(c) $\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\cos h^2 - 1}{h^3} = 0$

$$\lim_{(h,k)\to(0,0)} \frac{f(0+h,0+k) - f(0,0) - f_x(0,0)\,h - f_y(0,0)\,k}{\sqrt{h^2+k^2}} =$$

$$\lim_{(h,k)\to(0,0)} \frac{f(h,k)}{\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \frac{\cos(h^2+k^2)-1}{(h^2+k^2)^{3/2}} =$$

$$= \lim_{u\to 0} \frac{\cos u - 1}{u^{3/2}} \underset{\substack{\uparrow \\ \text{L'Hospital}}}{=} - \lim_{u\to 0} \frac{\operatorname{sen} u}{\frac{3}{2} u^{1/2}} =$$

$$= -\frac{2}{3} \lim_{u\to 0} \left(u^{1/2} \, \frac{\operatorname{sen} u}{u} \right) = -\frac{2}{3} \cdot 0 \cdot 1 = 0 .$$

Logo f é diferenciável em $(0,0)$.