2. (**2,5**) Seja $f : \mathbb{R}^2 \to \mathbb{R}$ uma função diferenciável tal que $4x + y + 2z = 4$ é o plano tangente ao gráfico de f no ponto $(3, 4, f(3, 4))$. Seja

$$g(u, v) = uf(u^2 - v^2, 2uv).$$

Encontre a equação do plano tangente ao gráfico de g no ponto $(2, 1, g(2, 1))$.

O plano procurado é $z - g(2,1) = \frac{\partial g}{\partial u}(2,1)(x-2) + \frac{\partial g}{\partial v}(2,1)(y-1)$

$$4x + y + 2z = 4$$
$$2z = -4x - y + 4$$
$$z = -2x - \frac{1}{2}y + 2$$
$$z = -2(x-3) - \frac{1}{2}(y-4) + 2 - 6 - 2$$
$$z + 6 = -2(x-3) - \frac{1}{2}(y-4) \Rightarrow \begin{cases} f(3,4) = -6 \\ \frac{\partial f}{\partial x}(3,4) = -2 \\ \frac{\partial f}{\partial y}(3,4) = -\frac{1}{2} \end{cases}$$

$$\frac{\partial g}{\partial u}(u,v) = f(u^2-v^2, 2uv) + u\left[\frac{\partial f}{\partial x}(u^2-v^2, 2uv)\,2u + \frac{\partial f}{\partial y}(u^2-v^2, 2uv)\,2v\right]$$

$$\frac{\partial g}{\partial u}(2,1) = f(3,4) + 2\left[\frac{\partial f}{\partial x}(3,4)\cdot 4 + \frac{\partial f}{\partial y}(3,4)\cdot 2\right] = -6 + 2\left[-2(4) - \frac{1}{2}(2)\right] = -24$$

$$\frac{\partial g}{\partial v}(u,v) = u\left[\frac{\partial f}{\partial x}(u^2-v^2, 2uv)(-2v) + \frac{\partial f}{\partial y}(u^2-v^2, 2uv)\,2u\right]$$

$$\frac{\partial g}{\partial v}(2,1) = 2\left[\frac{\partial f}{\partial x}(3,4)(-2) + \frac{\partial f}{\partial y}(3,4)\,4\right] = 2\left[-2(-2) - \frac{1}{2}(4)\right] = 4$$

$$g(2,1) = 2f(3,4) = -12$$

$$\boxed{z + 12 = -24(x-2) + 4(y-1)}\ //$$