

4. (2, 5) Seja  $g: \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $g = g(x, y)$ , uma função de classe  $C^2$  em  $\mathbb{R}^2$ . Seja

$$f(t, u) = g(u - 2t, tu).$$

(a) Calcule  $\frac{\partial^2 f}{\partial t \partial u}(t, u)$  em termos das derivadas parciais de  $g$ .

(b) Calcule  $\frac{\partial^2 f}{\partial t \partial u}(1, 2)$ , sabendo que  $\frac{\partial^2 g}{\partial x^2}(0, 2) = \frac{\partial^2 g}{\partial y^2}(0, 2)$  e que  $\frac{\partial g}{\partial y}(0, 2) = 10$ .

$$a) f(t, u) = g(x(t, u), y(t, u)) \quad x(t, u) = u - 2t, \quad y(t, u) = tu$$

$$\frac{\partial f}{\partial u}(t, u) = \frac{\partial g}{\partial x}(x, y) \cdot \frac{\partial x}{\partial u}(t, u) + \frac{\partial g}{\partial y}(x, y) \cdot \frac{\partial y}{\partial u}(t, u) = \frac{\partial g}{\partial x}(x, y) \cdot 1 + \frac{\partial g}{\partial y}(x, y) \cdot t$$

$$\frac{\partial^2 f}{\partial t \partial u}(t, u) = \frac{\partial}{\partial t} \left( \frac{\partial g}{\partial x}(x, y) \right) + \frac{\partial}{\partial t} \left( \frac{\partial g}{\partial y}(x, y) \right) \cdot t + \frac{\partial g}{\partial y}(x, y) \cdot 1$$

$$= \frac{\partial^2 g}{\partial x^2}(x, y) \cdot \frac{\partial x}{\partial t}(t, u) + \frac{\partial^2 g}{\partial y \partial x}(x, y) \cdot \frac{\partial y}{\partial t}(t, u)$$

$$+ \left[ \frac{\partial^2 g}{\partial x \partial y}(x, y) \cdot \frac{\partial x}{\partial t}(t, u) + \frac{\partial^2 g}{\partial y^2}(x, y) \cdot \frac{\partial y}{\partial t}(t, u) \right] \cdot t + \frac{\partial g}{\partial y}(x, y) \cdot 1$$

$$= -2 \frac{\partial^2 g}{\partial x^2}(x, y) + \frac{\partial^2 g}{\partial y \partial x}(x, y) \cdot u - 2t \frac{\partial^2 g}{\partial x \partial y}(x, y)$$

$$+ \frac{\partial^2 g}{\partial y^2}(x, y) \cdot ut + \frac{\partial g}{\partial y}(x, y)$$

sendo  $f$  de classe  $C^2$ ,  $\frac{\partial^2 g}{\partial y \partial x}(x, y) = \frac{\partial^2 g}{\partial x \partial y}(x, y)$

$$\therefore \frac{\partial^2 f}{\partial t \partial u}(t, u) = -2 \frac{\partial^2 g}{\partial x^2}(x, y) + (u - 2t) \frac{\partial^2 g}{\partial x \partial y} + tu \frac{\partial^2 g}{\partial y^2}(x, y) + \frac{\partial g}{\partial y}(x, y)$$

b)  $t=1, u=2 \Rightarrow x=2-2=0, y=2 \cdot 1=2$

$$\therefore \frac{\partial^2 f}{\partial t \partial u}(0, 2) = -2 \frac{\partial^2 g}{\partial x^2}(0, 2) + (2-2) \frac{\partial^2 g}{\partial x \partial y}(0, 2) + 2 \frac{\partial^2 g}{\partial y^2}(0, 2) + \frac{\partial g}{\partial y}(0, 2)$$

$$= \frac{\partial g}{\partial y}(0, 2) = 10$$