1. (3,0) Seja $f(x,y) = \begin{cases} \dfrac{x^2 \operatorname{sen}(x+y)}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0). \end{cases}$

(a) É f contínua em $(0,0)$?

(b) Calcule $\nabla f(0,0)$.

(c) Calcule $\dfrac{\partial f}{\partial \vec{\mathbf{u}}}(0,0)$ e $\langle \nabla f(0,0), \vec{\mathbf{u}} \rangle$, onde $\vec{\mathbf{u}} = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2} \right)$ e $\langle\ ,\ \rangle$ denota o produto escalar.

(d) É f diferenciável em $(0,0)$? Explique.

A1 (a) Para $(x,y) \neq (0,0)$, temos $f(x,y) = \dfrac{x^2}{x^2+y^2} \operatorname{sen}(x+y)$.

$\dfrac{x^2}{x^2+y^2}$ é limitada, pois $0 \leq \dfrac{x^2}{x^2+y^2} \leq 1 \ \forall (x,y) \neq (0,0)$

$\displaystyle\lim_{(x,y)\to(0,0)} \operatorname{sen}(x+y) = \lim_{\theta \to 0} \operatorname{sen}\theta = 0$

Logo, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$. Logo f é contínua em $(0,0)$

(b) $f(x,0) = \operatorname{sen} x \ \forall x \Rightarrow f_x(0,0) = \cos 0 = 1$

$f(0,y) = 0 \ \forall y \Rightarrow f_y(0,0) = 0$. Logo $\nabla f(0,0) = (1,0)$

(c) $\dfrac{\partial f}{\partial \vec{u}}(0,0) = \lim\limits_{t\to 0} \dfrac{f\left(\frac{\sqrt{3}}{2}t, \frac{1}{2}t\right) - f(0,0)}{t} =$

$= \lim\limits_{t\to 0} \dfrac{1}{t} \dfrac{\frac{3}{4}t^2}{\frac{3}{4}t^2 + \frac{1}{2}t^2} \operatorname{sen}\left(\dfrac{\sqrt{3}+1}{2} t\right) =$

$= \dfrac{3}{4} \dfrac{\sqrt{3}+1}{2} \cdot \lim\limits_{t\to 0} \dfrac{\operatorname{sen}\left(\frac{\sqrt{3}+1}{2} t\right)}{\frac{\sqrt{3}+1}{2} t} = \dfrac{3}{8}(\sqrt{3}+1)$

$\langle \nabla f(0,0), \vec{u} \rangle = \langle (1,0), (\frac{\sqrt{3}}{2}, \frac{1}{2}) \rangle = \dfrac{\sqrt{3}}{2}$

(d) Se fosse, valeria $\langle \nabla f(0,0), \vec{u} \rangle = \dfrac{\partial f}{\partial \vec{u}}(0,0)$.

Vimos no item c que $\langle \nabla f(0,0), \vec{u} \rangle \neq \dfrac{\partial f}{\partial \vec{u}}(0,0)$

Logo, f não é diferenciável em $(0,0)$.