1. (3,0) Seja $f(x,y) = \begin{cases} \dfrac{y^2 \operatorname{sen}(x+y)}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0). \end{cases}$

(a) É f contínua em $(0,0)$?

(b) Calcule $\nabla f(0,0)$.

(c) Calcule $\dfrac{\partial f}{\partial \vec{\mathbf{u}}}(0,0)$ e $\langle \nabla f(0,0), \vec{\mathbf{u}} \rangle$, onde $\vec{\mathbf{u}} = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right)$ e $\langle\ ,\ \rangle$ denota o produto escalar.

(d) É f diferenciável em $(0,0)$? Explique.

B1 (a) Para $(x,y) \neq (0,0)$, temos $f(x,y) = \dfrac{y^2}{x^2+y^2} \operatorname{sen}(x+y)$

$\dfrac{y^2}{x^2+y^2}$ é limitada, pois $0 \leq \dfrac{y^2}{x^2+y^2} \leq 1 \quad \forall (x,y) \neq (0,0)$.

$\lim\limits_{(x,y)\to(0,0)} \operatorname{sen}(x+y) = \lim\limits_{\theta \to 0} \operatorname{sen} \theta = 0$

Logo, $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$. Logo f é cont. em $(0,0)$

(b) $f(x,0) = 0 \ \forall x \Rightarrow f_x(0,0) = 0$

$f(0,y) = \operatorname{sen} y \ \forall y \Rightarrow f_y(0,0) = \cos 0 = 1$

Logo $\nabla f(0,0) = (0,1)$

(c) $\frac{\partial f}{\partial \vec{u}}(0,0) = \lim_{t \to 0} \frac{f\left(\frac{\sqrt{3}}{2}t, \frac{1}{2}t\right) - f(0,0)}{t} =$

$= \lim_{t \to 0} \frac{1}{t} \frac{\frac{1}{4}t^2}{\frac{3}{4}t^2 + \frac{1}{4}t^2} \operatorname{sen}\left(\frac{\sqrt{3}+1}{2}t\right) =$

$= \frac{1}{4} \frac{\sqrt{3}+1}{2} \cdot \lim_{t \to 0} \frac{\operatorname{sen}\left(\frac{\sqrt{3}+1}{2}t\right)}{\frac{\sqrt{3}+1}{2}t} = \frac{\sqrt{3}+1}{8}$

$\langle \nabla f(0,0), \vec{u} \rangle = \left\langle (0,1), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) \right\rangle = \frac{1}{2}$

(d) Se fosse, valeria $\langle \nabla f(0,0), \vec{u} \rangle = \frac{\partial f}{\partial u}(0,0)$

Vimos no item c que $\langle \nabla f(0,0), \vec{u} \rangle \neq \frac{\partial f}{\partial \vec{u}}(0,0)$.

Logo, f não é diferenciável em $(0,0)$.