A

Questão 3. Calcule, caso exista, ou mostre que não existe

a) (0,5 ponto) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^4-y^4)}{x^2-y^2}$

b) (1 ponto) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2-x^2}{\sqrt{x^2+y^4}}$

c) (1 ponto) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2-2xy^3}{x^2+y^6}$

a) $$\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^4-y^4)}{x^2-y^2} = \lim_{(x,y)\to(0,0)} (x^2+y^2)\frac{\operatorname{sen}(x^4-y^4)}{x^4-y^4}$$

Como $\displaystyle\lim_{(x,y)\to(0,0)} x^4-y^4=0$, temos $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\operatorname{sen}(x^4-y^4)}{x^4-y^4} = \lim_{u\to0}\frac{\operatorname{sen} u}{u} = 1.$

Logo, $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2)\frac{\operatorname{sen}(x^4-y^4)}{x^4-y^4} = 0 \cdot 1 = 0.$

b) $$\lim_{(x,y)\to(0,0)} \frac{xy^2-x^2}{\sqrt{x^2+y^4}} = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^4}}(y^2-x)$$

Temos: $0\le x^2 \le x^2+y^4 \underset{(x,y)\neq(0,0)}{\Longrightarrow} 0\le \dfrac{x^2}{x^2+y^4}\le 1 \Rightarrow \left|\dfrac{x}{\sqrt{x^2+y^4}}\right|\le 1$

e $\displaystyle\lim_{(x,y)\to(0,0)} y^2-x=0.$

Logo, $\displaystyle\lim_{(x,y)\to(0,0)} \underbrace{\frac{x}{\sqrt{x^2+y^4}}}_{\text{limit.}}\overset{\to 0}{(y^2-x)} = 0$

c) Sejam $\gamma_1(t)=(t,0)$, $\gamma_2(t)=(t^3,t)$ e $f(x,y)=\dfrac{x^2y^2-2xy^3}{x^2+y^6}$

Temos: $\gamma_1(0)=(0,0)$, $\gamma_2(0)=(0,0)$, γ_1 e γ_2 são contínuas, $\gamma_1(t)\neq(0,0)$ e $\gamma_2(t)\neq(0,0)$ se $t\neq0$ e $\gamma_1(t), \gamma_2(t)\in \operatorname{Dom} f$, $\forall t\neq 0$.

Como $\displaystyle\lim_{t\to0} f(\gamma_1(t)) = \lim_{t\to0} 0 = 0$ e

$$\lim_{t\to0} f(\gamma_2(t)) = \lim_{t\to0}\frac{t^8-2t^6}{2t^6} = \lim_{t\to0}\frac{t^2-2}{2} = -1,$$

não existe $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^2-2xy^3}{x^2+y^6}$