3. **(1,5)** Seja $f(x,y) = (x^2+y^2)^{\frac{2}{3}}$. É a derivada parcial $\frac{\partial f}{\partial y}$ contínua em $(0,0)$? JUSTIFIQUE.

$$\frac{\partial f}{\partial y}(x,y) = \frac{2}{3}(x^2+y^2)^{-1/3} \cdot 2y \quad \text{se } (x,y) \neq (0,0)$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{f(0,y) - f(0,0)}{y - 0}$$

$$= \lim_{y\to 0} \frac{y^{4/3}}{y} = \lim_{y \to 0} y^{1/3} = 0$$

Assim,

$$\frac{\partial f}{\partial y}(x,y) = \begin{cases} \frac{4}{3}(x^2+y^2)^{-1/3} y & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0). \end{cases}$$

Para verificar se $\frac{\partial f}{\partial y}$ é contínua em $(0,0)$, vamos calcular

$$\lim_{(x,y)\to(0,0)} \frac{4}{3} \frac{y}{(x^2+y^2)^{1/3}}$$

$$\lim_{(x,y)\to(0,0)} \frac{4}{3} \frac{y}{(x^2+y^2)^{1/3}} = \lim_{(x,y)\to(0,0)} \frac{4}{3} \frac{y^{2/3}\, y^{1/3}}{(x^2+y^2)^{1/3}}$$

$$= \lim_{(x,y)\to(0,0)} \frac{4}{3} \underbrace{y^{1/3}}_{\to 0} \underbrace{\left(\frac{y^2}{x^2+y^2}\right)^{1/3}}_{\text{limitada}} \overset{(*)}{=} 0 = \frac{\partial f}{\partial y}(0,0)$$

(*) ~~pois~~ $0 \leq \frac{y^2}{x^2+y^2} \leq 1$, implica $0 \leq \left(\frac{y^2}{x^2+y^2}\right)^{1/3} \leq 1$

Logo $\frac{\partial f}{\partial y}$ é contínua em $(0,0)$.