + u\cos v)(3u^2 - v^2) + \frac{\partial f}{\partial y}(u^3 - uv^2, 1 + u\cos v)(\cos v)$$

$$+ \frac{\partial^2 f}{\partial x \partial y}(vu, e^{2u+3v})v + \frac{\partial^2 f}{\partial y^2}(vu, e^{2u+3v})e^{2u+3v}2$$

$$\frac{\partial g}{\partial v}(u,v) = \frac{\partial f}{\partial x}(u^3 - uv^2, 1 + u\cos v)(-2uv) + \frac{\partial f}{\partial y}(u^3 - uv^2, 1 + u\cos v)(-u\,\mathrm{sen}\, v)$$

$$+ \frac{\partial^2 f}{\partial x \partial y}(vu, e^{2u+3v})u + \frac{\partial^2 f}{\partial y^2}(vu, e^{2u+3v})e^{2u+3v}3$$

(b) Suponha que

(I) $2x - 4y + 2z = 5$ é o plano tangente ao gráfico de f no ponto $(0, 1, f(0,1))$.

(II) $\frac{\partial^2 f}{\partial y^2}(0,1) = -3$

Determine o valor máximo da derivada direcional de g no ponto $(0,0)$.

$$2x - 4y + 2z = 5 \;\Rightarrow\; z = -x + 2y + \frac{5}{2} \;\Rightarrow\; \begin{cases} \frac{\partial f}{\partial x}(0,1) = -1 \\ \frac{\partial f}{\partial y}(0,1) = 2 \end{cases}$$

$$\nabla g(0,0) = \left(\frac{\partial g}{\partial u}(0,0), \frac{\partial g}{\partial v}(0,0)\right) = \left(\frac{\partial f}{\partial y}(0,1) + 2\frac{\partial^2 f}{\partial y^2}(0,1), 3\frac{\partial^2 f}{\partial y^2}(0,1)\right) = (-4, -9)$$

$$\frac{\partial g}{\partial \vec{u}}(0,0) = ||\nabla g(0,0)|| = ||(-4,-9)|| = \sqrt{16+81} = \sqrt{97}.$$