A

4ª Questão: (2,0 pontos) Considere a função

$$f(x,y) = y^4 + 4x^2y - 4x^2 - 8y^2$$

Determine os pontos críticos de f e classifique-os.

- PONTOS CRÍTICOS:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = 0 \\ \dfrac{\partial f}{\partial y}(x,y) = 0 \end{cases} \iff \begin{cases} ① \; 8xy - 8x = 0 \\ ② \; 4y^3 + 4x^2 - 16y = 0 \end{cases}$$

$x = 0 \overset{②}{\Rightarrow} 4y^3 - 16y = 0$

- $y = 0$
- $y \neq 0 \Rightarrow 4y^2 - 16 = 0 \Rightarrow y = \pm 2$

$x \neq 0 \Rightarrow 8y - 8 = 0$

$\Rightarrow y = 1$

$\overset{②}{\Rightarrow} 4 + 4x^2 - 16 = 0$

$\Rightarrow 4x^2 = 12$

$\Rightarrow x = \pm\sqrt{3}$

$\therefore$ $\boxed{(0,0),\ (0,2),\ (0,-2),\ (\sqrt{3},1),\ (-\sqrt{3},1) \text{ SÃO OS PONTOS CRÍTICOS!}}$

- CLASSIFICAÇÃO:

$$H_f(x,y) = \begin{vmatrix} 8y-8 & 8x \\ 8x & 12y^2-16 \end{vmatrix} \quad e \quad \frac{\partial^2 f}{\partial x^2}(x,y) = 8y - 8$$

$\dfrac{\partial^2 f}{\partial x^2}(0,0) = -8 < 0$ e $H_f(0,0) = (-8)(-16) = 128 > 0 \Rightarrow \boxed{(0,0) \text{ máx. local.}}$

$\dfrac{\partial^2 f}{\partial x^2}(0,2) = 8 > 0$ e $H_f(0,2) = 8 \cdot 32 = 256 > 0 \Rightarrow \boxed{(0,2) \text{ mín. local}}$

$\dfrac{\partial^2 f}{\partial x^2}(0,-2) = -24 < 0$ e $H_f(0,-2) = (-24) \cdot 32 = -768 < 0 \Rightarrow \boxed{(0,-2) \text{ SELA}}$

$\dfrac{\partial^2 f}{\partial x^2}(\sqrt{3},1) = 0$ e $H_f(\sqrt{3},1) = -192 < 0 \Rightarrow \boxed{(\sqrt{3},1) \text{ SELA}}$

$\dfrac{\partial^2 f}{\partial x^2}(-\sqrt{3},1) = 0$ e $H_f(-\sqrt{3},1) = -192 < 0 \Rightarrow \boxed{(-\sqrt{3},1) \text{ SELA}}$